Energy Changes of a Sailplane in Moving Air

Herbert Winter
Eberhardstr. 11, 71679 Asperg, Germany
Herbert.Winter@t-online.de

Abstract

The energy state of a sailplane is described in terms of its total energy, i.e. the sum of potential and kinetic energy. Total energy (TE) may be defined either in an air-fixed reference frame (TE_AIR) or in an earth-fixed reference frame (TE_EARTH). Both definitions are of importance here. The fundamental physical laws are deduced here and, depending on the actual application, one will opt for one or the other definition. In a practical example, the optimum flight path of a straight-line flight through an ascending air current (updraft) is computed and discussed. There are two types of mechanism that allow an energy gain of the sailplane: static soaring in updrafts, and dynamic soaring with changes of wind speed. Even though during "normal" flights updrafts are the primary source of energy, the exploitation of dynamic effects is likely to make a noticeable improvement of performance. In soaring practice, however, dynamic effects are seldom used in order to gain energy these days, and the main reason for this seems to be the absence of an appropriate measuring instrument. Therefore, some aspects of the question of how an "ideal variometer" should be designed are briefly discussed. It turns out that the creation of an ideal variometer also would open up the possibility to measure the magnitude of dynamic effects.

Nomenclature

$\vec{a}_a$	apparent acceleration, $\vec{a} = d\vec{u}/dt - \vec{g}$
b	maximum updraft gradient in updraft model Eq. (4.1)
$\vec{D}$	drag force
E_a	total energy in an air-fixed reference frame (TE_AIR)
e_a	specific total energy in an air-fixed reference frame (TE_Air)
E_g	total energy in an earth-fixed reference frame (TE_EARTH)
e_g	specific total energy in an earth-fixed reference frame (TE_Earth)
$\vec{g}$	acceleration due to gravity
h	height above ground
$\vec{I}$	inertial force in an air-fixed reference frame
$\vec{L}$	lift force
M	mass of aircraft
n	load factor
p_s	static pressure
R	radius of updraft in updraft model Eq. (4.1)
q	dynamic pressure ($q = ½ \rho v^2$)
$\vec{u}$	velocity of aircraft with respect to the ground
u	ground speed
u_{MC}	MacCready value (expected climb rate in the next updraft)
$\vec{v}$	velocity of aircraft with respect to the air
v	true airspeed
v_s	intrinsic sink rate of the sailplane, caused by drag
V_{ideal}	indicated value of ideal variometer (same for other variometer types)
$\vec{w}$	velocity of wind (with respect to the earth)
w^a_{dyn}	dynamic energy component in an air-fixed reference frame
w^g_{dyn}	dynamic energy component in an earth-fixed reference frame
w_0	intensity of updraft in updraft model Eq. (4.1)
w_h	horizontal component of wind velocity
w_v	vertical component of wind velocity
$\vec{W}$	weight of aircraft
α_x	angle of attack (related to the aircraft longitudinal axis)
γ	flight path angle (between $\vec{v}$ and the horizontal)
θ	pitch angle
ρ	density of air

Definition of total energy

In still air, the sailplane cannot gain energy and, because of the drag, only gliding flight is possible. We shall, therefore, immediately look at the movement of an aircraft in moving air (Fig. 1). If the aircraft moves at a velocity of $\vec{v}$ with respect to the air, and $\vec{w}$ is the wind velocity, an observer on the stationary ground will notice the superposition of these velocities:

$$\vec{u} = \vec{v} + \vec{w} \tag{1.1}$$

The aircraft's energy is characterized by its total energy which is composed of the potential energy (altitude) and the kinetic energy (speed). Two definitions are possible:

- Total energy in an air-fixed reference frame (TE_AIR):

$$E_a = M g h + \frac{1}{2} M v^2 \tag{1.2}$$

- Total energy in an earth-fixed reference frame (TE_EARTH):

$$E_g = M g h + \frac{1}{2} M u^2 \quad (1.3)$$

The designations "TE_AIR" and "TE_EARTH" were introduced here in order to distinguish between these two definitions. If the term "total energy" were applied to describe both models, this would lead to misunderstanding and confusion.

Specific total energy

TE_AIR and TE_EARTH are often calibrated to the weight W = M g. This results in the following definitions:

- Specific total energy in an air-fixed reference frame (TE_Air):

$$e_a = \frac{E_a}{M g} = h + \frac{v^2}{2g} \quad (1.4)$$

- Specific total energy in an earth-fixed reference frame (TE_Earth):

$$e_g = \frac{E_g}{M g} = h + \frac{u^2}{2g} \quad (1.5)$$

The term "energy height" also is used commonly as a synonym for "specific total energy".

The term "total energy" became common among glider pilots with the introduction of the "total energy variometer" which measures changes of TE_Air. When the flight speed is changed, potential and kinetic energy are converted into one another, whereas the total energy remains constant (in still air and if one neglects the energy loss due to drag).

