ON THE FINE STRUCTURE OF ATMOSPHERIC TURBULENCE MEASUREMENT, PILOTING, FATIGUE TESTS

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SUMMARY

Exact assessment and correct simulation of atmospheric turbulence is of prime importance for soaring. Raw power spectra of turbulence should be processed using natural-parameter spectrum formula and evaluating phase angle relationships, too. This makes it possible to calculate turbulence-induced sailplane loads and motions using input spectral vectors instead of the usual spectral matrices. Records of thermal traverses are best modelled as regular nonstationary processes. Exploitation possibilities of these new developments in soaring include dynamic thermaling and a more realistic fatigue load simulation. Boundary layer research, too, may benefit from it.

1. INTRODUCTION

Flow turbulence is in every respect of prime importance in gliding. We have threefold reasons to know as much of it as possible:

— every kind of updraft, our prime source of energy, is strongly connected with air turbulence;

— unwanted turbulence in the sailplane boundary layer diminishes the performance;

— the major part of glider fatigue loads is caused by atmospheric turbulence.

One of these problems would be quite enough to justify our interest. Successes and failures in the past emphasize the leading role of basic research in the efficient handling of turbulence problems. This is the reason why, according to the humble opinion of the author, some recent developments in the theory of stochastic processes deserve our due consideration. But, theory alone cannot give us the practical results required, so basic and applied research should both be carried out.

Innovations in theory should start with writing spectrum and correlation functions in terms of natural parameters. They have been used in turbulence work for several decades (see e.g. refs. 4, 10, 11) but full utilization of their possibilities is still lacking. Furthermore, amending the individual or sample spectra with phase angle data gives us a very efficient tool for assessment and simulation of atmospheric turbulence. Adding the phase angle to the present-day atmospheric turbulence measurement/assessment methods promises to give quite a new picture of the fine structure of convection. The next step toward the full understanding of atmospheric dynamics may be then the investigation of the so-called regular nonstationary type of stochastic processes.

When adapting these new theoretical relationships to practice, we shall be able to design, dimension and fly our machines for better advantage in turbulent environments.

2. NATURAL-PARAMETER SPECTRUM FUNCTIONS

Intelligent and rational stochastic measurement and calculation developments come invariably to the adoption of the spectral method. Raw power spectra — while containing in principle all the necessary information — are poorly suited to exact and economical additional processing and calculation. It is worth, therefore, calculating a least-squares approximation to them using an appropriate analytical function. There are several types giving acceptable formal fit to the spectra over the measured frequency range, but it will pay to select a special group.

Turbulence theory inherited the concept of "Mischungsweg" or mixing length from thermodynamics. Splendid work of Prandtl, Dryden, Taylor, Karman and others evolved from it two sorts of the so-called scale length. The first one is the integral scale and is defined as follows:

$$L = \lim_{\zeta_1 \to \infty} \int_0^{\zeta_1} R_w(\zeta)\, d\zeta \qquad /1/$$

The second one, Taylor's scale, reads:

$$\lambda = \frac{\sqrt{2}\ \sigma_w}{\left[-\left(\frac{d^2 R_w(\zeta)}{d\zeta^2} \right)_{\zeta = 0} \right]^{1/2}} \qquad /2/$$

The integral scale of turbulence L is used in every turbulence formula by tradition. Its general importance in the theory of stationary stochastic processes has been pointed out by Kovasznay/(3) pp. 91-94./. Taylor's scale length values above zero are indicating a finite frequency upper bound for the spectrum.

The full list of the so-called natural parameters of stochastic processes starts with the well-known standard deviation σ_w followed by the scale parameters L and λ ending with the spectrum exponent α. Definitions and calculation formula are shown on Table 1.

Natural-parameter turbulence spectrum equations are of the general form

$$G_w(\Omega) = G_w(0) \cdot \Phi(L\Omega, L/\lambda, \alpha) \qquad /3a/$$

It seems that for atmospheric turbulence $L/\lambda >> 1$ or perhaps even $\lambda/L \to 0$ so we can work with

$$G_w(\Omega) = G_w(0) \cdot \Phi(L\Omega, \alpha) \qquad /3b/$$

Zero values for such one-sided spectra are according to Kovasznay (3):

$$G_w(0) = \frac{2L}{\pi} \sigma_w^2 \qquad /4/$$

The first formula for the vertical component of air turbulence has been given by Dryden (2):

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1 + 3L^2\Omega^2}{(1 + L^2\Omega^2)^2} \qquad /5/$$

