BOUNDARIES FOR WORLD CLASS SAILPLANES

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Summary

This paper examines likely boundaries of span and aspect ratio for sailplanes conforming with the "World Class" specification [1]. Empty masses are in accordance with Stender's empirical formula [2] while the performance is assumed to correspond with the simplest analytical expressions [3]. The variation of aerodynamic characteristics with span and aspect ratio are generally similar to those proposed in Ref. 4.

By varying a constant in the Stender formula, it is possible to consider "heavy", "medium" and "light" structures. By inspecting the data of Ref. 5, it is also possible to estimate "high, "medium" and "poor" values of the maximum lift coefficient.

Spans from 10m to 18m and aspect ratios from 10 to 22 are considered. Boundaries are then drawn on span-aspect ratio axes relating to the stall, a minimum lift/drag ratio of 30 and, if relevant, a minimum sinking speed of 0.75 m/s and a reasonable value of the lift coefficient at the minimum sink condition (C_{LMS}), for interesting combinations of structural mass and maximum lift coefficient.

As one would expect, the combination of a light structure and a high C_{Lmax} provides the best combination. The span could be as low as 11m but there is then no room for error.

Spans greater than 13m appear to be more satisfactory. The performance improves as the span and aspect ratio are increased, the latter being limited by neither the stall boundary or the C_{LMS} boundary.

At the other extreme, a heavy structure and a poor C_{Lmax} give very little room for maneuver. The span must be more than 17m and even then there is only a very restricted range of possibilities. While it is unlikely that a designer would deliberately use such an unfavorable combination of characteristics, these results show that one must be very careful not to sacrifice too much to simplicity of construction.

A likely combination is a "medium" structural mass and a "high" C_{Lmax}. The characteristics of such sailplanes are examined in some detail. It seems that achieving a high C_{Lmax} is more important than minimizing the structure weight.

Since the specification of the "World Class" imposes no restriction on the span, other than by implication, there is an incentive for designers to consider large spans.

The specification

Those parts of the "World Class" specification which are relevant to the present paper are:

Best glide ratio: not less than 30.
Minimum sinking speed: not more than 0.75 m/s.
Stalling speed: not more than 65 km/h.
Wing span: no restriction.
Landing gear: fixed.
Flaps: not permitted.
Water ballast: not permitted.
Tip winglets: not more than 10 cm up or down.
Boundary layer blowing or sucking: not permitted.
Mass of pilot + parachute: 70 to 110 kg.
Minimum instruments and equipment are also specified.

The mass of the sailplane

Reference 1 gives the following empirical expression for the empty mass of a sailplane:

$$W_E=C_E \cdot K_E^{3/8} \quad (1)$$
$$\text{where } K_E=n \cdot S \cdot b^3 \quad (2)$$

(See the list of symbols. The original symbols have been retained although, when Stender refers to "weight", he strictly means "mass").

In Figure 4 of Ref. 1, the lower boundary of the diagram for single-seaters corresponds to C_E=1.3 and the upper boundary to C_E=2.15. These values have been taken for the "light" and "heavy" structures of the present paper, while the "medium" structure corresponds to the mean of these values, viz., 1.725.

To find the maximum sailplane masses on which the subsequent calculations are based, the maximum load is assumed to consist of 110 kg for the pilot + parachute together with a further 18 kg for instruments and equipment. The total mass is then (W_E+128) or, expressing W_E in terms of the span, b, and the aspect ratio, A, from (1) and (2):

$$m=128 + K_E \cdot (b^5/A)^{3/8} \quad (3)$$

where K_E+$C_E \cdot n^{3/8}$ and, in accordance with JAR 22, n=8.

Hence we obtain the following values for K_E:

Structure	K_E
Light	2.835
Medium	3.762
Heavy	4.689

The calculated total masses are tabulated in Appendix I.

