THE OPTIMUM CENTER OF GRAVITY POSITION FOR MINIMUM OVERALL ENERGY LOSS

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SUMMARY

The effect of center of gravity position on the additional induced drag due to the tail lift force is considered for both the circling and gliding phases of a cross-country flight. The loss of energy height per hour is then derived as a function of CG position for soaring conditions requiring various gliding speeds, assuming the usual MacCready theory to apply. The optimum CG position to minimize the loss of energy height per hour is found to be a function of the gliding speed (or of the corresponding rate of climb). However, if typical Standard and 15-Meter sailplanes are considered, it is found that a single CG position will provide near-optimum conditions over a reasonable range of gliding speeds. The optimum CG position, in the cases considered, was somewhat forward of the likely aft limit. Varying the CG position in flight to maintain zero tail load at all times does not appear to be worthwhile.

INTRODUCTION

It is common knowledge amongst soaring pilots that a tail lift force produces some extra induced drag, since the tail is simply a small wing. It is also common to suppose that down-loads are more unfavorable than up-loads, on the argument that up-loads relieve the wing lift whereas down-loads increase it. On this basis, pilots have tended to think in terms of reducing the down-load on the tail at high speeds by ballasting the machine to get the center of gravity to the aft limit, or perhaps even further aft.

In fact, a consequence of the mutual interference between the wing and the tail is that the direction of the tail lift force is of no consequence. Other things being equal, a certain up-load on the tail produces the same increment in induced drag as a down-load of the same amount.

A good starting point for a detailed analysis is the splendid article in SOARING, October 1979, by that famous aerodynamicist, Robert T. Jones.[1] He explains, inter alia, Munk's analysis of the total induced drag of a pair of lifting surfaces in tandem, such as a wing and a tail, taking into account their mutual interference. It turns out that if the tail is producing a lift force then, for the same total lift, the induced drag is always greater than with zero tail lift and, moreover, the direction of the tail lift is of no consequence. Also, the relative fore-and-aft location of the surfaces is of no consequence: the result for a canard aircraft is the same as for a conventional layout. (These results assume that the trailing vortex systems of the two surfaces are close to the same horizontal plane: with a T-tail, all of the results quoted in this article need slight modification.)

The consequence of this result is that upward tail lift is just as undesirable as downward tail lift. If we consider a Standard Class sailplane for the sake of simplicity, whose center of gravity position cannot be altered in flight, then there could be a small up-load on the tail in slow circling flight and an appreciable down-load in fast straight flight. Both will produce an increment in the induced drag. Percentage-wise, the increment may well be greater at the higher speed but, since the induced drag is then a smaller proportion of the total drag, the actual drag increment in newtons or pounds could well be smaller than at low speed. What really interests the pilot is the loss of energy due to the induced drag increments: in effect, how much further he has to climb in the course of a flight. These considerations suggest that there may be an optimum CG position.

ANALYSIS

From Ref. 1, the total induced drag of the wing and tail of an aircraft, assuming the vortex wakes of the two surfaces are close to the same horizontal plane, is

$$D_{1+2} = \frac{W^2}{\pi q b_1^2}\left\{1 + \left[(b_1/b_2)^2 - 1\right]\left[L_2^2/W^2\right]\right\} \qquad (1)$$

Since $W^2/\pi q b_1^2$ represents the induced drag when $L_2 = 0$, the increment in induced drag due to the tail load is obtained by subtracting this quantity from (1), leaving

$$\Delta D_i = \left[L_2^2/\pi q b_1^2\right]\left[(b_1/b_2)^2 - 1\right] \qquad (2)$$

It should be noted that if the lift is nW in circling flight, equation (2) will still apply if the effect of the vortex wakes becoming helical is neglected. L_2 must, of course, have the value appropriate to circling flight.

If the sailplane flies for a time t at speed V, then the loss of energy height due to ΔD_i will be

$$\Delta h_e = \Delta D_i Vt/W \qquad (3)$$

The symbol V denotes equivalent airspeed, so most of the subsequent equations should stricly include some sort of mean relative density. The quoted figures for loss of energy height per hour will only apply if the flight takes place near sea-level, but the conclusions on optimum CG positions are unaffected by the mean altitude.

