Some New Developments in Atmospheric Turbulence and Terrain Surface Description

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ABSTRACT

Atmospheric turbulence spectra follow either the von Karman formula or a modified form of the Lappe/Lockheed-Georgia equation. The integral scale parameter L is not only a turbulence concept but a basic parameter common to all stationary stochastic processes.

Because of the finite measuring base length, the standard deviation of the turbulence δ_{wm} as calculated directly from flight records is always less than the theoretical value δ_{wo} in the spectrum formulae. Spatial spectra can be directly transformed into time spectra by calculation of the time scale $T = L/V$.

Bulk processing of long flight records is inadvisable because of the limited lengths of the homogeneous sections in atmospheric turbulence.

Runway-respective grass airfield surface spectra follow closely the modified Lockheed-Georgia equation with the exception of the exponent being a third parameter α to be measured individually.

NOTATION

f	frequency	1/s
h	time interval between samples	s
n	wave number, reciprocal of the wavelength	1/m
t	time	s
w	turbulent velocity component	m/s
x	surface elevation above mean	m
G ( )	power spectral density function	
H	height	m
L	/integral/scale parameter	m
R ( )	autocorrelation function	
S	sample length	m
T	time scale	s
V	air speed	m/s
α	exponent	
χ	cutoff ratio	
δ	standard deviation	
τ	time lag	

ξ	space coordinate parallel to the flight speed	m
ζ	space lag	m
ω	circular frequency	rad/s
Δ	relative error	
Ω	space frequency	rad/m

SUBSCRIPTS

m	measured
max	maximal
w	for turbulence velocity
x	for surface elevation
o	theoretical, without frequency cutoff
1	low frequency cutoff
2	high frequency cutoff

INTRODUCTION

Aeronautical research has been a leader in the theory and measurement of stochastic phenomena. All the same, even recent full scale fatigue tests on sailplanes have been run employing only sinusoidal load block programs[9,15] and do not include the type of spectrum-generated stochastic time histories current, for example, in the automotive industry[18]. Even quite modern counting equipment reflects this quasi-static philosophy[3]. In view of the advantages, one can not help wondering if something might be done to further the introduction of spectal methods for sailplane fatigue test load generation. Will we meet some fundamental difficulties in doing so?

Basic data measurement and analysis procedures are well known[1,2]. Similarly, modern magnetic recording equipment works reliably and it can be made to fit into the confined space available in modern high-performance sailplanes[16].

Of course, no sailplane manufacturer can afford to buy a full set of servohydraulic fatigue test equipment including an on-line control computer. But glider tests are run mostly in research institutes where general purpose test equipment is available.

The most fundamental restriction is perhaps the necessity to employ a loading lever system for simulation of the airloads by hydraulic cylinders. This arrangement precludes the use of higher load frequencies.

There is also some uncertainty in the correct fatigue damage calculation procedure for stochastic load sequences. This difficulty may be partially overcome by using Neuber's Rule (see e.g. Galliart et. al.[7]).

Summing up our review, we can say that a switch over to stochastic load programs seems to be realizable and consequently advisable. In order to promote this, let us analyse some parts of the stochastic service load calculation processes.

1. Turbulence Spectra, Scale of Turbulence

As regards the starting point for an atmospheric turbulence description, we are in a most fortunate situation. The laws of turbulent fluid motions being universal, formulae worked out e.g. for wind tunnel or for boundary layer research are equally well applicable for our purposes. Dynamic air load calculations have to be started from the power spectral density function[11,13,14] as given e.g. by von Karman[10]:

$$G_w(\Omega) = \frac{2}{\pi}\sigma_w^2 L\,\frac{1 + \frac{8}{3}(1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}} \qquad (1)$$

wherein

$$\sigma_w = \left[\lim_{S\to\infty}\frac{1}{S}\int_0^S w^2(\xi)\,d\xi\right]^{1/2} \qquad (2)$$

is for the standard deviation of turbulence[1,2] and L is for the so called integral scale of turbulence defined by Taylor[19] through the autocorrelation function

$$R_w(\zeta) = \lim_{S \to \infty} \frac{1}{S} \int_0^S w(\xi)\, w(\xi+\zeta)\, d\xi \qquad (3)$$

using the equation

$$L = \frac{1}{\sigma_w^2} \int_0^\infty R_w(\zeta)\, d\zeta \qquad (4)$$

(see Figure 1)