In 1948 Karman (lo) revised this formula giving:

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1 + \frac{8}{3}(1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}} \qquad /6/$$

According to some authors, low-altitude turbulence has a special character necessitating the use of special formula. Lappe (11) proposed for it the spectrum equation:

$$G_w(\Omega) = \sigma_w^2 \frac{L}{(1 + L\Omega)^2} \qquad /7/$$

The Lappe spectrum has been modified at Lockheed-Georgia/Firebaugh (4)/ into:

$$G_w(\Omega = \sigma_w^2 \frac{0.8L}{(1 + 0.8L\Omega)^{1.8}} \qquad /8/$$

There is a little problem concerning the constants in Eqs. (7) and (8) because they do not meet all theoretical requirements. This induced the author to propose instead of Eq. (8) the following/Gedeon (5)/:

$$G_w(\Omega) = \frac{2}{\pi}\sigma_w^2 \frac{L}{(1 + \frac{12}{5\pi} L\Omega)^{11/6}} \quad /9/$$

The peculiarity of these formula of being written in terms of the natural parameters gives us several practical advantages. It facilitates the calculation of frequency cutoff errors, spece-frequency domain conversion, etc. /See e.g. Gedeon (5,6)/ Let us go now a step further and examine the problem of phase angle relationships.

3. TURBULENCE FIELD MEASUREMENT AND SIMULATION

Atmospheric turbulence is acting not on one single point, but on the whole surface of our sailplanes. Scalar-type functions as given in Eqs. /1/-/9/ are insufficient for calculating such input-output relationships. It is, therefore, customary to set up a discrete element model of the plane and to describe the turbulence field by the corresponding n × n dimension spectral matrix

$$\underline{\underline{G}}_{ww}(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) \dots & G_{1n}(f) \\ G_{21}(f) & G_{22}(f) \dots & G_{2n}(f) \\ \dots & \dots & \dots \\ G_{n1}(f) & G_{n2}(f) \dots & G_{nn}(f) \end{bmatrix} \quad /10/$$

Let us think and speak in terms of a sailplane flying at an air speed V through turbulence. For dynamic calculations we have to compose a discrete element vortex/mass (aerodynamic/mechanical) model of the plane/see e.g. Fig. 1 and Mai (12)/.

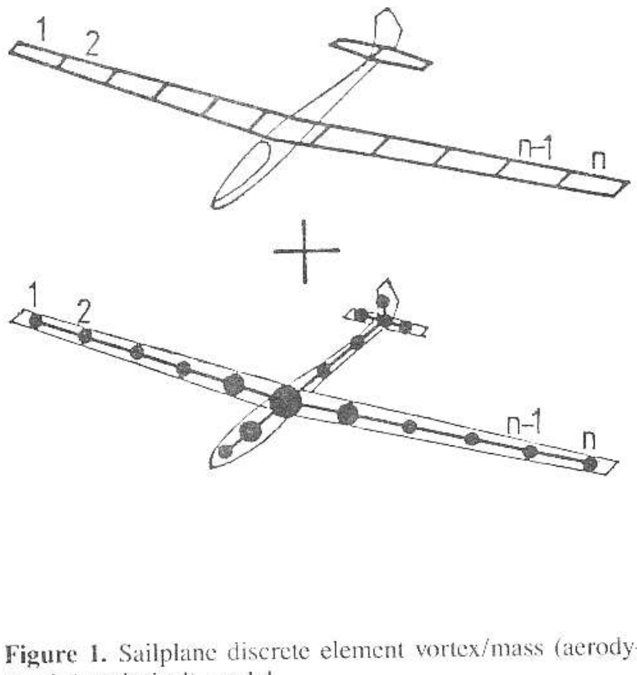

Figure 1. Sailplane discrete element vortex/mass (aerodynamic/mechnical) model.

The diagonal auto-spectra elements $G_{ii}(f)$ in matrix /10/ can be calculated from one of the space-domain spectra /5/-/9/ in the following way. First, we have to calculate the time scale/see e.g. Gedeon (5,6)/as:

$$T = \frac{L}{V} \quad /11/$$

The zero value of spectra $G_w(f)$ is /Gedeon (6)/:

$$G_w(0) = 4T\sigma_w^2 \quad /12/$$

and for Eq. /3b/ we should write:

$$G_w(f) = G_w(0) \cdot \Phi(2\pi Tf, \alpha) \quad /13/$$

So the time-domain variant of the Karman spectrum is:

$$G_{ii}(f) = G_w(f) = 4T\sigma_w^2 \frac{1 + \frac{8}{3}(8.4132\ Tf)^2}{[1 + (8.4132\ Tf)^2]^{11/6}} \quad /14/$$

Calculation of the nondiagonal cross-spectra $G_{ik}(f)$ of the matrix is not so clear. They can be determined from the corresponding auto-spectra using the appropriate coherence functions, but the present stochastic process theory does not give theoretical coherence values, so individual empirical values are used.