Maximum lift coefficient

The maximum lift coefficient of the complete sailplane is related to that of the "aircraft-less-tail" by:

$$C_{Lmax}=(C_{LWmax} + C_{Mo}\bar{\bar{c}}/1_t)/(1-(h-h_o)(\bar{\bar{c}}/1_t) \quad (4)$$

It is clearly advantageous to place the CG as far aft as possible, so as to maximize ($h-h_o$). A likely value for this quantity is 0.2. For wing sections likely to be of interest, C_{Mo} is in the range -0.9 to -0.10. We will assume the latter value, neglecting any contribution from the fuselage. The value of $\bar{\bar{c}}/1_t$ can obviously vary considerably. The value for an average Standard-Class sailplane is about 0.18 and this value will be assumed to apply in all cases.

Subject to these assumptions:

$$C_{Lmax}=1.037C_{LWmax}-0.019 \quad (5)$$

Reference 5 suggests that the following values would be appropriate:

Rating	C_{LWmax}	C_{Lmax}
"Poor"	1.20	1.225 (1.23)
"Medium"	1.35	1.381 (1.38)
"High"	1.50	1.537 (1.54)

The rounded-off values in parentheses are those actually used in the following calculations.

The stall boundary

The specification states that the stalling speed shall not exceed 65 km/h. Since the stalling speed with airbrakes extended normally exceeds the "clean" value, it has been assumed that the above figure relates to the "brakes-open" case and that the corresponding "clean" stalling speed would be 62 km/h. (Clearly, there is some encouragement for designers to use brakes which do not increase the stalling speed when extended).

The maximum mass corresponding to a certain maximum stalling speed is then:

$$m_{max}=C_{Lmax}\rho_o(V_{Smax})^2S \qquad (6)$$

Inserting the appropriate numerical values:

$$m_{max}=K_S(b^2/A) \qquad (7)$$

where K_S is as follows:

C_{Lmax}	K_S
"High"	28.51
"Medium"	25.55
"Poor"	22.77

It is therefore possible to prepare tables of m_{max} as functions of b and A as in Appendix II.

The stalling boundaries can then be derived by plotting the mass as a function of aspect ratio for a given span (i) according to Appendix I and (ii) according to Appendix II. The intersections of the various lines then indicate the values of a corresponding to the stalling boundaries for various combinations of structural heaviness and maximum lift coefficient for the given span. An example of such a plot for b=15m is shown in Fig. 1.

(The same result could be obtained by solving equations (3) and (7) simultaneously. However, this does not lead to a simple analytical result. No doubt an iterative program could be devised).

These values of maximum aspect ratio on the stalling boundary are tabulated in Appendix III and some are plotted in Fig. 2. This diagram indicates the large changes in the stalling boundary which occur as a result of various structure weights and maximum lift coefficients.

A	SPAN, m.								
	"Light" Structure			"Medium" Structure			"Heavy" Structure		
	10	14	18	10	14	18	10	14	18
10	218	296	398	247	352	486	276	407	574
16	203	269	354	228	315	428	252	362	502
22	195	253	329	217	294	394	238	335	460

APPENDIX I

Maximum Laden Masses of World Class Sailplanes

The empty mass is given by Stender's formula and the maximum load os taken as 128 kg. The figures are masses in kg. Some of them are unlikely to be relevant to the present study.

A	SPAN, m.								
	"High" C_{Lmax}			"Medium" C_{Lmax}			"Poor" C_{Lmax}		
	10	14	18	10	14	18	10	14	18
10	285	559	924	256	501	828	228	446	738
16	178	349	577	160	313	517	142	279	461
22	130	254	420	116	228	376	104	203	335

APPENDIX II

Maximum Laden Masses of World Class Sailplanes if the Clean Stalling Speed is not to Exceed 62 km/h.

The figures are masses in kg. Some of them are unlikely to be relevant to the present study.

Drag coefficients

For a typical wing section (e.g., FX 61-168), C_{DOw} would be 0.0075 at $R_\infty=1.5X10^6$. The Reynolds number at max. L/D will clearly be a function of sailplane geometry and weight. It would be possible to devise a program which took these effects into account but it would be quite lengthy. For the present purposes, it was thought to be sufficient to take a mean value as above.

As in Ref. 4, the fuselage length and its wetted area were assumed to be proportional to the span, so the miscellaneous drag coefficient" becomes $0.0012/\bar{c}$.