Below a height of about 600-800 meters Lappe has found a slightly different turbulence spectrum[12]. His formula was modified later at Lockheed-Georgia[6] into

$$G_w(\Omega) = \sigma_w^2 \frac{0.8L}{(1 + 0.8L\Omega)^{1.8}} \qquad (5)$$

Basic dynamic load calculation laws are well known[11,13,14,15] but as yet comparatively little has been written on correct and efficient data assessment and space-time conversion procedures.

To the author's knowledge it was Kovasznay[5] who proved first - as a special case of the Wiener-Khinchin relationships - that for every stationary stochastic process the zero value of the PSD function $G_w(\Omega)$ is:

$$G_w(0) = \frac{2}{\pi} \sigma_w^2 L \qquad (6)$$

This opens up new aspects in the interpretation of stochastic processes. Let us review some of them. It is well known that the zero value of the autocorrelation function is:

$$R_w(0) = \sigma_w^2 \qquad (7)$$

and the variance can also be calculated using the formula:

$$\sigma_w^2 = \int_0^\infty G_w(\Omega)\, d\Omega \qquad (8)$$

By comparing Eq. 6 to Eq. 7 and Eq. 4 to Eq. 8 it will be evident that the scale parameter L is not only a turbulence concept but a basic parameter common to all stationary stochastic processes. In this respect it is equal to and complementary with the standard deviation σw[5].

When comparing the Lockheed-Georgia formula as given in Eq. 5 to Eq. 6 it is seen that unlike the Karman formula this one does not meet the zero value requirement. This problem recently induced the author to propose the following alternate formula[8]:

$$G_w(\Omega) = \frac{2}{\pi} \sigma_w^2 \frac{L}{\left(1 + \frac{12}{5\pi} L\Omega\right)^{11/6}} \qquad (9)$$

The fact that such a problem can occur is to be explained by the quasi-stationary character of atmospheric turbulence and by problems associated with the finite record base lengths.

2. Consequences of a Finite Measuring Base Length

There is always an upper limit to the extension of the measuring/evaluation base length. It is imposed not so much by basic measuring accuracy but more by the limited lengths of homogeneous turbulence sections. For thermal atmospheric turbulence this upper limit has to be set to about $S_{max} \cong L \div 4L$

Measured spectra are therefore always truncated on their low-frequency boundary at

$$\Omega_1 = \frac{2\pi}{S} \qquad \text{(Figure 2)} \qquad (10)$$

Frequency transfer characteristics of the instrumentation and/or sampling rate also give a high-frequency cutoff. For digital processing with a sampling interval h and holding a 100% safety factor to the Nyquist frequency the high-frequency cutoff will be at

$$f_2 = \frac{1}{4h} \qquad (11a)$$

giving

$$\omega_2 = 2\pi f_2 = \frac{\pi}{2h} \qquad \text{(Figure 2)} \qquad (11b)$$

The corresponding space frequency is

$$\Omega_2 = \frac{\omega_2}{V} \qquad (12)$$

In assessing the differences in the as measured value of the standard deviation σ_w caused by the frequency cutoffs for a spectrum following Eq. 9 it will be practical to introduce the shorthand notations

$$\kappa_1 = \frac{12}{5\pi} L\omega_1 = \frac{24}{5} Ln_1 \quad (13)$$

and

$$\kappa_2 = \frac{12}{5\pi} L\omega_2 = \frac{24}{5} Ln_2 \quad (14)$$

respectively. Integration of Eq. 9 between these limits will give

$$\sigma_m^2 = \sigma_o^2 \left[\frac{1}{(1+\kappa_1)^{5/6}} - \frac{1}{(1+\kappa_2)^{5/6}}\right] \quad (15)$$