For turbulence air load simulation n input time functions $w_i(t)$ /i=1÷n/ have to be generated from the matrix $\underline{\underline{G}}_{ww}(f)$ by inverse Fourier transformation. While the calculation procedure for the absolute values of the Fourier components is straightforward, there is no text-book formula for Fourier component phase angle calculation. It is, therefore, usual to assign random phase angle values to the calculated absolute values of the Fourier components. Although statistically not a big source of errors, this is theoretically not correct.

Input-output relationships can be written in the classical way as

$$\underline{\underline{G}}_{yy}(f) = \underline{\underline{H}}^*_{yw}(f)\ \underline{\underline{G}}_{ww}(f)\ \underline{\underline{H}}^T_{yw}(f) \quad /15/$$

We can improve on both these relationships in the following way. Fourier series theory indicates that phase angle values for every finite-length representation have to increase in proportion to the frequency. Using this theorem

— we can give more realistic phase angle values:

— we can discard the spectral matrix form of representation and can switch over to the complex auto-spectrum vector $\underline{G}_w(f)$ (for details see Appendix 2).

In this concept we can write for Eq. /15/:

$$[\underline{G}_y(f)]^{1/2} = \underline{\underline{H}}_{yw}(f)\ [\underline{G}_w(f)]^{1/2} \quad /16/$$

Summing up briefly, the essence of our method is the following. If there are n pieces of finite length samples $w_i(t)$ from a stationary and ergodic stochastic process/ see Fig. 2/ then finite base length individual auto-spectra do have phase angle values as well, so they can be expressed in the form:

$$G_w(f)\ e^{-j\varphi_i(f)} \qquad /17/$$

The classical auto-spectrum $G_w(f)$ — i.e. a spectrum $G_{ii}(f)$ in the diagonal of the spectral matrix $\underline{\underline{G}}_{ww}(f)$ — is a real type ensemble spectrum of the whole process $\{w(t)\}$. The complex auto-spectrum vector $\underline{G}_w(f)$ composed from n complex auto-spectra contains all information on the individual and relative phase angle values as well. It is a full substitute for the spectral matrix $\underline{\underline{G}}_{ww}(f)$ and it makes possible the correct reconstruction of individual time histories $w_i(t)$, too. substitution of Eq. /16/ for Eq. /15/ yields significant computer time and space savings.

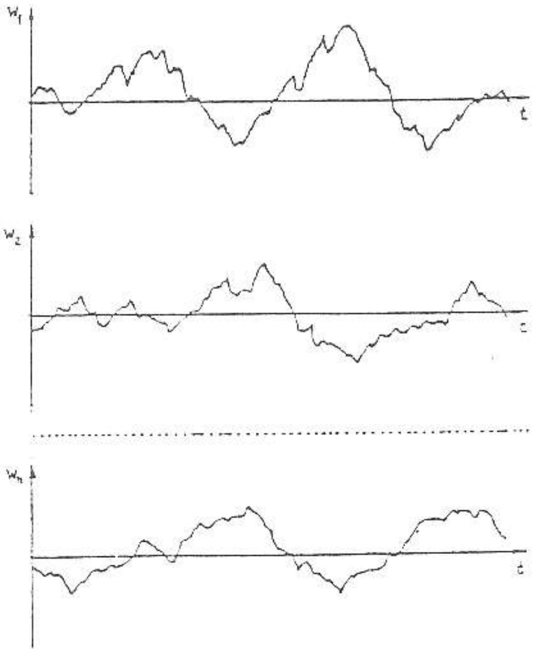

Figure 2. Sample time-histories

4. TRANSIENT TURBULENCE

The stationary stochastic process model of turbulence is correct only when flying in steady turbulence, e.g. in thermaling. Homogeneous turbulence field lengths in the order of 5L are necessary to justify such an assumption. In straight thermal traverses, the magnitude of turbulence is transient. Nevertheless, it is customary to treat even long mixed records as if they were stationary inasmuch as bulk processing methods are applied. Results of such processing methods can be interpreted as some form of extended range statistics, but a more correct method would be the following.