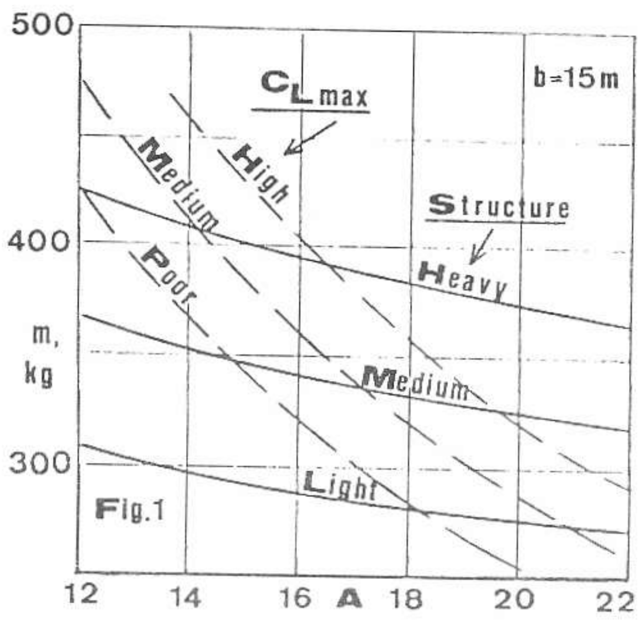

FIGURE 1. The full lines show the total mass of the sailplane (Stender). The dashed lines show the maximum mass, as limited by the stalling speed requirement. Intersections of the sets of lines are points on the stalling boundaries.

b, m.	C_{Lmax}	STRUCTURE		
		Heavy	Medium	Light
10	High Medium Poor	10.45 ------ ------	12.3 10.55 ------	14.1 12.15 10.55
14	High Medium Poor	15.25 13.1 11.35	18.25 15.65 13.55	22.05 19.35 16.85
18	High Medium Poor	19.4 16.6 14.15	------ 20.7 17.65	------ ------ 22.3

APPENDIX III

Values of Aspect Ratio on the Stalling Boundaries
Blank spaces indicate values outside the range under consideration.

Likewise, as in Ref. 4, the contribution of the tail to the profile drag coefficient was taken to be 0.00112.

The above figure for the "miscellaneous" drag coefficient relates to a sailplane with a retracting wheel. Tests on the Slingsby "Sky" [6] suggested that replacing the short landing skid plus wheel by a longer skid decreased the profile drag coefficient by about 0.0015, this figure being related to a wing area of 17.7 m². This difference is not, of course, the same as the effect of an isolated wheel such as a current sailplane would have, but it indicates an order of magnitude. Let us assume that, with careful fairing, half of this figure could be achieved. This represents a further contribution to the profile drag coefficient of 0.0133/S.

The total profile drag coefficient is therefore:

$$C_{DO}=0.0075 + 0.00112 + 0.0012/\bar{c} + 0.0133/S$$

$$\text{ie, } C_{DO}=0.00862 + 0.0012A/b + 0.0133A/b^2 \quad (8)$$

A	k_v	k_p	k	K
10	1.012	0.0660	1.078	2.699
16	1.028	0.1056	1.134	3.329
22	1.043	0.1452	1.188	3.814

APPENDIX IV

Induced Drag Factor as a Function of Aspect Ratio
For the induced drag factor k, see equation (9) and (10). The quantity K is defined in equation (13).