It is also convenient to note that, if V_0 is the speed at which the lift/drag ratio is a maximum and q_0 corresponds to V_0,

$$\tfrac{1}{2}D_{min} = \frac{W}{2E_m} = \frac{W^2}{\pi q_o b_1^2} \qquad (4)$$

where E_m is, strictly, the maximum lift/drag ratio with zero tail load. The effect of tail load on E_m will be second-order so far as the final result below is concerned. From (2) and (4), and putting $q/q_o = V^2/V_o^2$,

$$\Delta D_i = (L_2^2 V_o^2/2E_m W V^2)\left[(b_1/b_2)^2 - 1\right] \qquad (5)$$

and introducing (3)

$$\Delta h_e = (L_2^2 V_o^2 t/2E_m W^2 V)\left[(b_1/b_2)^2 - 1\right] \qquad (6)$$

If the proportion of time spent in circling flight is P_c, then the loss of energy height per hour will be

$$\delta h_e/\mathrm{hr} = (1800 V_o^2/E_m W^2)\left[(b_1/b_2)^2 - 1\right]\left[(L_{2c}^2/V_c)P_c + (L_{2g}^2/V_g)(1-P_c)\right] \qquad (7)$$

where suffix 'c' refers to circling flight and 'g' to gliding flight.

For a given V_g, the corresponding rate of sink is fixed and so is the appropriate mean rate of climb in the thermals. Hence, by a simple extension of the MacCready theory it may be shown that

$$P_c = \left[(V_g/V_o)^4 + 1\right]/\left[3(V_g/V_o)^4 - 1\right] \qquad (8)$$

assuming a parabolic drag polar. (See Ref. 2, Appendix 7.)

The tail lift is given by

$$L_{2c} = \left[C_{Mo}\tfrac{1}{2}\rho_o V_c^2 S\bar{\bar{c}} + (h - h_o)\bar{\bar{c}}nW\right]/\ell_T' \qquad (9)$$

where n is the load factor when circling, or

$$L_{2g} = \left[C_{Mo}\tfrac{1}{2}\rho_o V_g^2 S\bar{\bar{c}} + (h - h_o)\bar{\bar{c}}W\right]/\ell_T' \qquad (10)$$

The procedure for finding the loss of energy height per hour is therefore as follows for a sailplane of given characteristics.

a. Choose a dimensionless center of gravity position, h.
b. Estimate a likely speed V_c and load factor n in circling fight.
c. Hence find L_{2c}, the tail load in circling flight, from (9).
d. Choose a gliding speed V_g.
e. Hence find L_{2g}, the tail load in gliding flight, from (10).
f. Also find P_c from (8).
g. Substitute these values of L_{2c}, V_c, P_c, L_{2g} and V_g in (7) to find δh_e/hr.
h. Repeat for different values of h, keeping the same V_g and then plot δh_e/hr against h.
i. Repeat the whole procedure for a new value of V_g.

These calculations have been carried out for a typical Standard-class glider whose characteristics are given in Appendix I.

It was assumed that, when circling in thermals, the speed was 47 kts (87 km/h) and the angle of bank 35^o, giving a load factor of 1.22.

For a gliding speed of 80 knots (148 km/h), the losses in energy height are as follows:

CG position h	δh_e/hr, metres Circling	Gliding	Total
0.25	3.36	51.95	55.31
0.30	0.07	36.49	36.56
0.35	1.67	23.75	25.42
0.40	8.15	13.74	21.89
0.45	19.52	6.45	25.97
0.50	35.77	1.88	37.65

It will be seen that when the CG is well forward, the energy loss in the straight glide is predominant whilst, when the CG is far aft, the energy loss in circling flight is the greater component.

