A New Approach to Fatigue Damage Calculation

József Gedeon
Budapest University of Technology and Economics
Budapest, H-1521, Hungary
joska@kme.bme.hu

Presented at the XXIX OSTIV Congress, Lüsse-Berlin, Germany

Abstract

The paper presents a study of improving the accuracy of fatigue damage accumulation calculation. Weibull probability distribution is used for primary assessment of test data. A set of single-stress-level fatigue life distributions gives the basis of Manson-style two-step test series. Imaginary homogeneous test series are generated from the single stress-level Wohler curves. The equivalent load cycle number N* is introduced to damage calculation. High-low and low-high load orders require different formulae. After full development and trial analysis of a number of two-step test results, the procedure promises to give substantial improvement in the accuracy of fatigue life calculation, e.g. in finite-element programs.

Nomenclature

x fatigue life ratio for nominal stress level σ
D index of fatigue damage
N number of load cycles
N_0 nominal safe life (P=0)
N_s number of load cycles to break (with P probability)
N* equivalent load cycles for fatigue damage calculation
P probability of failure
β standard deviation
ε safe life ratio
κ exponent
φ effective conversion rate

Index
a first stress-level
b second stress-level
hi high
lo low
1 for stress-level σ_1
2 for stress-level σ_2
3 for stress-level σ_3

Introduction

Knowledge of the physical aspects and efficient calculation procedures are needed for safe fatigue design and operation of sailplanes. The theory of fatigue has to deal with two difficulties: the wide spread of individual service lives and the strongly nonlinear character of the respective physical laws.

At present the weakest part of the theory is the efficient modeling of fatigue damage accumulation. Even the name may be deceptive for the uninitiated because it does not mean the decrease of strength but the wear of the safe service life.

Originating from the observations of Grover[1], Manson and Freche[2] initiated a double linear damage calculation method based on two stress-level fatigue test series. Wanting a simple and reliable safe-life calculation method, the proposal seemed to promote more exact calculation, but the paper did not give enough information for decision and direct application. In order to make a trial, rotating bending tests were run on Al Cu Mg 2 type specimens. The results were reported on at the Seventh Congress of the International Council of the Aeronautical Sciences[3].

The linear modeling did not prove sufficiently accurate in these tests even if fatigue lives were divided into two parts. On lower stress levels for part of the fatigue life, temporary augmentation of the fatigue tolerance may occur. Analysis of the results using the habitual $0 \leq D \leq 1$ index-number seemed to indicate that the fatigue damage level cannot be reliably expressed by a scalar index.

Similar tests at Technion by Buch and Weinstein[4] gave similar results referring to our work. Wanting a satisfying interpretation for this result, the problem was shelved.

Recently the problem was reviewed starting from the idea that the best calculation model is the simplest one making allowance for *all significant parameters of the process including the correct values for the initial conditions.*

Adopted basic methods and procedures

As a rule, current fatigue damage calculations suppose two common fatigue damage conditions of the test pieces: the factory-new one at the beginning and that at breaking, scaled e.g. to D = 0 and to D = 1, respectively. Would this be correct, the standard deviation of the fatigue life at a single-level test series would amount only to a few per cent of the mean. Against this, a proportion of 1 to 3 or 1 to 4 of the worst-to-best life in a test series is by no means a sign of bad quality. Consequently, pieces of a series should not be scaled at the start of service to have equal fatigue capacity. Starting from this perception, we work on a fatigue damage scale and calculation procedure not requiring homogeneous test series.