Example

A surplus in airspeed shall be converted into altitude in still air. If the aircraft mass is M = 400 kg, and the airspeed is reduced from v_0 = 35 m/s to v_1 = 25 m/s, the kinetic energy changes by

$$\Delta E_{kin}^{a} = \frac{1}{2} M \left(v_1^2 - v_0^2\right) = -120 \text{ kJ}.$$

The potential energy rises by the same amount (neglecting the drag); the gain of altitude is

$$\Delta h = \frac{\Delta E_{pot}}{M g} = -\frac{\Delta E_{kin}^{a}}{M g} = -\frac{v_1^2 - v_0^2}{2g} = 30.6 \text{ m}.$$

If the same flight maneuver is carried out with a constant tailwind of w = 15 m/s, nothing will change from the viewpoint of an air-related observer, because the evenly flowing air-stream may be regarded as an inertial system as well. However, an earth-fixed observer will measure the initial speed u_0 = 50 m/s and the final speed u_1 = 40 m/s; for them the change of kinetic energy is

$$\Delta E_{kin}^{g} = \frac{1}{2} M \left(u_1^2 - u_0^2\right) = -180 \text{ kJ}.$$

The resultant gain of altitude is the same in both instances: Δh = 30.6 m. The question of how the energy difference

$$\Delta E_{kin}^{g} - \Delta E_{kin}^{a} = -60 \text{ kJ}$$

can be explained will be answered later (see Eq. (3.6)).

Occasionally the question may arise whether the traditional TE_Air variometer should not better be replaced by a TE_Earth variometer (which measures changes of TE_Earth). The example, however, shows that this would not be unproblematic: During speed reduction in a tailwind the TE_Earth variometer indicates an energy loss; it behaves here like an over-compensated TE_Air variometer. As will be seen later (Eq. (3.3)), whenever the air is moving compensation does not work properly.

In the following two sections, let us investigate what effects air movements have on TE_Air and TE_Earth.

Total energy in an air-fixed reference frame (TE_Air)

For the process of deducing the energy equation that is of interest here, we start with Newton's Fundamental Law of Dynamics. Applying it to an earth-fixed reference frame, it states:

$$M \frac{d\vec{u}}{dt} = \vec{L} + \vec{D} + \vec{W} \quad (2.1)$$

M stands for the aircraft mass and $\vec{u}$ for the velocity with respect to the ground. The forces acting upon the aircraft are lift $\vec{L}$, drag $\vec{D}$, and weight $\vec{W}$.

When Newton's Law is applied to an air-fixed reference frame we must consider that the system accelerates, and therefore we have to make allowances for an inertial force $\vec{I}$. Thus, we arrive at:

$$M \frac{d\vec{v}}{dt} = \vec{L} + \vec{D} + \vec{W} + \vec{I} \quad (2.2)$$

If we include Eq. (1.1) into Eq. (2.1), it becomes apparent that

$$\vec{I} = -M \frac{d\vec{w}}{dt} \quad (2.3)$$

In order to arrive at the energy equation, we must multiply Eq. (2.2) on both sides in scalar fashion with the vector $\vec{v}$ ("dot product"). Equation (2.2) is then transformed into

$$M v \frac{dv}{dt} = \vec{v}\left(\vec{L} + \vec{D} + \vec{W} + \vec{I}\right) \quad (2.4)$$

The change of kinetic energy equals the scalar product of the velocity vector $\vec{v}$ and the sum of all forces acting on the aircraft. Next, if one applies Eqs. (1.1), (1.4) and (2.3) one arrives at the following important result:

$$\begin{aligned} \frac{de_a}{dt} &= -\frac{v D}{M g} + w_v - \frac{\vec{v}}{g} \frac{d\vec{w}}{dt} \\ &= \quad v_s \quad + w_{stat} + w_{dyn}^{a} \end{aligned} \quad (2.5)$$

The first component

$$v_s = -\frac{v D}{M g} \quad (2.6)$$

is the intrinsic sink rate of the glider. It is caused by the fact that in order to overcome the air's inevitable drag, energy is permanently used up. The two remaining components offer a chance to gain energy. In order to conform to conventions, they shall be designated as static and dynamic energy components.

The static energy component

$$w_{stat} = w_v \tag{2.7}$$

results from updrafts and downdrafts. Updrafts are the main source of energy for soaring flight.

The aircraft's dynamic change in energy is defined by

$$w^a_{dyn} = -\frac{\vec{v}}{g}\frac{d\vec{w}}{dt} \tag{2.8}$$

Here $-\vec{v}$ is the flow rate of the air, as observed from the aircraft. It is multiplied in scalar fashion (dot product) with the change of wind velocity.

Therefore, if the wind contains a component of acceleration which has the same direction as the air flow (with respect to the aircraft) this will lead to a dynamic gain of energy.