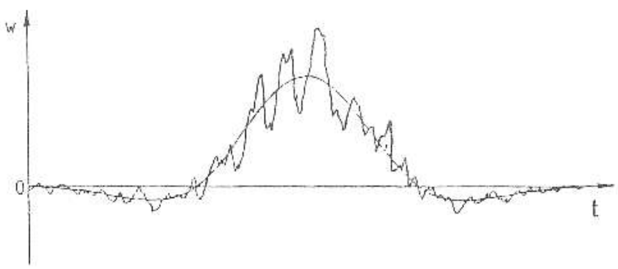

Figure 3. Vertical velocity record of a thermal traverse.

Let us take e.g. a record from a traverse trough a thermal core and its near surroundings /see Fig. 3/. This part of the record $w=w(t)$ is obviously nonstationary, even after subtracting the updraft velocity profile from it. But, the transient turbulence can be regarded as a case of the so-called regular nonstationary processes. That means that while the time function $w=w(t)$ is nonstationary, there is a transformation function $g(t)$ making the product

$$g(t) \cdot w(t)$$

stationary /Gedeon (7)/. Using the complex sample spectrum concept and an appropriate weighing function $g(t)$ full record assessment can be achieved by otherwise standard procedures. Even reconstruction of the original or equivalent time functions $w(t)$ from the complex sample spectrum $G_w(f)$ for simulation is possible.

Data collection is the most difficult and problematical part of air turbulence investigations. An ideal solution would be the simultaneous sampling over short intervals Δt over say an $n \times n \times n$ matrix cube mesh of uniform h division. What we have at present is a formation of three motorgliders recording continuous time histories see e.g. Hutter (8)/ from which sampling in time intervals Δt will give a spatial sampling division

$$h = \Delta t \cdot V\ [m] \qquad /18/$$

How and to what extent such isolated traverses can be substituted for a matrix data set is an interesting problem beyond the scope of the present paper.

5. SEMI-DYNAMIC THERMALING

The knowledge which will be gained on the fine structure of atmospheric turbulence could be put to practical use as follows. First of all, we want to extract as much energy as possible from the atmosphere for staying up and making way. Would it be possible to utilize atmospheric turbulence for dynamic soaring? There are quite a number of treatises on the dynamic energy exchange between the sailplane and a non-steady atmosphere but full-scale utilization, except for dolphin-style thermal traverses, is still lacking. In view of the wind shear rates necessary to support the permanent flying of a sailplane there is little hope of ever attaining this ideal. But we can perhaps, improve rate of climb values in thermal circling by utilizing turbulence components.

Recently Sto'kovic (13) recomı ·nded doing a series of alternate dynamic beats through the thermal core. He did his calculations on an exponential updraft profile /Fig. 3, mean line/ without taking turbulence components into account.

Instead of this, the author prefers another method. While thermaling, most pilots fly through turbulence components in the 0.5-2 Hz frequency / 10-50 m wavelength/ range. When we shall know more about phase angle relations in turbulence, it will be possible to do some intelligent aileron and elevator/rudder dynamic fine work in circling for a little extra rate of climb. The author did some instinctive practical experimenting in this way. In weak evening conditions a difference in the order of 0.5 m/s seemed to be attainable. Maybe it is worthwhile making the effort for such an improvement.

6. FATIGUE TEST LOADS

A substantial proportion of glider fatigue loads is caused by atmospheric turbulence. Simulating them for fatigue tests is based either on service load statistics or on spectral calculations. Let us investigate the latter case.

