OPTIMAL FLIGHT STRATEGY IN A GIVEN SPACE-DISTRIBUTION OF LIFTS WITH MINIMUM AND MAXIMUM ALTITUDE CONSTRAINTS

by

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1. INTRODUCTION

In recent years, several researchers have examined the various problems connected with the optimization of the flight strategy for a sailplane soaring cross-country. Some of the most significant work can be found in (1) - (10). From the point of view of the theorist in optimization, these problems have a perfectly well defined objective. Indeed, it is always the total time which must be minimized, either because the sailplane is involved in a contest or because, when lift is due to thermals alone, the useful time for distance flight is limited to the day time period during which the sun produces thermals. On the other hand, an aspect of the problem which does not seen to have been completely resolved at the present time, is the atmospheric model that should be used. Some attempts (11) - (12) have been made to model the shape of a thermal. We shall assume concentrated lift. We shall see later that this assumption is justified in the framework of the problem that we are treating.

The problem that we are examining in this paper is completely deterministic: no stochastic aspects are considered. *We are essentially concerned with the problem of determining, for a given sailplane, the optimal flight strategy which corresponds to travelling a given distance in minimum time with zero net altitude loss.* We assume that lift regions are concentrated at specified locations unequally spaced along the trajectory. The locations of these lift regions, as well as their characteristics, are constant with time; strengths of the various lift regions are not generally equal. We suppose that the air mass between lift regions is stationary, i.e., there is no sinking zone surrounding the core of the thermal. Finally, the flight must stay within two given flight levels: the lowest one corresponds to safety with respect to ground clearance and the highest one means that no cloud flying is allowed. Thus, the sailplane's flight is divided into steps, each of which consists of an ascent in a thermal and a glide at constant speed to the next thermal. The flight starts and ends at the given minimum flight level. We assume that there is no wind. The projection of one glide on the ground is rectilinear; however, the projections of the glides of successive steps can be inclined to one another. The pilot must make two kinds of decisions sequentially: how much to climb in each lift region and what speed to fly in between two lift regions.

By its very definition, the problem prohibits soaring in the so called "dolphin" or "essing" mode. We are well aware, as was rightly pointed out in (2), that..."It is generally recognized that many of the very fast cross-country flights achieved in recent years have been made under conditions where the latter two modes were utilized and relatively little time was spent in thermaling". However, we still think that the problem treated here is of interest: first, because the atmospheric conditions do not always allow for the "dolphin" or essing" mode and second, because - to the best of our knowledge - it is the first time that a problem has been solved which involves not just one step but a whole flight, taking into account altitude constraints.

Although, in general, the results cannot be used by a pilot on an actual flight since he must know the characteristics of all the thermals that he will encounter later, one still can use these results for many purposes. Two applications are: simulation experiments can be conducted with competition pilots to enable them to compare their strategies with the optimal one; performance of sailplanes can be compared with respect to a given standardized space-distribution of lift regions.

2. STATEMENT OF THE PROBLEM

As stated in the introduction, we divide the sailplane's flight into steps, each of which consists of an ascent in a thermal at minimum sink rate, followed by a glide at constant speed to the next thermal.

The situation is best illustrated in Figure 1 which is drawn in a vertical plane. The positive direction of the y axis indicates the direction of travel of the sailplane and the vertical z axis is positive upwards. Accordingly, all horizontal speeds will be considered positive to the right and all vertical speeds will be considered positive upwards.

For reference, the minimum altitude is taken equal to zero and the maximum altitude is denoted by h.

The sailplane starts at point A_0 corresponding to y = 0 and to altitude $h_0 = 0$, then climbs into the first thermal at altitude h_1 and then glides at constant forward speed v_0 (corresponding to sinking rate w_0) to the second thermal which is reached at altitude h_2, having travelled the horizontal distance ℓ_1. Generally, the sailplane reaches the i-th lift with a forward speed v_{i-1} and at an altitude h_{2i}, then climbs to altitude h_{2i+1} with an achieved rate of climb a_i and leaves the thermal with a forward speed v_i. Finally, the sailplane must reach the point A_n of coordinates $y = \ell$ and $z = 0$. Of course, for the problem to make sense, all distances ℓ_i must be such that they can be travelled by the sailplane flying at the speed of maximum lift-to-drag ratio with a loss of altitude at most equal to h.

Recall that we assumed that there is no wind. We shall neglect the transient dynamical effects occuring when entering or leaving a thermal, hence the only characteristic of the sailplane that we shall need will be the polar equation relating the forward speed v to the sinking rate w.