If one divides the wind velocity in Eq. (2.8) into a horizontal component w_h and a vertical component w_v, the result for straight and steady flight is

$$w^a_{dyn} = -\frac{v}{g}\frac{dw_h}{dt}\cos\gamma - \frac{v}{g}\frac{dw_v}{dt}\sin\gamma \tag{2.9}$$

whereby γ represents the angle between the horizontal and the velocity of flight (positive value for ascent).

Flight in a field of wind gradients (wind shear)

Let us observe a landing approach with strong head wind. Due to friction, the lower layers of air are slowed down; the course of the wind may look as shown in Fig. 2. Much higher wind gradients may occur than might be expected from the graph if turbulence is encountered. A particular point of the landing approach is picked, for which the following numerical values shall apply:

$$v = 126 \text{ km/h} = 35 \text{ m/s}; \quad \gamma = -3°; \quad \frac{dw_h}{dh} = -0.2\frac{m/s}{m}$$

From Eq. (2.9) and $dh/dt = v\sin\gamma$ then follows:

$$w^a_{dyn} = -\frac{v^2}{g}\frac{dw_h}{dh}\sin\gamma\cos\gamma = -1.3\text{ m/s} \tag{2.10}$$

The total energy variometer displays a loss energy of -1.3 m/s which is in addition to the aircraft's intrinsic sink rate v_s. Despite the absence of any downdraft this should not be interpreted as an instrument error. If you fly at a constant airspeed, the loss of height expected, due to the variometer reading, actually occurs.

Energy also can be gained in a wind shear. The elegance and perfection displayed by albatrosses in performing the art of dynamic soaring is a strong incentive for trying to follow their example one day, perhaps in the boundary layer of a low level jet.

Total energy in an earth-fixed reference frame (TE_Earth)

The energy equation is deduced in a similar way as in the previous section, i.e. by multiplying Newton's equation of motion Eq. (2.1) on either side in scalar fashion by $\vec{u}$. If one still uses Eq. (1.1) and the equation of definition Eq. (1.5), the result initially will be

$$\frac{de_g}{dt} = -\frac{vD}{Mg} + \frac{1}{Mg}\vec{w}\left(\vec{L}+\vec{D}\right) \tag{3.1}$$

where $\vec{w}\left(\vec{L}+\vec{D}\right)$ is the energy per time unit (power) extracted from the ambient air. Applying Eq. (2.1) once more, we arrive at

$$\frac{de_g}{dt} = -\frac{vD}{Mg} + \frac{1}{g}\vec{w}\left(\frac{d\vec{u}}{dt}-\vec{g}\right) \tag{3.2}$$

and finally

$$\frac{de_g}{dt} = -\frac{vD}{Mg} + w_v + \frac{1}{g}\vec{w}\frac{d\vec{u}}{dt}$$
$$= v_s + w_{stat} + w^g_{dyn} \tag{3.3}$$

The intrinsic sink rate of the glider v_s and the static energy component w_{stat} agree with Eq. (2.5). For the dynamic energy component we see that

$$w^g_{dyn} = \frac{1}{g}\vec{w}\frac{d\vec{u}}{dt} \tag{3.4}$$

To interpret this result we can use Eq. (3.2). The term

$$\vec{a}_a = d\vec{u}/dt - \vec{g} \tag{3.5}$$

denotes the "apparent" acceleration which is measured by accelerometers (and which is experienced as an acceleration by a human body); $\vec{a}_a = \vec{0}$ signifies weightlessness, during a non-accelerated straight-line flight one has $\vec{a}_a = -\vec{g}$.

A (static or dynamic) energy gain is the result of apparent acceleration $\vec{a}_a$ coinciding with the direction of the wind-flow $\vec{w}$. At this point energy is exchanged between the sailplane and the surrounding air. (An energy gain of a sailplane is possible only if the energy of the atmosphere is diminished by the same amount.) The speed of the air will slow down. To put it in other words: "One must attempt to equalize the fluctuations in the wind" (Prandtl[1]).

As a consequence of this, during a straight-line flight through an updraft one must fly with an increased load factor, through a downdraft one must fly with a decreased load factor. Prandtl's advice, however, is universally true; for example it also may be applied to lateral wind gusts encountered in turning flight. Much consideration is bestowed on this kind of energy-gain in the context of microlift gliders and small

UAVs. To what extent this can play a role for modern high-performance sailplanes is difficult to say. Because of the fact that there is no instrument on the market that (unequivocally) indicates dynamic energy changes, the pilot of the sailplane is left guessing.

Following Peter Riedel[2], one can divide dynamic soaring into two categories. The type of dynamic soaring just described (exploiting small-scale wind fluctuations) is classified by Riedel as Category I. The flight of the albatross (in the extensive field of gradients of shear wind) is classified as Category II.

With the help of Eq. (3.4) we can answer now the question put in the introductory example. If one reduces flight speed with respect to the ground by Δu = -10 m/s in the presence of constant tailwind of w_h = 15 m/s, TE_EARTH is reduced by

$$\Delta E_g = M\, w_h\, \Delta u = -60 \text{ kJ} \qquad (3.6)$$

(The aircraft mass is 400 kg.) According to the law of preservation of energy, energy of the sailplane is transferred to the ambient air.