Figures for the total loss of energy height per hour for various gliding speeds are plotted in Fig. 1. Each curve has a minimum, and the higher the speed during the glides, the further aft is the optimum CG position, as one would expect. But the significant feature of the results is that they show that there is no point whatsoever in getting the CG aft of 0.4 $\bar{c}$ for speeds up to 80 knots (148 km/h), corresponding to an average rate of climb of a little over 4 1/2 knots (2.3 m/s) for this sailplane. Altering the CG position in accordance with forecast thermal strengths seems a somewhat improbable occupation, but if the CG were fixed at about 0.37 $\bar{c}$, the loss of energy height per hour would be within a few feet of the minimum for any of the conditions considered here.

When the sailplane has flaps, the calculations become a little more complicated because of the different flap settings for circling and gliding. In equations (9) and (10), C_{M_0} has different values in the two conditions of flight. Some calculations for a 15-meter sailplane with flap settings, deduced from Ref. 5, lead to the curves of Figure 2. The effect of the flaps is to reduce the tail loads during the glide, and hence the overall energy loss. Indeed, with the CG at 0.04 $\bar{c}$ and with a glide speed of 60 knots (111 km/h), the minimum loss of energy is quite negligible since, as it happens, the tail loads in both conditions of flight are very small. For this machine, the optimum CG position moves forward as the glide speed increases, due to the differing flap deflections at the various gliding speeds. Once again, the most aft optimum CG position is about 0.4 c; if it were fixed at 0.37 c, the departure from optimum would be negligible.

DISCUSSION

The most important conclusion which emerges from these calculations is that, in the case of the Standard class sailplane, the optimum CG position is reasonably well aft, but by no means extremely so. Very aft CG positions lead to an excessive loss of energy due to the up-load on the tail in circling flight. In the case of the flapped 15-meter machine, the effect of the flaps is to alter the tail loads in the favorable sense. The engergy loss is generally very small indeed and can be almost zero. There is no point in flying with excessively aft CG position.

It is worth saying that, in performing these calculations, no attempt was made to obtain results which would satisfy those with fairly conventional views on desirable handling characteristics. The

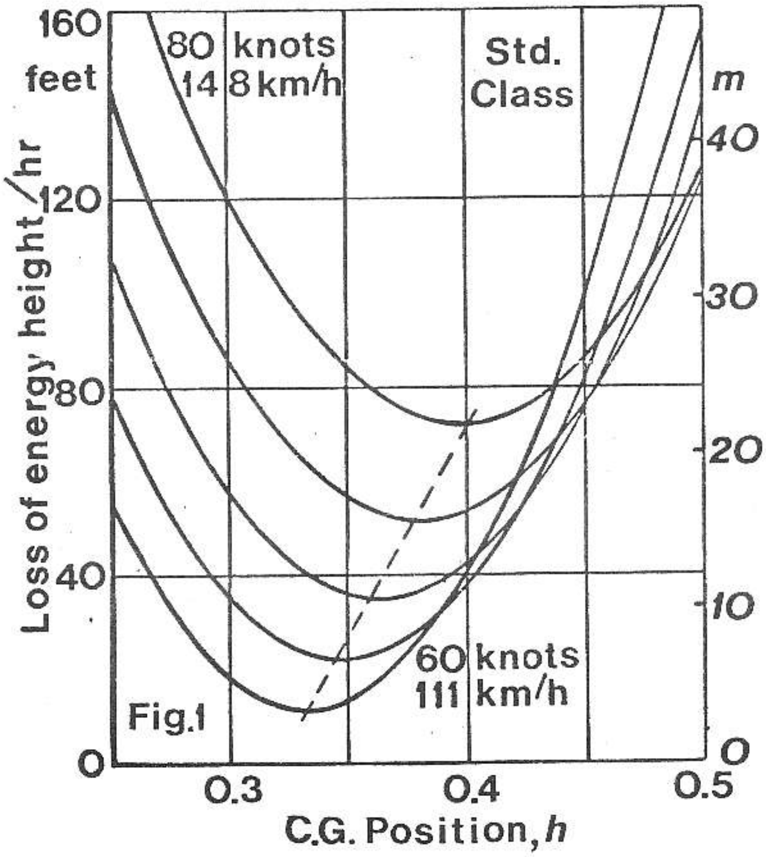

Fig. 1

Loss of energy height per hour due to the effect of tail lift on the induced drag for a typical Std. Class sailplane. The curves are drawn at 5-knot intervals of gliding speed between the thermals. The C.G. position is expressed as a multiple of the mean aerodynamic chord.