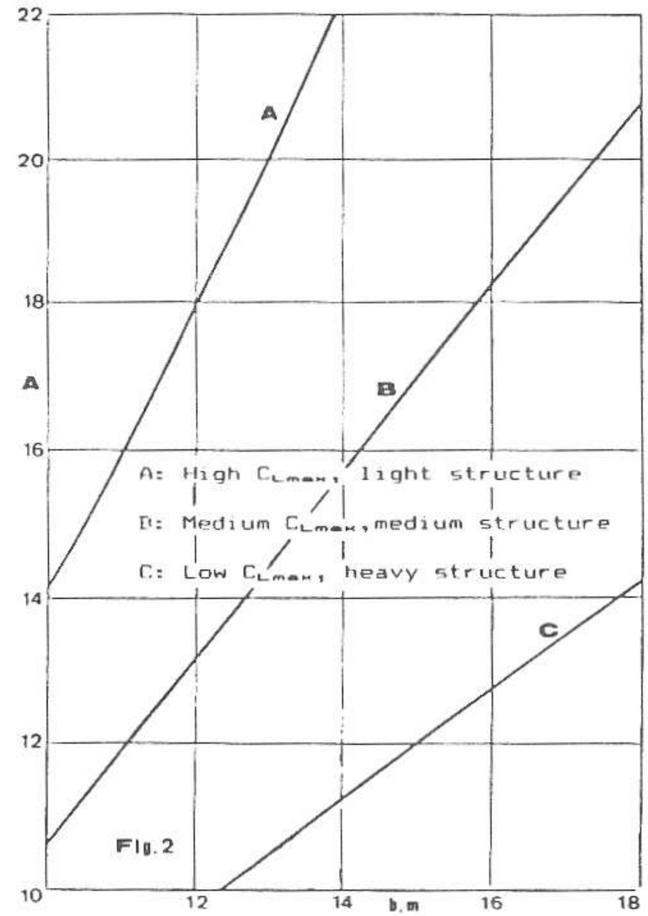

FIGURE 2. Some stalling boundaries derived from plots such as Figure 1. Only three of nine possibilities are shown. Spans and aspect ratios of feasible sailplanes lie below the appropriate line.

Hence, C_{DO} can be found for various values of A and b, as in Appendix V.

As explained in Ref. 4, following Goodhart [7], the induced drag factor K may be considred as including the usual vortex drag factor together with a further contribution which depends on the variation of wing profile drag coefficient with lift coefficient, whence:

$$k=k_v + \pi A(dC_{Dp}/dC_L^2). \quad (9)$$

The value of the above derivative varies from one wing section to another and is only constant if the curve of profile drag coefficient vs. lift coefficient is parabolic. For the present purposes, a value of 0.0021, corresponding to the section FX61-168, has been taken so equation (9) becomes:

$$k=k_v + 0.0066A. \quad (10)$$

Values of k_v have been obtained from Ref. 8, assuming a straight-tapered wing of taper ratio 0.5. The induced drag factor k is therefore a function of A only, as tabulated in Appendix IV.

Maximum lift/drag ratio

If the complete sailplane has a linear C_D-C_L^2 curve then the maximum lift/drag ratio [3] is:

$$(L/D)_{max}=(\pi\ A/4kC_{DO})^{1/2} \qquad (11)$$

$$\text{or } (L/D)_{max} = K/(C_{DO})^{1/2}, \qquad (12)$$

$$\text{where } K=0.8862\ (A/k)^{1/2} \qquad (13)$$

and K is a function of aspect ratio only. Values of K are also tabulated in Appendix IV.

A	SPAN, m.								
	10			14			18		
	C_{DO}	$(L/D)_{max}$	C_{LMS}	C_{DO}	$(L/D)_{max}$	C_{LMS}	C_{DO}	$(L/D)_{max}$	C_{LMS}
10	0.0112	25.56	0.992	0.0102	26.78	0.946	0.0097	27.41	0.921
16	0.0127	29.58	1.301	0.0111	31.63	1.216	0.0103	32.73	1.168
22	0.0142	32.02	1.575	0.0120	34.82	1.447	0.0110	36.38	1.386

APPENDIX V

Performance Characteristics as Functions of Span and Aspect Ratio

Hence, knowing C_{DO} from equation (8), $(L/D)_{max}$ can be found as a function of span and aspect ratio as in Appendix V. Hence, by interpolation, the aspect ratio corresponding to fixed values of the maximum lift/drag ratio can be found for various spans, as in Appendix VI.

Lines of constant max. L/D can then be plotted on (b,A) axes as in Fig. 3. The $(L/D)_{max}$=30 line is one of the World Class boundaries.

Lift coefficient at minimum sink

Assuming that the sailplane has a linear C_D-C_L^2 curve extending even to the high lift coefficient will be:

$$C_{LMS}=(3\pi AC_{DO}/k)^{1/2}. \qquad (14)$$

Hence, from (14) and (11):

$$C_{LMS}=3.4641C_{DO}(L/D)_{max}. \qquad (15)$$

Values of C_{LMS} are also tabulated in Appendix V.

b, m.	$(L/D)_{max}$						
	30	31	32	33	34	35	36
10	16.85	19.14	21.94	------	------	------	------
14	13.64	15.06	16.59	18.33	20.26	------	------
18	12.58	13.74	14.98	16.35	17.86	19.50	21.29

APPENDIX VI

Aspect Ratio as a Function of Span and $(L/D)_{max}$. Blank spaces indicate values outside the range under consideration.