For want of imperfection-free test pieces, imaginary primary data are generated by analytical smoothing of the fatigue test results. The following calculation methods and processes are based on the three-parameter Weibull distribution function (Fig. 1):

$$\frac{1}{1-P(N_s)} = \exp\left[\left(\frac{N_s - N_0}{\beta}\right)^{\alpha}\right] = \exp\left[\left(\frac{N_s}{\beta} - \varepsilon\right)^{\alpha}\right] \quad (1a)$$

$$\varepsilon = \frac{N_0}{\beta} \qquad \text{(Fig. 1)} \qquad (1b)$$

The fatigue life of P probability reads according to Eq. 1a:

$$N_s = \left[\left(\ln \frac{1}{1-P}\right)^{1/\alpha} + \varepsilon\right]\beta \qquad (1c)$$

The probable nominal safe-life is given by the best linear fit value of N_0 on Weibull coordinates (Fig. 2). A sufficient number of such single level test series gives the Wholer map (Fig. 3). Each one of the P = constant Wohler curves can be regarded as representative of an imaginary test series of the same fatigue endurance. The most significant one will be the P = 0 nominal safe-life limit.

As before, the damage calculation procedure starts with the introduction of the fatigue life ratio:

$$x = \frac{N}{N_s} \qquad (2)$$

Manson[2] uses two-level test series for analysis of the character of high-low versus low-high fatigue load orders. The results of such investigations can be summed up on so-called Manson diagrams $x_b = f(x_a)$:

$$x_a = \frac{N_a}{N_{sa}}; \quad x_b = \frac{N_b}{N_{sb}} \quad \text{(See e.g. Figs. 4-6)} \qquad (2a)$$

A simple linear damage law presented by

$$D = \sum_{i=1}^{m} \frac{N_i}{N_{si}}$$

would give a Manson diagram $x_b = 1 - x_a$ irrespective of the load order as indicated by the diagonal dotted line on the graphs.

The damage accumulation calculation formulae

After experimenting with some possible theoretical approaches the simple power-function approximation

$$\left(\frac{N_{s,lo}}{N_{s,hi}}\right)^{\kappa}$$

was chosen for giving a common base of the stress intensities. Introducing an equivalent/imaginary fatigue damage cycle number N* the damage index reads:

$$D = x^* = \frac{N^*}{N_s} \qquad (3)$$

Using the short notation

$$\varphi = \left(\frac{N_{s,lo}}{N_{s,hi}}\right)^{\kappa-1} \qquad (4)$$

the analysis goes on as follows.

Dual (high-low and low-high) demonstration is required and the fatigue life in each demonstration is separated into three sections.

Load order high-low: $\left(N^*_{hi} > N_a\right)$

The three fatigue-life sections are as follows:
$0 \le N \le N_a$, $N_a \le N \le N^*$, $N^* \le N \le N_s$

The surplus damage absorbed in the first section diminishes the linear damage capacity in the second one. Therefore, the balance reads as:

$$\frac{N^*_{hi}}{N_{s,hi}} = 1 - \frac{\left(N^*_{hi} - N_a\right)\left(\frac{N_{s,lo}}{N_{s,hi}}\right)^{\kappa}}{N_{s,lo}} - \frac{N_{b,lo}}{N_{s,lo}}$$

$$\frac{N^*_{hi}}{N_{s,hi}} = 1 - \frac{N^*_{hi}}{N_{s,hi}}\left(\frac{N_{s,lo}}{N_{s,hi}}\right)^{\kappa-1} + \frac{N_a}{N_{s,hi}}\left(\frac{N_{s,lo}}{N_{s,hi}}\right)^{\kappa-1} - \frac{N_{b,lo}}{N_{s,lo}}$$

$$D_{hi} = x^*_{hi} = \frac{1 + x_a\varphi - x_{b,lo}}{1+\varphi} \qquad (5)$$

Load order low-high: $\left(N^*_{lo} < N_a\right)$

The three fatigue life sections are as follows:
$0 \le N \le N^*$, $N^* \le N \le N_a$, $N_a \le N \le N_s$