One last thing we must discuss, before writing down the equations, is the vertical characterizations $a_i(z)$ of the thermals. We shall examine two cases:

a) The strength of the thermal is constant with the altitude [$a_i = a_i(z) = \text{constant}$],

b) The strength of the thermal at first increases with altitude up to a maximum and then decreases [$a_i = a_i(z)$].

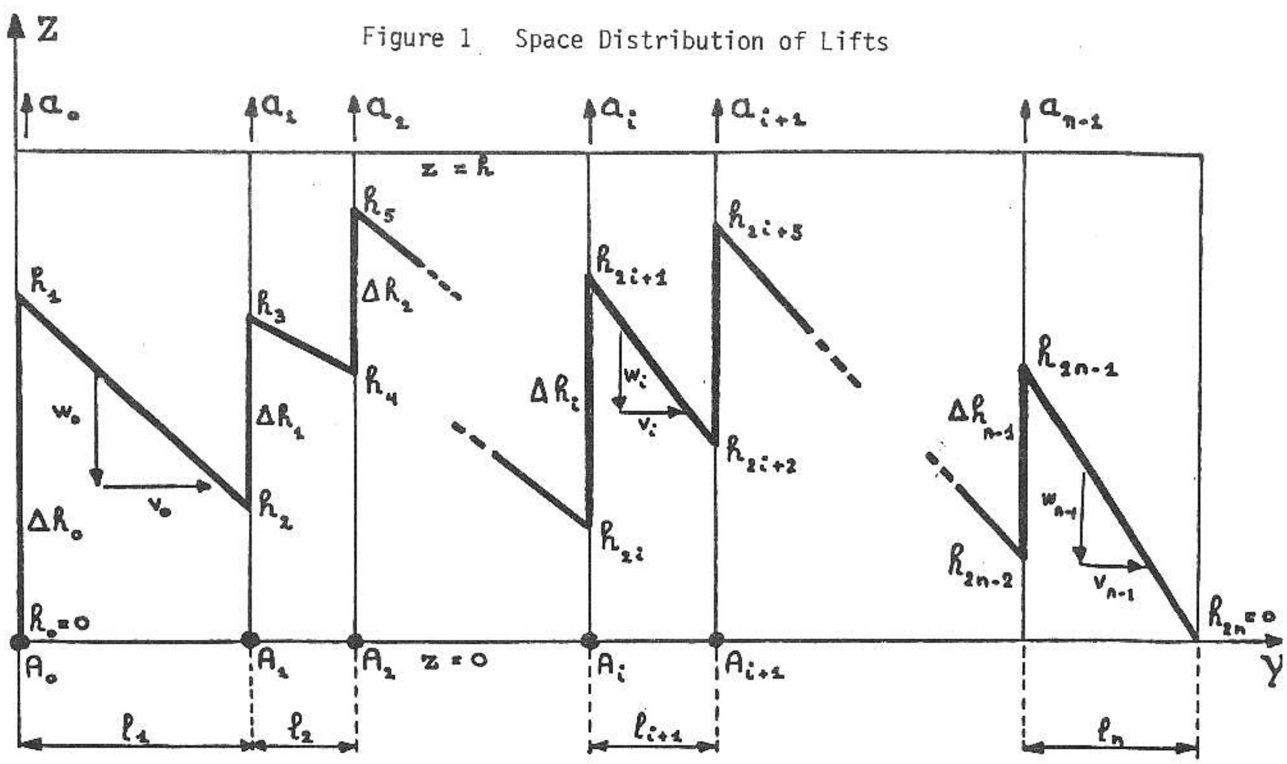


Figure 1 Space Distribution of Lifts

The pilot has two controls at his disposal:

a) Δh_i: the gain of altitude in the i-th thermal,

b) v_i: the speed to fly after leaving the i-th thermal.

If we further denote by t_{2i} the time to climb in the i-th thermal and by t_{2i+1} the time to travel the distance ℓ_{i+1}, we can write the following relations:

Finite difference equations for the altitudes:

$$h_{2i+1} - h_{2i} = \Delta h_i$$

$$h_{2i+2} - h_{2i+1} = \frac{w_i(v_i)}{v_i} \ell_{i+1}$$

$$i = 0, 1, 2, \dots n-1 \quad [1]$$

where $w_i(v_i)$ is given by the polar equation.