From (14) it is clear that C_{LMS} increases as A and C_{DO} increase. It follows that, with certain configurations, the theoretical value of C_{LMS} given above may become close to, or may even exceed, the available C_{Lmax}. In practical terms, this means that the minimum sink condition will occur at some lift coefficient less that the theoretical C_{LMS}, probably with a high drag coefficient due to the onset of the stall. This may not matter much, but it seems to be a condition to be avoided if possible.

If it is decided, rather arbitrarily, that C_{LMS} should not exceed 0.9 times C_{Lmax}, then a further set of boundaries is defined. By interpolating the figures of Appendix V it is possible to find, for a given span, the aspect ratio satisfying the above condition for "high", "medium" and "poor" values of C_{Lmax}. These results are tabulated in Appendix VII.

Minimum Sinking Speed

Again assuming that the sailplane has a linear C_D-C_L^2 curve even at the high lift coefficient of the minimum sink condition then, introducing K from equation (13) and inserting the various constants, the minimum rate of sink will be:

$$V_{smin}=0.7927C_{DO}^{1/4}K^{-1.5}w^{1/2}. \qquad (16)$$

Values of the wing loading w can be deduced from Appendix I, C_{DO} from Appendix V and K from Appendix IV.

b, m.	C_{Lmax}		
	Poor	Medium	High
10	12.12	14.79	17.79
14	13.43	16.64	20.35
18	14.34	17.73	22.00

APPENDIX VII

Aspect Ratios for Which $C_{LMB} = 0.9\ C_{Lmax}$
Reducing the aspect ratio reduces C_{LMB}.

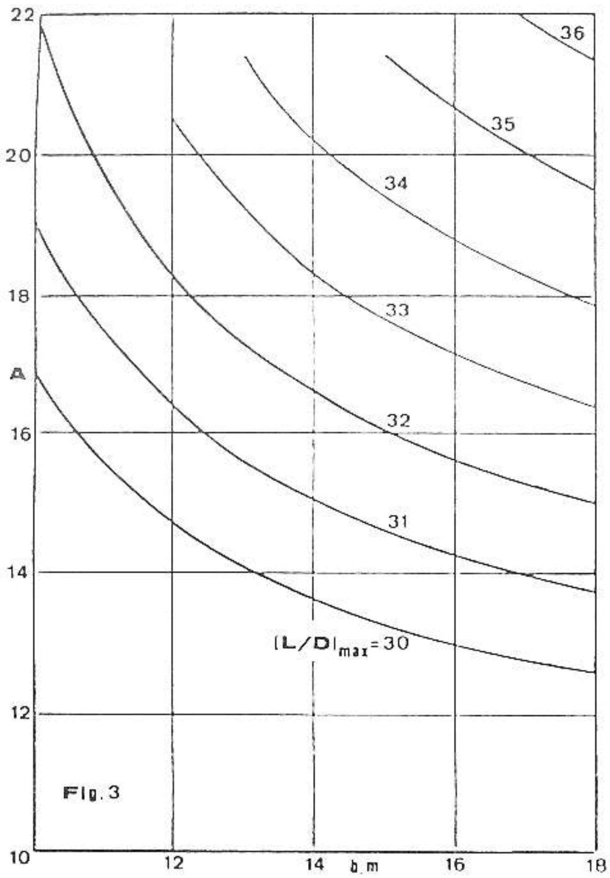

FIGURE 3. Maximum Lift/Drag ratio as a function of span and aspect ratio.

The minimum rate of sink can therefore be found as a function of span and aspect ratio for the three categories of structure weight. By interpolation, the aspect ratio corresponding to V_{smin}=0.75 m/s can be found for various spans for each type of structure weight. The results are presented in Appendix VIII.

b, m.	STRUCTURE		
	Heavy	Medium	Light
10	------	------	23.18
12	20.00	14.28	8.50
16	10.97	------	------

APPENDIX VIII

Aspect Ratios for Which V_{smin} = 0.75 m/s

Blank spaces indicate values outside the range under consideration. Spans above 16 m . lead to aspect ratios below 10.