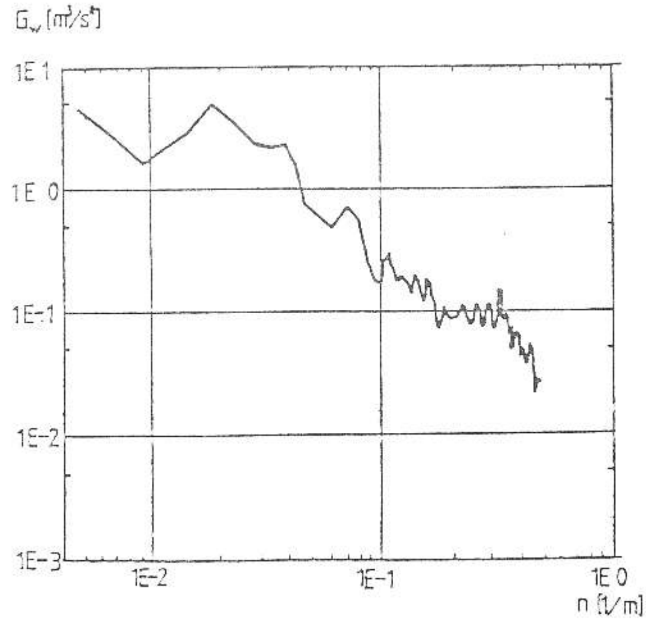

Figure 4. Atmospheric turbulence spectrum /Jochum (9)/

It is practical to divide the service life of the glider into characteristic flight modes. Each of them has its own block in the fatigue test load program. Some suggestions on input spectrum parameter determination can be found e.g. in Ref. (6). Starting points for them are raw air turbulence spectra as shown e.g. on Fig. 4. They are to be processed using one of the standard formulas, e.g. Eq. /6/ or Eq. /9/. Which one of them is to be preferred is a question of further statistical investigations. No matter how, phase angle data are to be preserved during the processing. The aerodynamic/mechanical model on Fig. 1 is also the pattern for the aerodynamic load vector function $\underline{F}(t)$ calculation.

The transfer matrix $\underline{\underline{H}}_{Fw}(f)$ necessary for this can be calculated e.g. by the Low Frequency Aeroelastic Element Method developed by Mai (12). Current fatigue load simulation methods are rudimentary in two ways. In calculations it is usual to presume constant input vector component phase angle values all along the wing span, i.e. to assume orthotropic turbulence instead of isotropic. Moreover, loading in our fatigue rigs is by a single hydraulic linear actuator through a mechanical lever system. This practically precludes any possibility of phase differences in loading.

A uniform spanwise loading may cause overloading in some respect in proportion to the service loads, but on the other hand, all the antisymmetric load components are lacking. In possession of the complex sample spectra in the form of Eq. /17/ this can be corrected. According to Eq. /16/ the complex loading spectral vector is:

$$[\underline{G}_F(f)]^{1/2} = \underline{\underline{H}}_{Fw}(f)\,[\underline{G}_w(f)]^{1/2} \qquad /19/$$

Load time histories can be composed by the formula

$$F_i(t) = \sum_{k=1}^{m} [|G_{Fi}(f)|\,\Delta f]^{1/2}\, e^{j\,[2\pi k\, f_1\, t + \varphi_i(f)]} \qquad /20/$$

Full simulation of the local load fluctuations for each wing/tailplane section requires a separate load actuator for every section and a corresponding control system. How far this can be simplified within acceptable error margins is still an open question, but surely worthy of serious consideration.

7. LAMINAR FLOW CONTROL

Last but not least, the new concepts could be applied to the classical boundary layer and laminar profile problems as well. Their consequent use in the assessment and evaluation of multiple simultaneous hot-wire anemometer and laser Doppler anemometer records may contribute to the discovery of new concepts in flow structure interpretation. Improvements in sailplane design as well as in flow mechanics in general would be the net result of this. At present, we cannot predict for sure the practical improvements, but a better understanding of the laws of nature has always given ample dividends.

8. CONCLUSIONS

It is advisable to use natural-parameter spectra for atmospheric turbulence description and simulation. Contrary to the prevalent belief, sample autospectra do have non-zero phase angle values as function of the frequency. Individual sample spectra should be written, therefore, in a complex form. This simplifies considerably the calculation and simulation of turbulence-induced sailplane motion and of turbulence fatigue loads. Transient turbulence can be modelled as a special case of regular nonstationary stochastic processes. Practical use of these new theses is in dynamic thermaling, in the compilation of fatigue test programs and maybe in laminar flow research, too.

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Tab.1: Natural parameters of stationary stochastic processes