The first load step now leaves room for part of the second-step damage. Therefore, the damage balance reads:

$$\frac{N^*_{lo}}{N_{s,lo}} = 1 - \frac{\left(N_a - N^*_{lo}\right)\left(\frac{N_{s,hi}}{N_{s,lo}}\right)^{\kappa}}{N_{s,hi}} - \frac{N_{b,hi}}{N_{s,hi}}$$

$$\frac{N^*_{lo}}{N_{s,lo}}\left[1 - \left(\frac{N_{s,hi}}{N_{s,lo}}\right)^{\kappa-1}\right] = 1 - \left[x_a\left(\frac{N_{s,hi}}{N_{s,lo}}\right)^{\kappa-1} + x_{b,hi}\right]$$

$$D_{lo} = x^*_{lo} = \frac{1 - \left(\frac{x_a}{\varphi} + x_{b,hi}\right)}{1 - \frac{1}{\varphi}} \qquad (6)$$

A first control gives $x_a = 0$: $D_a = 0$ respective $x_a = 1$: $D_a = 1$ for both formulae.

The damage accumulation calculation procedure

The analysis process is compiled from equation-twins (Eqs. 5 and 6) giving approximately equal $D_a = f(x_a)$ pairs if and only if the correct φ was selected for the calculation. By way of illustration, the lower curve on Fig. 6, σ_2-σ_3, and the upper one on Fig. 4, σ_2-σ_1, can be selected for determining $D_2(x_a)$ for the test piece quality level P = 0.5.

Obviously the P = 0 case will be the safest for actual stressing calculations but e.g. the P = 0.5 level, too, may help in the development process.

Our 1970 test series proved to be insufficient for final testing of the process because it gives only one full evaluation stress level. The first manual trial and error calculations looking for the correct value of κ with respect to φ seem to indicate the following situation:

- the basic idea seems to work but the need of further refinements, too, cannot be excluded;
- in the first part of the fatigue life and under a certain load level, the alternate loading gives negative damage (i.e. a longer fatigue life in the second part of the test);
- at least 7-8 two-step fatigue test series are needed for giving acceptable error margins;
- the two-step test series should be made at least with 16 pieces;
- an appropriate optimizing computer program is necessary for full evaluation of the test results.

Only the practical use can prove the correctness of the calculations. Anyone joining the research is welcome in this situation.

By-passing damage calculations

Anybody interested in fatigue design can tell stories of excessive costs. It is, therefore, a matter of course to look for cheaper substitute methods. Sorry to remark, but absolute protection can be given exclusively by full-scale tests and special safe-by-inspection procedures. By-passing them would require exact modeling of the service load conditions and equally exact cheap fatigue model design.

Based on the post-war test series of Johnstone and Payne[5] a simple and cheap equivalent light metal plate model was recommended by Schütz[6]. But it is valid only for the contemporary riveted light metal designs. Loaded with a good reproduction of the probable service forces, it can give realistic life estimations.

In general, no kind of simple model can cover all influences. Key issues are quality of design, material and technology. In the present analysis, the quality of the product is represented by the probability index P. In some instances the product quality or the damage condition is indicated by the safe life ratio ε. For example, ε is dominant in the choice of safety factors for single piece fatigue tests[7]. A low value of ε means bad material or technology for new products while it signals the end of the safe life for aged sailplanes.

Concluding remarks

The analysis of a two stress-level fatigue test series is reviewed and extended in order to improve the accuracy of fatigue damage calculations. The problem of homogeneous test series is circumvented by reading the primary life data as given for a probability level P (e.g. P = 0 or P = 0.5).

The calculation procedure is a complex task based on the Manson diagrams. It sets out from the obvious fact that the damage level at the end of the first step x_a is independent of the second step load level, i.e. of the high-low or low-high load sequence and of the magnitude of the load difference. Formally it requires handling equation twins to give the desired equality.

The original moderate intention and limited resources prevented the full development and proof of the calculation procedure. Nevertheless, it seems to deserve further work and attention.

Results of the research may be utilized first of all in finite element preliminary stress and fatigue life calculation programs. Experience will show us the correctness or the failure of the basic concept. The evaluation can be summarized as follows: any improvement in accuracy will be highly dependent upon the relevance of the basic data to the structure which is undergoing a fatigue life calculation.

Acknowledgment

The author thanks the Head of the Department, Assistant Professor Alexander Horváth, C.Sc.. for providing the computer facilities and for consultations. Comments by an anonymous reviewer are appreciated.