The relative error is therefore:

$$\Delta_m^2 = \frac{\sigma_m^2 - \sigma_o^2}{\sigma_o^2} \quad (16)$$

or

$$\Delta_m^2 = \left[\frac{1}{(1+\kappa_1)^{5/6}} - 1\right] - \frac{1}{(1+\kappa_2)^{5/6}} = \quad (17)$$

$$= \Delta_1^2 + \Delta_2^2$$

The high-frequency cutoff error

$$|\Delta_2| = \sqrt{\frac{1}{(1+\kappa_2)^{5/6}}} \quad (18)$$

isn't serious. Using good instrumentation and with a little care it can be reduced to below 1%.

The difference due to low-frequency cutoff

$$|\Delta_1| = \sqrt{1 - \frac{1}{(1+\kappa_1)^{5/6}}} \quad (19)$$

is much more difficult to live with. There are simply no homogeneous turbulence sections for the direct measurement of the theoretical standard deviation σ_{wo} so we have to introduce the concept of the measured or effective standard deviation σ_{wm}8. For Lockheed-Georgia type turbulence, conversion can be made using Eqs. 13-15. Similar equations can be worked out for Karman-type spectra.

3. Space Spectra and Time Spectra

Theoretical turbulence spectra, as given e.g. by Eqs. 1 and 9 respectively, refer to the turbulent velocity as a function of the space coordinate $w=w(\xi)$.

However, The aircraft structure experiences them as a function of the time t and so are accelerations or strains displayed in flight records. Every time a flight-measured spectrum is used for air load calculation a two-fold space-time conversion has to be accomplished.

It is not enough for a correct conversion to transform the time scale of the spectrum by

$$f = nV = \frac{\Omega V}{2\pi} \quad (20)$$

Theoretically the correct procedure would be to transform the space correlation function $R_w(\xi)$ by the substitution

$$\xi = \tau V \quad (21)$$

into the time autocorrelation function $R_w(\tau)$. Fourier transformation of the autocorrelation function will give

$$G_w(f) = 4\int_0^\infty R_w(\tau) \cos 2\pi f\tau \, d\tau \quad (22)$$

This lengthy and expensive process can be easily evaded. A direct equivalent time spectrum can be given in the following way[8]. Let us define the time scale, the time equivalent of the scale of turbulence L, as

$$T = \frac{1}{\sigma_w^2} \int_0^\infty R_w(\tau)\, d\tau \quad (23)$$

If the time spectrum is given as a function of the frequency f, then - observing Eq. 22 - Eq. 6 will turn into

$$G_w(0) = 4\sigma_w^2 T \quad (6a)$$

It follows from Eqs. 4, 21, and 24 that the time scale can be directly calculated as

$$T = \frac{L}{V} \quad (24)$$

Eqs. 20, 6a, and 24 give then the time variant of the Karman spectrum directly as

$$G_w(f) = 4T\sigma_w^2 \frac{1 + \frac{8}{3}(8.4132Tf)^2}{\left[1 + (8.4132Tf)^2\right]^{11/6}} \quad (25)$$

Formally the same has been done in Report FAA-ADS-53 by Nystrom and Mai[15]. Similarly the low-level turbulence Eq. 9 becomes

$$G_w(f) = \sigma_w^2 \frac{4T}{\left(1 + \frac{24}{5} Tf\right)^{11/6}} \quad (26)$$

4. A Numerical Example

Let us suppose that records of the atmospheric turbulence have been made in the flight modes and weather conditions given in Table 1. Strength of turbulence values are rounded off from typical data as given by Steiner[17] and scale of turbulence values follow the trends found by Lappe[12]. The data sampling interval should be h=0.005 s, giving a sampling frequency of 200 Hz. The theoretical Nyquist frequency is therefore 100 Hz giving a practical high-frequency cutoff at f_2=50 Hz (Table 2).

In flight mode 1, a Lockheed-Georgia type turbulence spectrum following Eq. 9 is to expected while in all other cases the spectrum will be of the Karman type.