Flying through an updraft

We look at another example: the symmetric straight-line flight through updraft is to be investigated now. Particularly the peripheral zones of the updraft are subject to variations of vertical wind-speed, and the dynamic effects described in previous sections will appear now. First, the optimum flight path shall be computed and then be interpreted with the help of TE_Air and TE_Earth.

In order to exemplify the essence of this, the strongly idealized updraft model shown in Fig. 4 has been chosen. It is based on the equation

$$w_v(r) = 0.5\, w_0 \left(\tanh \frac{2b(R-r)}{w_0} + \tanh \frac{2b(R+r)}{w_0} \right) \qquad (4.1)$$

with

Intensity of updraft	w_0 = 3 m/s
Radius of updraft	R = 1000 m
Maximum updraft gradient	b = 0.03 (m/s)/m

The updraft can be divided into 3 phases:

i. Increase of vertical wind speed with gradient b
ii. Constant vertical velocity w_0
iii. Decrease of vertical wind speed with gradient -b

The cruising speed with an expected climb rate of u_{MC} = 3 m/s shall be optimized. The performance criterion (Eq. (I) in Fig. 3) is basically the same as in classical speed-to-fly theory, generalized to instationary flight. In Fig. 3 the optimization problem is formulated. It is a variational problem that can be solved by applying Pontryagin's maximum principle. (A two-point boundary value problem has to be solved for the system of state equations and adjoint equations.) The results are shown in Fig. 4.

For the sailplane, we assume the speed polar

$$v_s = -\left(10\frac{m^2}{s^2}\right)\cdot\frac{1}{v} - \left(81000\frac{m^2}{s^2}\right)^{-1}\cdot v^3 \qquad (4.2)$$

(lift/drag ratio 45 at a speed of 108 km/h). For instationary flight Eq. (V) in Fig. 3 is applied. This is an idealization. Rudder losses and a non-stationary air-stream around the wings are not considered here. However, one can still expect that the results of the calculations do indicate a clear tendency.

In the inner core of the updraft region (phase ii) the speed of the updraft remains constant. A stationary phase of flight is the result, at a speed of flight which follows from the speed-to-fly theory in a well-known manner. In the peripheral zones (phases i and iii) however, the classical theory (which is true only for stationary flight) can no longer be applied.

In the case of optimum flight, dynamic effects will lead to energy gain.

- Description with the help of TE_Air (see γ(s) and w^a_{dyn}(s) in Fig. 4):

  One has to concentrate on these areas of updraft where the vertical air flow changes speed. In case of an updraft gradient $dw_v/ds \neq 0$, the following is true:

  $$w^a_{dyn} = -\frac{1}{g} v^2 \frac{dw_v}{ds} \sin\gamma\cos\gamma \qquad (4.3)$$

  On entering the area of updraft, the vector of flight-speed must be directed downwards ($dw_v/ds > 0$, $\gamma < 0$), and on leaving the same, it must be directed upwards ($dw_v/ds < 0$, $\gamma > 0$).

- Description with the help of TE_Earth (see n(s) and w^g_{dyn}(s) in Fig. 4):

  Dynamic energy gain takes place here in the core area of the updraft (at $w_v > 0$). If one flies with a higher load factor (n > 1), the following is true:

  $$w^g_{dyn} = \frac{1}{g} w_v \frac{du_v}{dt} \approx w_v\left(n\cos\gamma - 1\right) \qquad (4.4)$$

  But flying with an increased load factor is only possible for a limited time. It means here an upward acceleration of the sailplane, and there must be a deceleration (in upward direction) before it or after it. An overall energy gain is only achieved if the phase of deceleration (n<1) lies outside the inner updraft region.

The optimization of flight path is not dependent on the definition of total energy. So it is only a question of usefulness, which of the two descriptions you will prefer. Since flight begins and ends in still air, the gain of energy over the entire flight route is the same: $\Delta E_a = \Delta E_g$.

In Fig. 5 the optimum flight through a narrow thermal is shown. The updraft model proposed by Gedeon is used[3].

Compared with the previous example there is no phase ii and the phases i and iii are partly merged. Notice that the flight speed v(s) is almost the opposite of what would be expected from speed-to-fly theory. The amount of energy gain due to dynamic effects is remarkable. However, the curves are non-causal: one must react to the updraft-gradients before reaching them. So, in flight practice only a sub-optimum flight is feasible at best. (If the variations of n(s) are done at the wrong places, great energy losses may occur as well. For today's soaring pilots it is probably the best advice to traverse small updrafts with nearly constant airspeed.)

Fig. 6 shows a further example. This time the four-cell type of Gedeon's updraft model is used.

The ideal variometer

In regard to the problem of finding suitable instrumentation, only the variometer will be looked at here. A variometer displaying the true vertical speed of the air stream w_v will be called the "ideal variometer". The history of the development of the variometer can be described as a step-by-step approach towards the ideal variometer.