Cost:

$$t_{2i} = \int_{h_{2i}}^{h_{2i} + \Delta h_i} \frac{dz}{a_i(z)}$$

$$t_{2i+1} = \frac{\ell_{i+1}}{v_i}$$

$$i = 0, 1, 2, \dots, n-1 \quad [2]$$

Control constraints:

$$\Delta h_i \geq 0,$$

$$i = 0, 1, 2, \dots, n-1 \quad [3]$$

Initial and terminal constraints:

$$h_0 = 0, \qquad h_{2n} = 0 \qquad [4]$$

Altitude constraints:

$$h_{2i+1} \leq h,$$

$$i = 0, 1, 2, \dots, n-1 \quad [5]$$

$$h_{2i} \geq 0,$$

$$i = 1, 2, \dots, n-1$$

The problem is thus to find the optimal strategy, i.e., the sequence(s)

$\Delta h_0, v_0, \Delta h_1, v_1, \dots, \Delta h_{n-1}, v_{n-1}$, which among all such sequences satisfying the relations [1] and [3] - [5], minimize(s) the total cost (i.e., the total time)

$$T = \sum_{i=0}^{n-1} \left[\int_{h_{2i}}^{h_{2i} + \Delta h_i} \frac{dz}{a_i(z)} + \frac{\ell_{i+1}}{v_i} \right] \quad [6]$$

obtained by summing all the partial costs (i.e., partial times given by [2]).

There may be more than one minimizing sequence but the minimum cost is either unique or does not exist.

As such, the problem is of course one of mathematical programming with equality and inequality constraints. However, the way we have set it up, it is in fact a *"discrete optimal control problem"*.

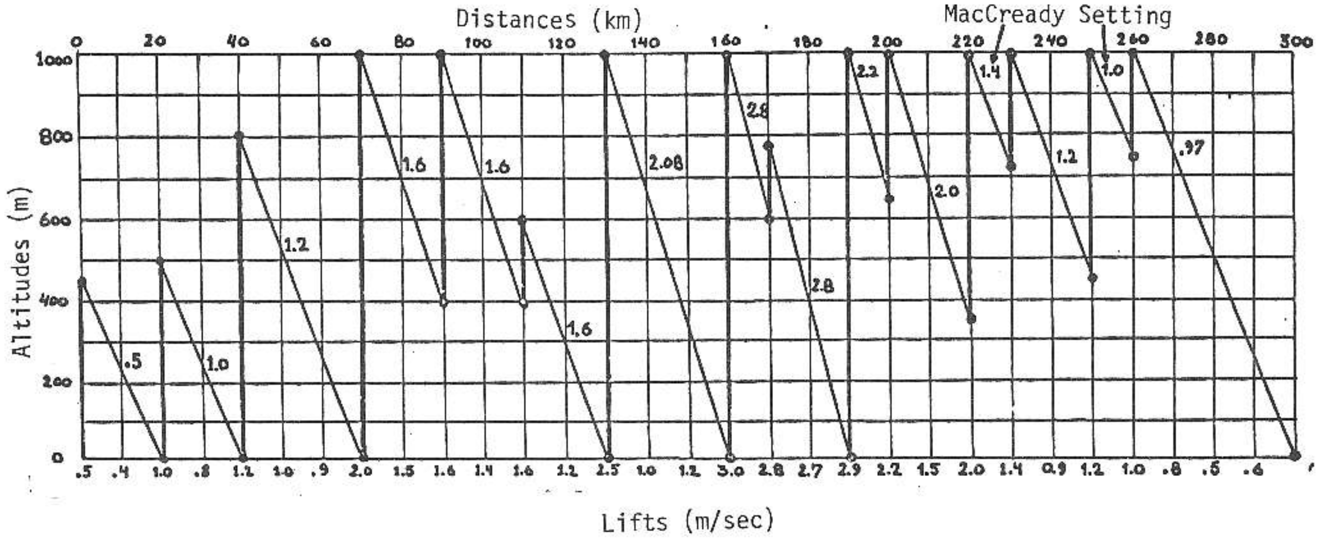
Full details of the mathematics associated with the problem specified and its solution may be found in (13) and the author's complete report (15). [Editor's Note: Further work on this problem by the present authors was reported in reference (16).]

3. EXAMPLES

As a simple example of the results of the optimization discussed, we have taken the 300 km flight schematized in Figure 2. The lift regions are equidistant (10 km) for simplicity although this is by no means required. The lift strengths are indicated in m/sec along the y axis. They increase progressively during the flight, then decrease, but are in general unequal. The altitude limits are 0 and 1000 m. We considered a sailplane having a polar equation given by

$$w = -0.0016409 \, v^2 + 0.061637 \, v - 1.02557$$

Figure 2 Optimum Flight Strategy



The optimal strategy for that lift distribution is illustrated in Figure 2 where the MacCready setting for each glide is indicated. It follows as a simple and systematic application of rules for optimality established in (15). Note that the flight strategy consists in hitting systematically the altitude constraints, except at 110 km and 170 km where we gain, in lift equal to the present MacCready setting, the altitude necessary to reach (at zero altitude) the next best lift. Note also that the MacCready setting is not always equal to the strength of the next lift used. Note finally that this example clearly justifies the practical rule of flying "low" where the lift is improving and flying "high" when it is deteriorating. The total time required for the flight is T = 13331 sec corresponding to a cruising speed of 81.01 km/h.