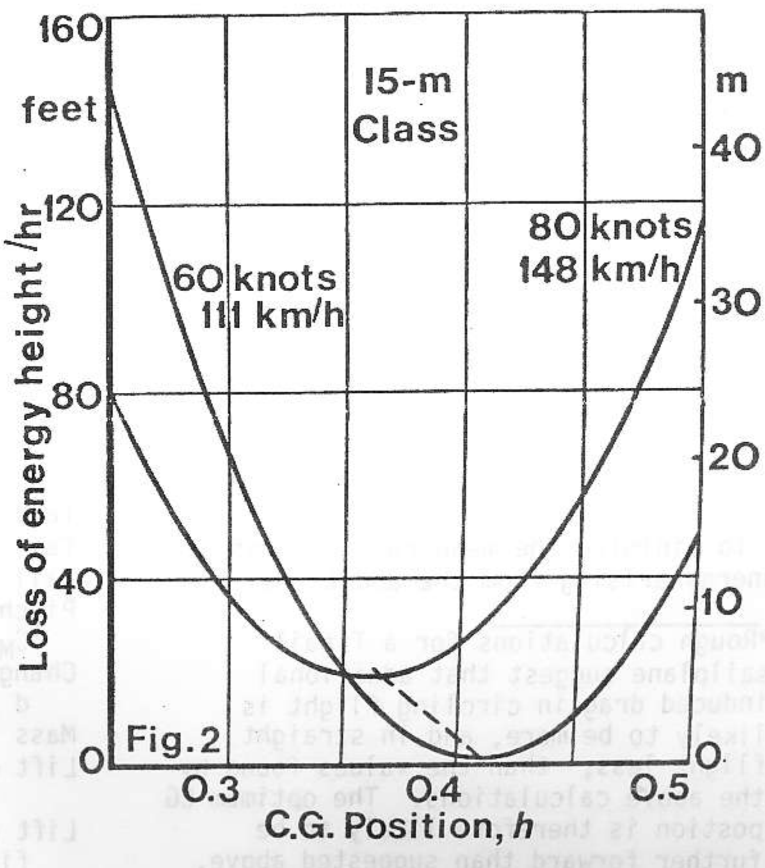

Fig. 2

Loss of energy height per hour due to the effect of tail lift on the induced drag for a typical 15-m class sailplane. Only two values of the gliding speed between the thermals are considered.

'typical' sailplane was chosen and the calculations were performed once only.

These considerations also lead one to conclude that the tail size of the 'typical' machines considered (corresponding to a tail volume of 0.57) is close to the optimum: the optimum CG position does not depend on the tail area but, with the CG at this position, the tail area would appear to be enough to provide adequate static margins.

In the case of the Standard machine, one is tempted to wonder whether it would be profitable to alter the CG position in flight. For example, if the gliding speed between thermals were 70 knots (130 km/h), the energy losses due to tail loads could be reduced to zero by circling with the CG at $h = 0.3$ and gliding with it at $h = 0.5$. The saving in energy height per hour, relative to the minimum loss with the CG fixed at $h = 0.35$, would be 35.7 ft (10.9 m) and, since the average rate of climb for this gliding speed is 2.9 knots (1.5 m/s), the saving in time would be about 7 seconds per hour or 0.02%. To produce this CG shift would involve moving a mass of 8 kg through a distance of nearly 5 m along the fuselage, doubtless by pumping water ballast. Also, with the CG at h = .0.5, the machine would be slightly unstable. To restore some stability, a slightly larger tailplane would be required, thus increasing the profile drag. Also, the CG shift would require a greater change of elevator angle between the two conditions of flight, compared with the fixed CG condition, again increasing the profile drag. Moving the center of gravity in flight appears to be a profitless occupation.