The original altitude variometer measures changes of the static air pressure p_s and displays changes of altitude:

$$\frac{dh}{dt} = -\frac{1}{\rho g}\frac{dp_s}{dt} \tag{5.1}$$

From Eq. (2.5) and (1.4) we arrive at:

$$V_{Altitude} = \frac{dh}{dt} = w_v + v_s + w^a_{dyn} - \frac{v}{g}\frac{dv}{dt} \tag{5.2}$$

$V_{Altitude}$ is the displayed value. We are still far away from the ideal variometer.

Since modern sailplanes operate over a wide speed range especially the last component has a very disturbing effect ("stick-thermal"). It may be compensated by an additional measurement of changes of dynamic pressure q. Thus, we arrive at the **total energy variometer**:

$$\frac{de_a}{dt} = -\frac{1}{\rho g}\left(\frac{dp_s}{dt} - \frac{dq}{dt}\right) = \frac{dh}{dt} + \frac{v}{g}\frac{dv}{dt} \tag{5.3}$$

The total energy variometer displays changes of TE_Air. From Eq. (2.5) we already know:

$$V_{TotalEnergy} = \frac{de_a}{dt} = w_v + v_s + w^a_{dyn} \tag{5.4}$$

(The term "total energy variometer" used here always refers to the traditional TE_Air-variometer.)

With the netto variometer the sailplane's intrinsic sink rate v_s is compensated by means of the speed polar (and eventually the load factor). One measures merely the sum of w_v and w^a_{dyn} :

$$V_{Netto} = w_v + w^a_{dyn} \tag{5.5}$$

During the search for updrafts and during centering a thermal, however, we just want to see the true speed of the updraft; the dynamic energy component w^a_{dyn} is only distracting. So, a consistent further development of the variometer would be to measure both the speed of the updraft and the dynamic energy component independently from one another. With that one arrives at the **ideal variometer**:

$$V_{Ideal} = w_v \tag{5.6}$$

It is not at all an easy task to build an ideal variometer that works in a satisfactory way. However, in sailplanes used for research purposes the measurement of the vector of the wind's velocity (measuring all three components) is possible already today (refer to BAT probe[4]). The measurement is based on Eq. (1.1):

$$\vec{w} = \vec{u} - \vec{v} \tag{5.7}$$

The velocities $\vec{u}$ and $\vec{v}$ must be measured; the wind velocity then can be calculated from the difference of these two values. For the ideal variometer, this can be restricted to the vertical components of the vectors. In order to get an impression of the difficulties of the measuring task, it perhaps suffices to look at the special case of symmetrical straight-line flight. The formula for the determination of w_v (which is more complicated in the general case) is then:

$$w_v = \frac{dh}{dt} - v\sin\gamma \tag{5.8}$$

where γ is the inclination of the vector $\vec{v}$ in relation to the horizontal, which can be computed from the pitch angle θ and the angle of attack α_x: $\gamma = \theta - \alpha_x$. Looking at Eq. (5.8), one gets easily convinced that even small errors in the determination of γ will have a pronounced effect on the end result.

There are a lot of other possible error sources which may corrupt the measurement. Hopefully they can all be managed in a sufficient way. Compared to the BAT probe there is no need to measure very short fluctuations of the wind, since the pilot cannot react quickly enough to these high frequency fluctuations. Thus, a filter can smooth the measurement result. But this almost automatically means a time delay and, on the other hand, the measurement must not be too slow. This problem is well-known from variometers.

Measurement expenditure can be reduced if one resorts to the laws of aerodynamics and flight dynamics (refer to e.g. Myschik, Sachs[5]). It is important here that the variables of performance and the behavior of the aircraft will be reproduced as realistic as possible by assumptions of the theoretical model.

Apart from the air's vertical velocity w_v, the change of total energy (TE_Air) is still of interest. If you imagine the ideal variometer and the total energy variometer combined in an instrument with two pointers, there is also the possibility to measure the magnitude of dynamic effects. The difference between the positions of both indicator hands will be:

$$V_{TotalEnergy} - V_{Ideal} = w^a_{dyn} + v_s \qquad (5.9)$$

or, if the total energy variometer is substituted with a netto variometer:

$$V_{Netto} - V_{Ideal} = w^a_{dyn} \qquad (5.10)$$

The ideal variometer and the total energy variometer (resp. the netto variometer) must show the same time behavior to make the signals comparable.

Conclusions

The question raised here is in what way dynamic effects can contribute to the improvement of the performance of a "normal" soaring flight. One can try to make model assumptions of atmospheric air flows that are as close to reality as possible, for which the optimum flight paths then can be calculated. A truly satisfying clarification, however, may be only achieved in practical flights. To achieve this, there must be at least the possibility to measure the dynamic energy component in the sailplane. The creation of an "ideal variometer" would be desirable because it would not only provide you with an impression of the dynamic changes of energy but would also give a true picture of the distribution of the atmospheric updrafts and downdrafts.