Digital data processing using a sample length of 4096h[1,2] will give a low-frequency cutoff at f_1=0.0488 Hz. Time scales calculated using Eq. 24 are given in the last column of Table 1. Respective space and time spectra will turn out therefore as in Figures 3 and 4. End products of the record evaluations will be the space spectra shown on Figure 3 and stochastc fatigue load time functions can be generated from the time spectra in Figure 4.

The appraisal of these results raises no questions regarding the correctness of the high-frequency cutoffs. But a comparison of the measuring base lengths S with the scales L reveals

$S \simeq 8.5$ L for flight mode 1 and

$S \simeq 11.4$ L for flight mode 2.

We cannot be sure of such long relative lengths of homogeneous turbulence. Therefore, it will be advisable to cut these records in half -

	Flight mode	V km/h	H m	Load factor g	σ_w m/s	L m	Spectr. type	T s
1	Straight flight in clear air	90	600	1.0	0.5	60	L-G	2.4
2	Glide between thermals	200	1500	1.0	1.0	100	K	1.8
3	Circling in thermals	80	1500	1.6	1.0	100	K	4.5
4	Circling in cloud	80	2000	1.6	2.0	200	K	9.0
5	Circling in Cb	80	3000	1.6	4.0	300	K	13.5

Table 1

Load Spectra

	Flight mode	f_2 Hz	f_1 Hz	S m	Ω_1 rad / m	Ω_2 rad / m
1	Straight flight in clear air	50	0.0488	512	0.0123	12.566
2	Glide between thermals	"	"	1138	0.0055	5.655
3	Circling in thermals	"	"	455	0.0138	14.137
4	Circling in cloud	"	"	"	"	"
5	Cicling in Cb	"	"	"	"	"

Table 2

Cutoff Frequencies

perhaps even split record 2 in four - and then see if they will give congruent results. Otherwise, turbulence strengths and scale parameters might turn out as undefiniably biased averages from different turbulence sections.