Parameter:	Calculation formula using $w(\xi)/\bar{w} = O/$	autocorrelation func. $R_w(\zeta)$	spectral density func. $G_w(n)$ resp. $G_w(\Omega)$
Standard deviation σ_w	Def.: $\sigma_w = \lim_{S\to\infty}\left[\frac{1}{S}\int_O^S w^2(\xi)\,d\xi\right]^{1/2}$	———	$\sigma_w^2 = \int_O^\infty G_w(n)\,dn = \int_O^\infty G_w(\Omega)\,d\Omega$
Scale parameter /integral/ L	———	Def.: $L = \lim_{\zeta_1\to\infty}\frac{1}{\sigma_w^2}\int_O^{\zeta_1} R_w(\zeta)\,d\zeta$	Regression analysis
Taylor's scale parameter λ	———	Def.: $\lambda = \frac{\sqrt{2}\,\sigma_w}{\left[-\left(\frac{d^2R_w(\zeta)}{d\zeta^2}\right)\zeta = O\right]^{1/2}}$	$\lambda = \frac{\sigma_w}{\sqrt{2\pi}}\left[\int_O^\infty n^2 G_w(n)\,dn\right]^{-1/2}$ $\lambda = \sqrt{2}\,\sigma_w\left[\int_O^\infty \Omega^2 G_w(\Omega)\,d\Omega\right]^{-1/2}$
Exponent α	———	———	Regression analysis

$R_w(O) = \sigma_w^2$

$G_w(n)$: $G_w(O) = 4L\sigma_w^2 n_{max} = \frac{1}{\lambda}$

$G_w(\Omega)$: $G_w(O) = \frac{2}{\pi} L\sigma_w^2 \Omega_{max} = \frac{2\pi}{\lambda}$

Appendix 1: Notation

f	frequency	1/s
g ( )	function	
h	interval between samples	m
n	space frequency	1/m
t	time	s
Δt	time interval	s
w	vertical component of turbulence velocity	m/s
y	induced time variable	
F	force /air load/	N
G ( )	power spectral density function	
$\underline{G}$ ( )	complex spectral vector	
$\underline{\underline{G}}$ ( )	spectral matrix	
$\underline{\underline{H}}$ ( )	transfer matrix	
L	integral scale parameter	m
R ( )	autocorrelation function	
S	sample length	m
T	time	s
T	time base	s
V	air speed	m/s
α	exponent	
φ	phase angle	
λ	Taylor's scale length	m
σ	standard deviation	
τ	time lag	
ξ	space coordinate parallel to flight speed	m
ζ	space lag	
ω	circular frequency	rad/s
$\omega_1 = 2\pi f_1$	base frequency	rad/s
Φ	shape function	
Ω	space circular frequency	rad/m

Superscripts:

T	transpose
*	complex conjugate

Appendix 2: Proof of the Spectral Vector Concept

We state that n pieces of finite-length /O $\leq$ t $\leq$ T/ stationary and ergidic time functions $w_i(t)$ /i=1÷n/ can be fully described and represented by the complexspectral vector $\underline{G}_w(f)$ instead of the traditional spectral matrix $\underline{\underline{G}}_{ww}(f)$.

Let us take an n degree of freedom aerodynamic/mechanical model of the glider /Fig. 1/. By use of the direct Fourier calculation method the complex spectrum at the i-th degree of freedom point reads:

$$G_{wi}(f) = G_{Re,i}(f) + j^{G}{}_{Im,i}(f) = |G_w(f)| \, [\cos \varphi_i(f) + j \sin \varphi_i(f)] \qquad /22/$$

Then

$$[G_{wi}(f)]^{1/2} = [|G_w(f)|]^{1/2} \, [\cos \varphi_i(f) + j \sin \varphi_i(f)] \qquad /23/$$

and the input spectral vector in Eq. /19/ reads:

$$[\underline{G}_w(f)]^{1/2} = \begin{bmatrix} [G_1(f)]^{1/2} \\ [G_2(f)]^{1/2} \\ \cdots\cdots \\ [G_n(f)]^{1/2} \end{bmatrix} = [|G_w(f)|]^{1/2} \begin{bmatrix} \cos \varphi_1 + j \sin \varphi_1 \\ \cos \varphi_2 + j \sin \varphi_2 \\ \cdots\cdots \\ \cos \varphi_n + j \sin \varphi_n \end{bmatrix} \qquad /24/$$

The spectral matrix is a dyadic product:

$$\underline{\underline{G}}_{ww}(f) = [\underline{G}_w^*(f)]^{1/2} \, [\underline{G}_w^T(f)]^{1/2} \qquad /25/$$

The diagonal elements of the matrix read:

$$G_{ii}(f) = |G_w(f)| = G_w(f) \qquad /26/$$

and the general formula for the elements reads:

$$G_{ik}(f) = |G_w(f)| \, [\cos (\varphi_i(f) - \varphi_k(f)) + j \sin (\varphi_i(f) - \varphi_k(f))] \qquad /27/$$

The spectral matrix is obviously hermitic:

$$G_{ki}(f) = G_{ik}^*(f) \qquad /28/$$