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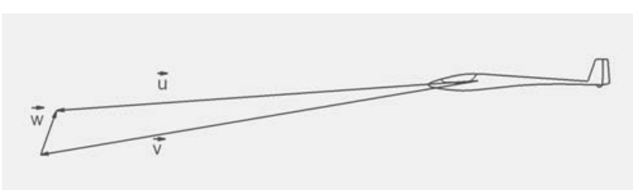

Figure 1 Vectors of velocity.

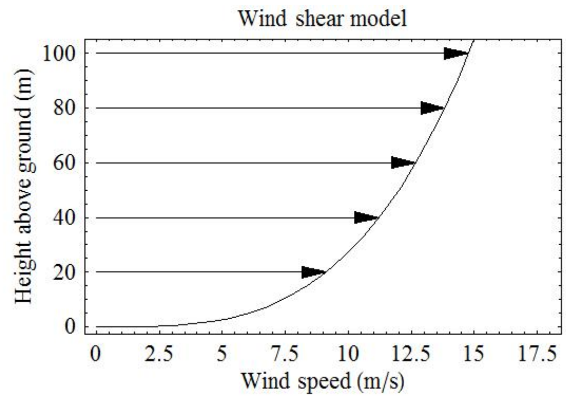

Figure 2 Wind shear during landing approach.

Optimization of Flight Path:

Performance criterion (time for energy-neutral flight):

$$\int_0^L \left(1 - \frac{1}{u_{MC}} \frac{dh}{dt}\right) \frac{ds}{v \cos\gamma} \rightarrow \min \qquad (I)$$

$$\text{with} \quad \frac{dh}{dt} = w_v + v \sin\gamma \qquad (II)$$

State equations (differential equations of flight path):

$$\frac{dv}{ds} = \left[-g \sin\gamma - \frac{dw_v}{ds} v \sin\gamma \cos\gamma + \frac{g}{v} v_s(v,n) \right] \frac{1}{v \cos\gamma} \qquad (III)$$

$$\frac{d\gamma}{ds} = \left[-g \cos\gamma - \frac{dw_v}{ds} v \cos^2\gamma + g\, n \right] \frac{1}{v^2 \cos\gamma} \qquad (IV)$$

$$\text{with} \quad v_s(v,n) = c_1 \frac{n^2}{v} + c_2 v^3 \qquad (V)$$

Control variable (load factor):

n

Boundary conditions :

$v(0) = v_0$

$v(L) = v_L$

$\gamma(0) = \gamma_0$

$\gamma(L) = \gamma_L$

Inequality constraints:

$0 \le n \le n_{max}$

$v_{min} \sqrt{n} \le v \le v_{max}$

Figure 3 Optimization problem for optimum cruise speed.