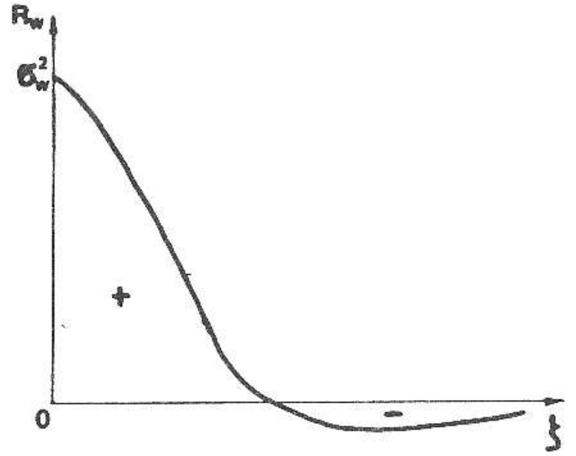

Figure 1: Calculation of the Scale Parameter L

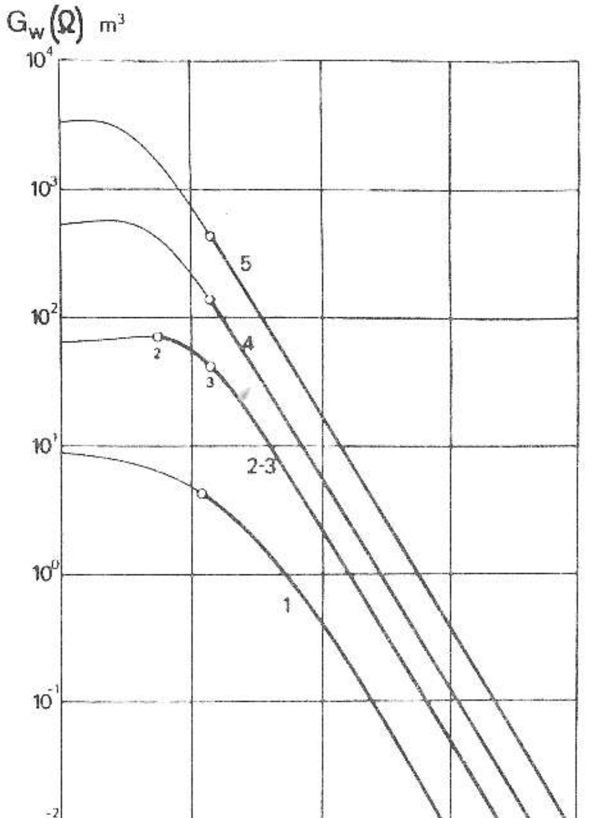

Figure 3: Turbulence Spectra for Different Flight Modes

5. Runway and Grass Field Surfaces

Fourier transformation of runway and soft terrain surface profile records give spectra of quite similar character to turbulence spectra following Eq. 9. As shown recently by the author[8], they

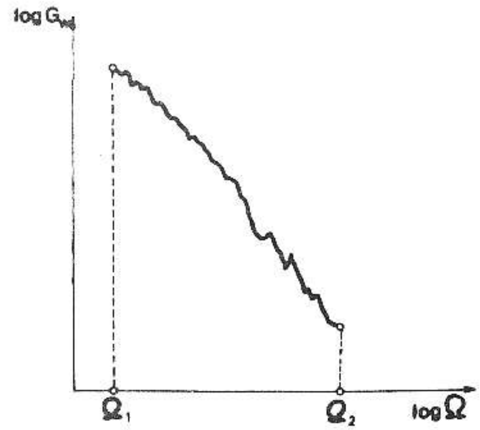

Figure 2: Low- and High-Frequency Cutoff

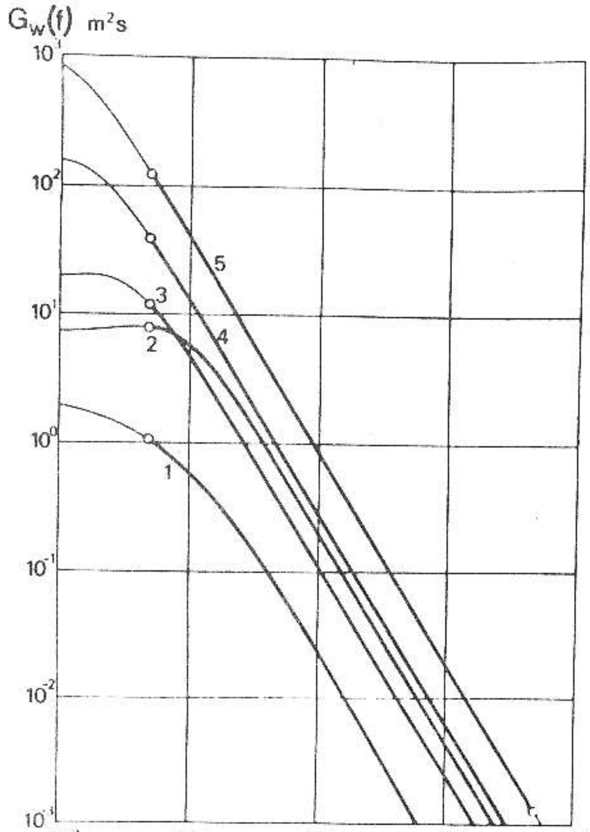

Figure 4: Time Spectra for Different Flight Modes

can be correctly evaluated using the following formula:

$$G_x(n) = \sigma_x^2 \frac{4L}{\left(1 + \frac{4}{\alpha - 1} Ln\right)^{\alpha}} \quad (27)$$

In some cases a twin formula of similar character will be required[8]. The value of the exponent α depends on the type of road surface with α =1.5-2.8 while L will vary between 1.8 m to 50-80 meters. These are preliminary results, however.

CONCLUSION

Evaluation of flight records as well as practical use of the spectral method will be faciliated if full use is made of the scale parameter L with respect to the time scale T. They are not only an attribute of atmospheric turbulence but basic parameters in all stationary stochastic processes.

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