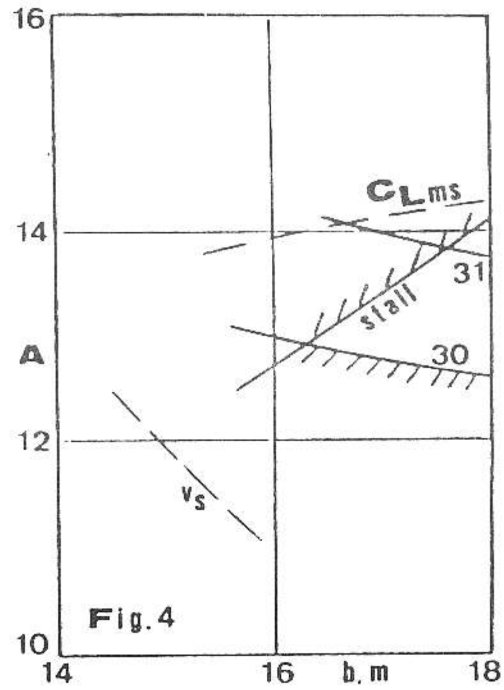

FIGURE 4. Boundaries for sailplanes with heavy structure, poor C_{Lmax}.

Overall boundaries

The range of values of span and aspect ratio considered in the preceding calculations are fairly arbitrary. The upper limits were based on (a) a feeling that a span of more than 18m would hardly be in accordance with the concept of the Class and (b) a feeling that aspect ratios exceeding 22 were unlikely if the structure was to be reasonably inexpensive. The lower span limit of 10 m turned out to be just below the most favorable boundaries.

There are nine possible combinations of C_{Lmax} and structure weight, and sufficient information is given in the appendices to enable all of them to be plotted. Four examples are considered in detail below

(a) Heavy structure, poor maximum lift coefficient. The boundaries are plotted on (b,A) axes, only the stall and maximum L/D lines being relevant (see Fig. 4). The region of viable sailplanes is very small since the span must be greater than about 16.5 m and the aspect ratio must be quite low. Values of max / L/D exceeding 31 are unlikely. Moral: the structure and the aerodynamics cannot bee too crude, or the sailplane will not be feasible.

(b) Light structure weight, high maximum lift coefficient. Here, the relevant boundaries are maximum L/D, the stall and the lift coefficient at minimum sink. The minimum sink itself is not a boundary. Even if it were, due perhaps to non-linearities, it would only affect the extreme left-hand side of the diagram . The extreme left-hand corner corresponds to

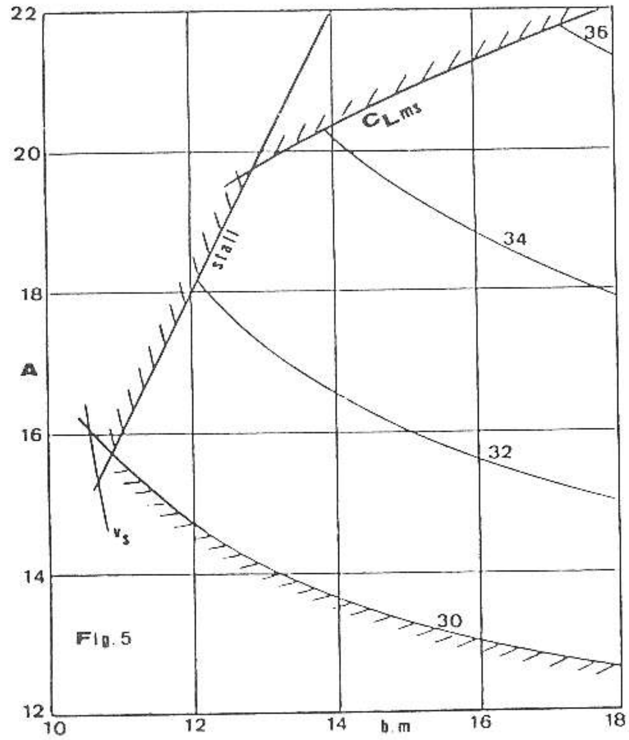

FIGURE 5. Boundaries for sailplanes with light structure, high C_{Lmax}.