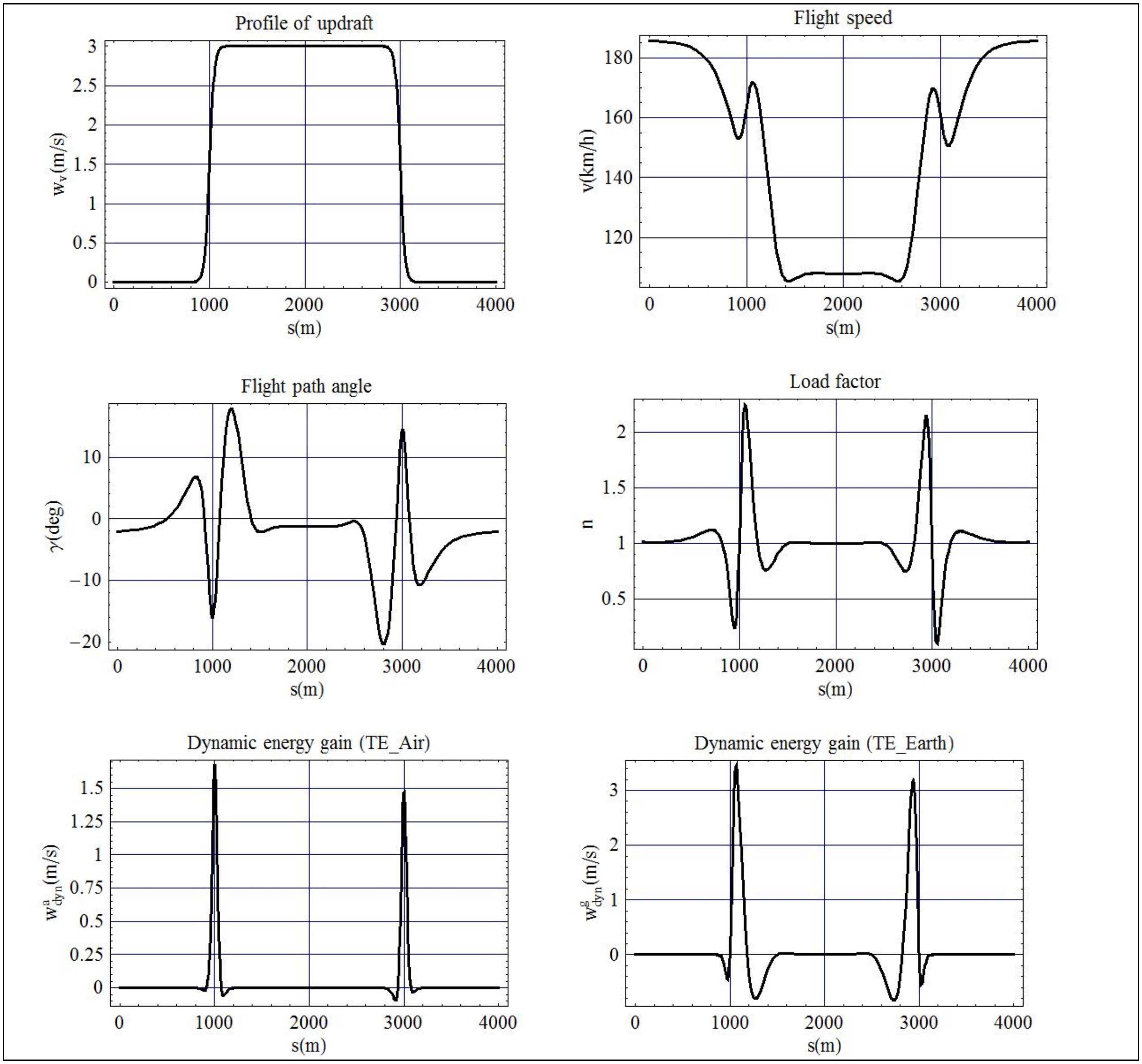

Figure 4 Optimum straight-line flight through an extended updraft region.