To illustrate and quantify on the same example the importance of the global flight strategy, we have compared the result obtained by a pilot flying according to the following rules.

- Decision to use a particular thermal:

  from h = 0 to h = 300 m take any lift $a_i > 0$

  from h = 300 m to h = 600 m take lift only if $a_i \geq 1$ m

  from h = 600 m to h = 1000 m take lift only if $a_i \geq 2$ m

- Altitude gained:

  climb up to 600 m if the lift is $a_i < 2$ m

  climb up to 1000 m if the lift is $a_i \geq 2$ m

- Speed to fly:

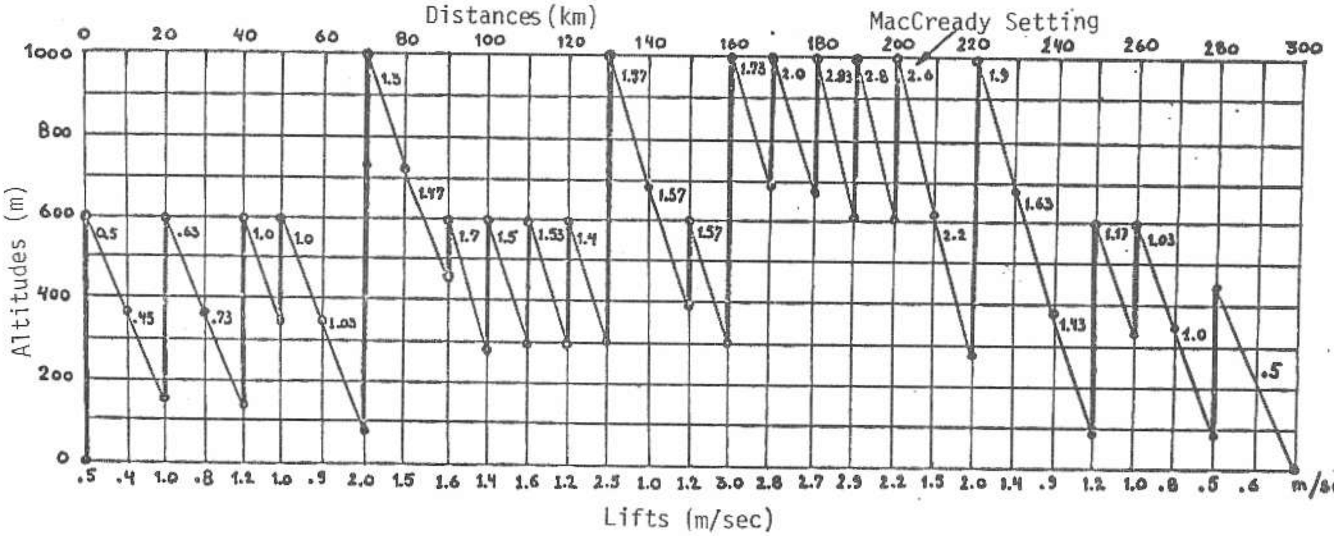
  Adopt a MacCready setting corresponding to the moving average of the last 3 lifts encountered (even if they are not used).

The result is illustrated in Figure 3 and leads to a total time of T = 14401 sec (cruising speed of 74.9 km/h).

4. CONCLUSIONS

Simple rules have been derived for finding the global optimal flight strategy in the case of unequally spaced lifts of variable strength taking into account altitude constraints. The assumption that the locations and strengths of the lift regions are known in advance makes the practical usefulness of the results questionable during an actual flight. However, it is now possible to determine optimal flight strategies in a set of given situations that are often encountered during a flight. The importance of giving due consideration to the altitude constraints is

Figure 3 Non Optimal Flight Strategy



evident. From various tests conducted by the authors in a community of experienced competition pilots it appears that the rules given here are, at best, intuitively approximated.

Improvements to the theory should take into account the size and structure of the thermals in order to allow for dolphin flight segments. This seems possible only if a numerical model is set up. It implies that no simple rules for optimality will be obtained in that case but that a catalog of optimal strategies in a given set of situations could be derived.

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