a span of 10.85 m and an aspect ratio of 15.7. The maximum all-up mass would be 217 kg, so the empty mass would be only 89 kg. This all seems rather unlikely: a very refined structure would be required and, at this extreme limit, there is no allowance for errors. With greater spans, there is much more freedom of maneuver. For example, at 15 m, viable sailplanes have aspect ratios between 13.3 and 20.8. Obviously, the greater the span and the higher the aspect ratio, the better is the maximum L/D. For a given span, the higher aspect ratios give lower total masses but higher wing loadings. The wing loading cannot exceed that corresponding to the stalling boundary (28.52 kg/m^2, or 5.84 lb/ft^2). This is quite a modest figure so, again, the higher aspect ratios would be desirable. At a span of 15 m, the upper limit to the aspect ratio is 20.85 on the C_{LMS} boundary. The corresponding total mass is 256 kg (empty mass=128 kg or exactly half the total) and the wing loading is 23.72 kg/m^2. It is worth noting that such a sailplane is not much different from the earlier versions of the Standard Libelle: the latter had an aspect ratio of 23 and the total mass was 34 kg greater, some of which could be debited to a retracting wheel and low structural stresses. The maximum L/D was claimed to be 38, while for the hypothetical World Class sailplane it would be about 34.7. Taking into account the fixed wheel and the lower aspect ratio, the comparison is close enough to lend credence to the present calculations.

(c) Medium structure weight, high maximum lift coefficient. In Fig. 6, the heavier structure moves the stalling line

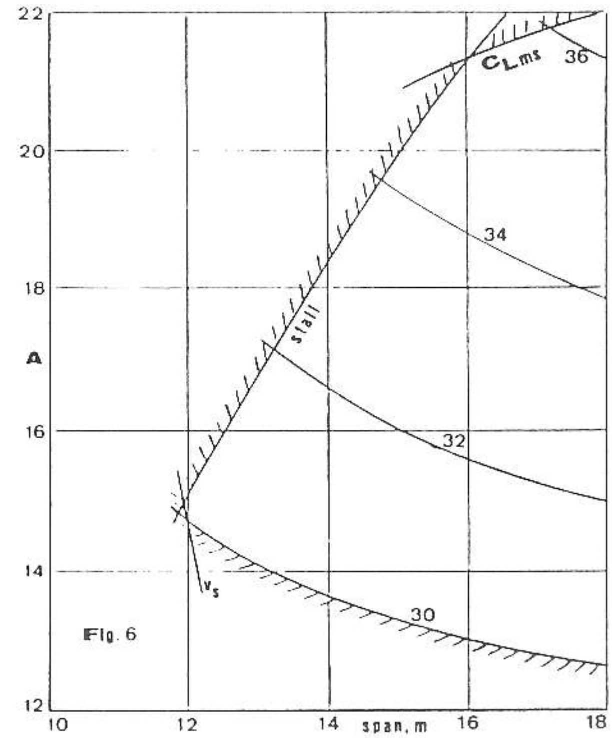

FIGURE 6. Boundaries for sailplanes with medium structure, high C_{Lmax}.

in the direction of greater spans and lower aspect ratios. The minimum sink boundary just appears at the left-hand corner of the diagram, where the minimum sailplane would have a span of 12 m and an aspect ratio of about 15. A 15 m span would permit an aspect ratio of 19.9, a maximum L/D of 34.3, a total mass of 276 kg (empty mass = 148 kg) and a wing loading of 28.52 kg/m^2. The relevant boundary is the stall. It may be felt that the loss in performance compared with the previous case is a small price to pay for the convenience of having another 20 kg available for the structure.