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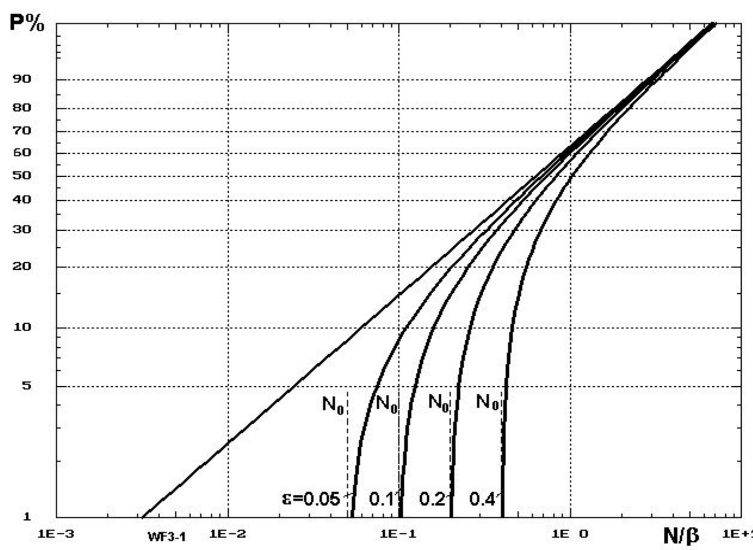