CONCLUSIONS

To minimize the mean rate of loss of energy arising from the additional induced drag caused by tail lift forces, the optimum center of gravity postion is found to be a function of the gliding speed between thermals (or of the corresponding rate of climb in the thermals). However, if typical Standard and 15-meter sailplanes are considered, it is found that a single CG positionwill provide near-optimum conditions over a reasonable range of gliding speeds. The optimum CG postion, in the cases considered, was somewhat forward of the likely aft limit.*

Varying the CG position in flight, to maintain zero tail load at all times, does not appear to be worthwhile.

*Rough calculations for a T-tail sailplane suggest that additional induced drag in circling flight is likely to be more, and in straight flight less, than the values found by the above calculations. The optimum CG postion is therefore likely to be further forward than suggested above.

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APPENDIX 1

Characteristics of a Typical Standard-Class Sailplane

Wing span, b_1 = 15 m
Wing area, S = 9.67 m^2
Mean chord, $\bar{c}$ = 0.64 m (assumed to be substantially the same as c).
Aerodynamic center postion, h_0 = 0.21
Tail area, S_T = 0.99 m^2
Tail span, b_2 = 2.5 m
Tail moment area, l_T' = 3.57 m
Pitching moment coefficient, C_{M_0} = -0.1
Change of downwash with incidence, d /d = 0.2
Mass = 295 kg (i.e. W = 2894 N)
Lift curve slope (without tail), = 5.73/radian
Lift curve slope of tail (elevator fixed), $_1$ = 3.72/radian

Hence,

$F = 0.0532$, $V' = 0.571$, $V_T = 0.542$. (See Appendix 2).

The stick-fixed neutral point position is

$$h_n = h_o + V_T(a_1/a)\left[1-(d\varepsilon/d\alpha)\right]$$

(Refs. 3 & 4), and hence has the value of 0.492.

(Assuming reasonable values for the other tail and elevator coefficients gives a stick-free neutral point position $h_n' = 0.456$. However, the use of springs in the circuit would bring h_n' close to h_n.) Likely CG limits would be 0.25 h 0.4.

APPENDIX 2

Symbols

a	Lift curve slope of the glider (without tail)
a_1	Lift curve slope of the tail (elevator fixed)
b	Span
$\bar{\bar{c}}$	Mean aerodynamic chord
D_{M_O}	Pitching moment coefficient of the glider (without tail) about its aerodynamic center
D	Drag
D_i	Increment in induced drag due to the tail load
E_m	Maximum lift/drag ratio
h_e	Energy height
$h\bar{\bar{c}}$	Distance of the center of gravity aft of datum
$h_o\bar{\bar{c}}$	Distance of the aerodynamic center of the glider (without tail) aft of datum
$h_n\bar{\bar{c}}$	Distance of the stick-fixed neutral point aft of datum
$h_n'\bar{\bar{c}}$	Distance of the stick-free neutral point aft of datum
T'	Distance between aerodynamic center of the glider (without tail) and the areodynamic center of the tail
L	Lift
n	Load factor
P_c	Proportion of total fight time spent circling in thermals
q	Dynamic head
S	Wing area
S_T	Tail area
t	Time
V	Equivalent airspeed
V_T	Effective volume coefficient stick fixed, given by $\bar{V}'/(1+F)$, where $\bar{V}' = S_T\ell_T'/S\bar{\bar{c}}$ and $F = \left[S_Ta_1/Sa\right]\left[1-(d\bar{\varepsilon}/d\alpha)\right]$.
W	All-up weight of the glider
	Angle of incidence
	Downwash angle at the tail
ρ_0	Standard sea-level air density

Suffices:

o	Refers to the max (L/D) condition, in conjunction with V and q
1	Refers to the wing } In conj. with D, b and L.
2	Refers to the tail } In conj. with D, b and L.
c	Refers to the circling condition
g	Refers to the glide condition between thermals □