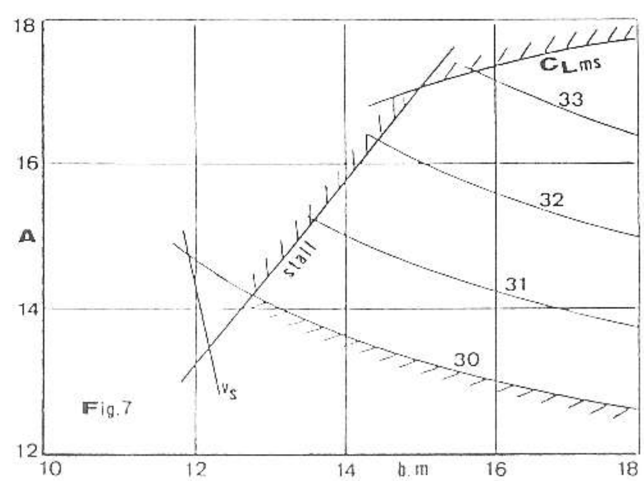

FIGURE 7. Boundaries for sailplanes with medium structure, medium C_{Lmax}.

(d)Medium structure weight, medium maximum lift coefficient. As Fig. 7 shows, the reduced maximum lift coefficient has an appreciable effect on both the stall and C_{LMS} boundaries, compared with those of Fig. 6. The left-hand side of the diagram corresponds to a span of 12.8 m and an aspect ratio of 14.2. At spans above 15 m, the C_{LMS} boundary has a very restricting effect on aspect ratio and hence on maximum L/D. At 15 m span, the maximum aspect ratio is 17, giving a maximum L/D of 32.7. The total mass would be 337 kg (empty mass = 209 kg).

Conclusions

1. As structures are made lighter and as C_{Lmax} improves, smaller spans become feasible. It would, however, be imprudent to design a minimum-span sailplane since small discrepancies relative to the above assumptions would place it outside the boundaries. The smallest feasible span is probably 12 m (with a light structure and a high C_{Lmax}), rising to 13 or 14 m as the structure gets heavier and C_{Lmax} worsens.

2. As the span is increased, higher aspect ratios and better values of maximum L/D become possible. There seems to be little point in trying to achieve very small spans. However, with a high C_{Lmax}, the C_{LMB} boundary becomes relevant at the higher spans, thus reducing the advantage of increasing span.

3. For any given span, it pays to use the highest available aspect ratio, as limited by either the stall boundary or the C_{LMB} boundary.

4. Airbrakes which do not reduce the maximum lift coefficient when open would be highly advantageous.

5. Achieving a good C_{Lmax} is much more important than attaining the lightest structure (see Figs. 6 and 7).

All of the above calculations are, of course, pretty approximate. Quoting masses within 1 kg and aspect ratios to two places of decimals lends a rather spurious air of accuracy. The numerical values in the various Figures should not be taken too literally but it seems likely that the general shape of the boundaries is correct and can provide at least first-order guidance to likely configurations. It also seems likely that the absence of a span limitiation could lead to future problems.

LIST OF SYMBOLS

A	Aspect Ratio
b	Wing Span
$\bar{c}$	Geometric Mean Chord
$\bar{\bar{c}}$	Mean Aerodynamic Chord
C_{DO}	Zero-lift Drag Coefficient
C_{DOw}	Minimum profile drag coefficient of the wing section.
C_{Dp}	Wing section profile drag coefficient
C_L	Lift Coefficient
C_{Lmax}	Maximum lift coefficient of the sailplane
C_{LWmax}	Maximum lift coefficient of the aircraft-less-tail
C_{LMS}	Lift coefficient at minimum sink
C_{MO}	Pitching moment coefficient of the aircraft-less-tail
D	Drag
g	Acceleration due to gravity
h	Dimensionless CG position
h_o	Dimensionless position of the aerodynamic center of the aircraft-less-tail
k	Induced drag factor
k_v	Vortex contribution to k
k_p	Contribution to k due to the variation of CDp with CL
K	A function of aspect ratio(see equation 13)
K_E	See equation 2
K_S	See equations 6 and 7
l_t	Tail moment arm: distance between the aerodynamic center of the aircraft-less-tail and the a.c. of the tail
L	Lift
m	Mass of the sailplane
n	Ultimate load factor
R_∞	Reynolds number
S	Wing area
V_{smax}	Maximum permitted stalling speed, EAS
V_{smin}	Minimum rate of sink
w	Wing loading (N/m^2 in calculations, kg/m^2 in the text)
W_E	Empty mass of the sailplane
ρ_o	Standard sea-level air density

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