Figure 1 Fatigue life distributions for different safe life ratios

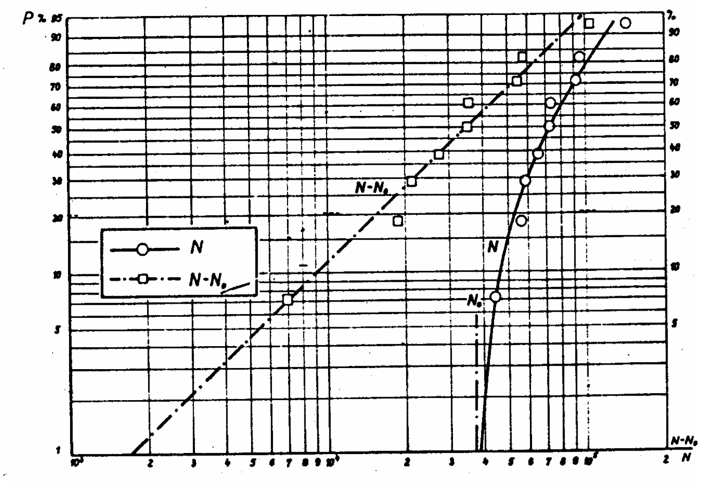

Figure 2 Calculation of the nominal safe life N_0

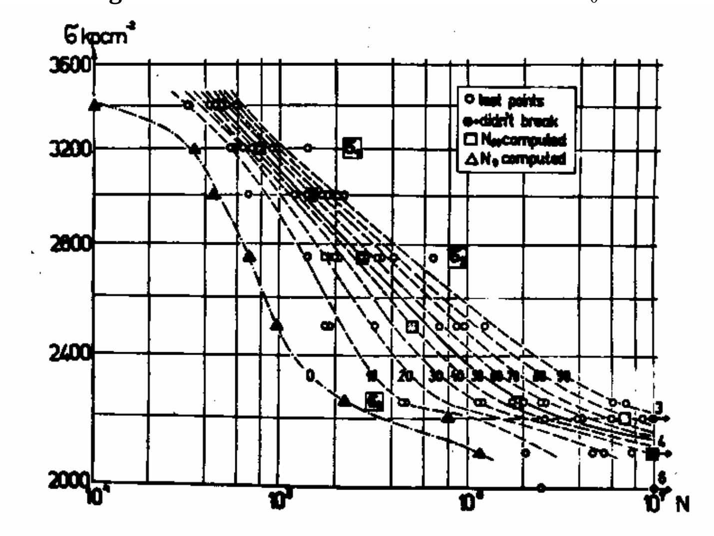

Figure 3 Wohler curves for different probability of failure

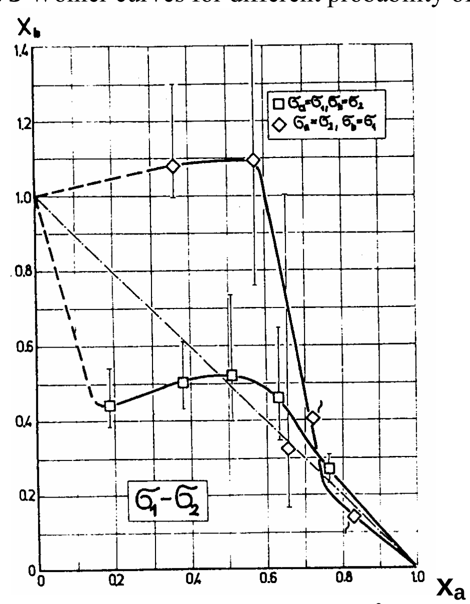

Figure 4 Manson diagram (Gedeon[3], P = 0.5)

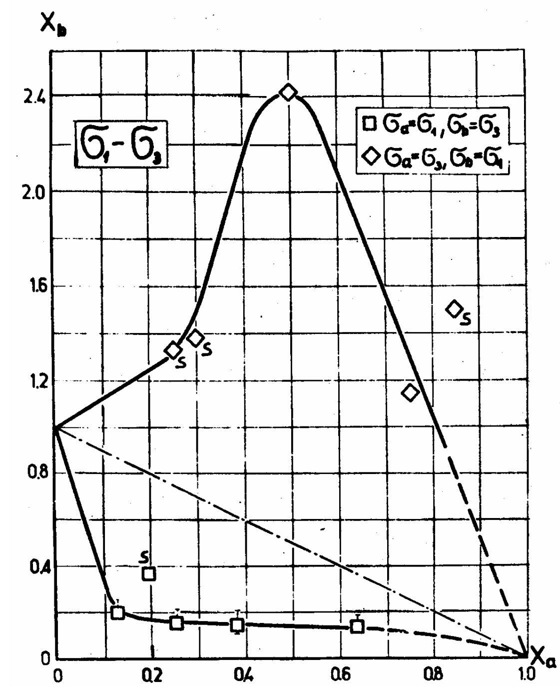

Figure 5 Manson diagram (Gedeon[3], P = 0.5)

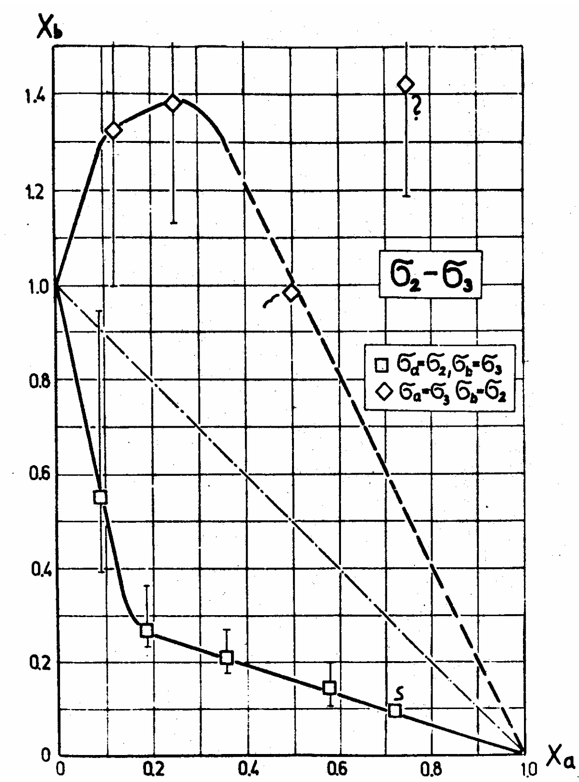

Figure 6 Manson diagram (Gedeon[3], P = 0.5)