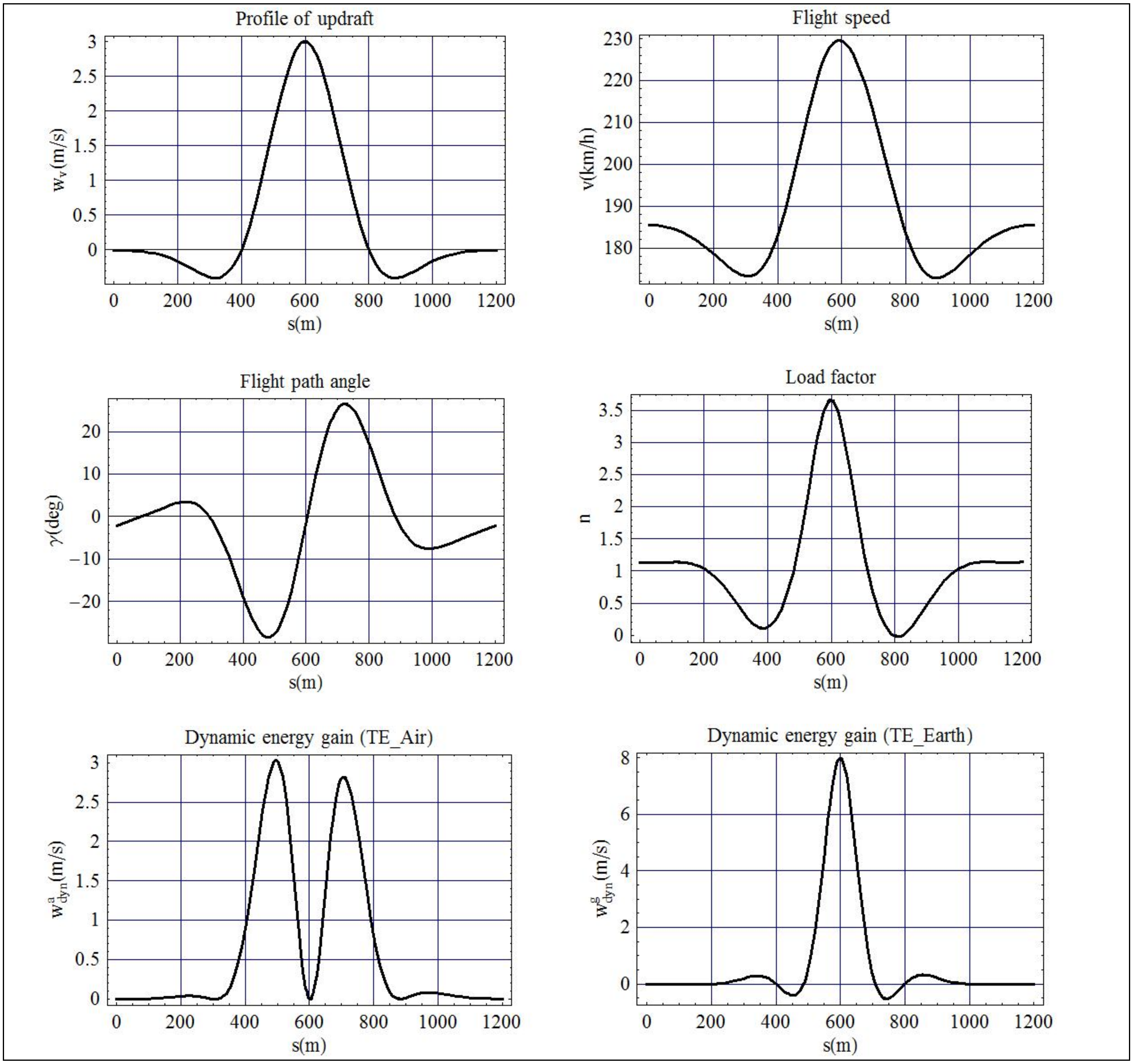

Figure 5 Optimum straight-line flight through a narrow thermal.

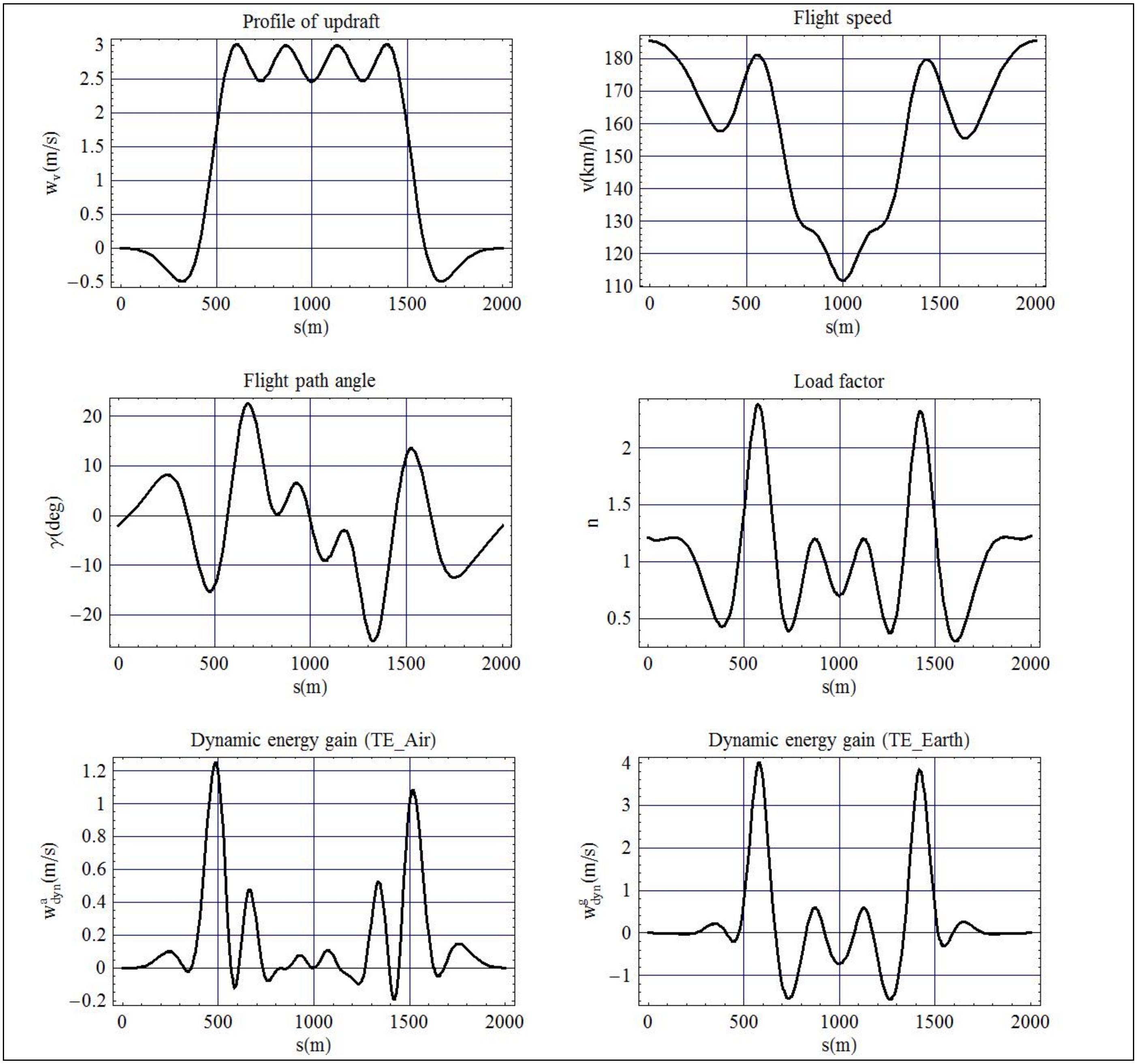

Figure 6 Optimum straight-line flight through a wide thermal.