TRIM DRAG

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SUMMARY

A simple method is given for determining trim drag coefficient, trim drag being the increase in induced drag of the wing-tailplane combination above the nominal value calculated from the usual approximation in which the effect of the tail load is ignored. The method has been used to derive trim drag coefficients for low- and t-tail locations for a range of parameters including C.G. position. For the t-tail it is necessary to know the location of the wing vortex sheet at the tailplane position, and a simple rule of thumb for determining this is derived. Comments on optimum wing-section camber and C.G. position are made.

1. INTRODUCTION

Much has been written on the profile drag of wings and other components, and on basic trailing vortex (induced) drag as affected by plan-form, etc. I have not been able to find many references to the increase in induced drag which occurs when the tailplane lift is not zero, that is, the difference between the total induced drag of the wing-tailplane combination and the nominal induced drag of the wing calculated on the ordinary assumption that the wing lift is equal to the total weight of the glider. This difference may be termed "trim drag" and comprises three parts:

(1) Change in wing induced drag arising from the change in wing lift coefficient;

(2) Tailplane induced drag arising from its own lift;

(3) Changes in wing and tailplane induced drag arising from the interference effects of each other's lift.

A formula for the total induced drag applicable to aircraft on which the tailplane is situated on the fuselage, that is, more or less straight behind the wing, is given by Jones (Reference 1) and this was used by Irving (Reference 2) as the basis for determining the loss of energy height per hour, consequent upon trim drag, for a hypothetical standard class glider, as affected by center of gravity position. Schmaljohann (Reference 3) considered interference effects of the wing on the tailplane drag and effectiveness, but not those of the tailplane on the wing drag. Kerlin (Reference 4) calculated many examples, expressing trim drag coefficient in terms of tailplane lift coefficient. Kroo (Reference 5) investigated performance penalties resulting from trim drag and the associated profile drag changes with variation of wing area, tail area and tail spin.

It was felt that a simple method of deriving trim drag coefficient as a function of total lift coefficient, that could be included in the general drag estimation, in terms of the appropriate parameters including C.G. position, would be useful. Such a method is presented herein; the three parts of the trim drag do not need to be determined individually, but are covered by a single drag coefficient. The t-tail case is included by means of an increment in trim drag coefficient, which depends on the same parameters as well as on the vertical position of the tailplane.

2. GENERAL BASIS OF METHOD

Symbols are defined as they are introduced; a general list is given in Appendix I.

Glauert (Reference 6) and Prandtl and Tietjens (Reference 7) give formulas for the total drag due to the trailing vortices, or induced drag, of two lifting planes. They were derived originally in connection with biplanes, but are also applicable to the case of a wing and a tailplane, or a wing and a foreplane.

If L_W and L_T are the wing and tailplane lifts b_W and b_T the wing and tailplane spans, σ an interference coefficient, and q is 1/2 V^2 we may write that the total induced drag D_i is given by

$$D_i = \frac{1}{\pi q}\left(\frac{L_W^2}{b_W^2} + 2\sigma\frac{L_W L_T}{b_W b_T} + \frac{L_T^2}{b_T^2}\right) \qquad \ldots 1$$

σ is a function of tailplane/wing span ratio b_T/b_W and of gap/mean span ratio $2Z_T/(b_W + b_T)$, where Z_T is the perpendicular distance between the tailplane and the vortex sheet of the wing, or loosely the vertical separation between tailplane and wing.

It is assumed that the effects of rolling up of the vortex sheet are negligible and that the spanwise load distributions on both surfaces are elliptical.

The latter assumption requires some comment. As regards the ordinary planform effect it is entirely justified; the increases in induced drag resulting from non-ellipticity of loading are unlikely to exceed 3%, and for the wing considerably less if the shape is trapezoidal or double-tapered as it often is. We are not, of course, discussing the so-called "induced drag factor" often used in performance studies, which covers changes of profile drag with incidence and the consequences of variation of Reynolds Number over the speed range as well as genuine trailing vortex effects.

However, we need to consider the influence of the fuselage, which among other things, magnifies the vertical component of the velocity locally. In potential flow, the magnification factor reaches a maximum of 2, and in real flow about 1.8. This and other interference effects must cause some disturbance to the lift distributions. For the wing only a small fraction of the span is affected, and the effects are normally counted as part of the fuselage drag. For the tailplane, if mounted on the fuselage, things are more complicated. Except at low incidence, the vertical velocity component is upwards, and close to the fuselage the magnification is sufficient to overcome the wing downwash completely and cause local upwash, as illustrated in Reference 3. The resulting disturbance is thus quite intense and the affected region, while still local, is less so, relative to the span, than in the case of the wing. The disturbance is distinctly lift-dependent and must cause some addition to the tail's own induced drag. However, I would guess the magnitude of the increase to be not much more than 10%, and in view of the other approximations involved in the whole process no correction for the effect, even if one could be derived, seems to be justified. Pessimists could assume the tailplane span to be a few percent less than its actual value. For t-tails the fuselage influence is almost absent and the interference of the tail lift on the wing is virtually unaffected by it since the tail span is so small compared to that of the wing.

2.1 TAILPLANE ON FUSELAGE, OR LOW TAIL

Here the vertical separation may be neglected for all lift coefficients, and for this case σ is equal to b_T/b_W. Substituting this in 1. and putting L_W equal to $W - L_T$, where W is the weight, gives

$$D_i = \frac{W^2}{\pi q b_W^2}\left(1 + \left[\left(\frac{b_W}{b_T}\right)^2 - 1\right]\frac{L_T^2}{W^2}\right) \qquad \ldots 2$$

Now $W^2/\pi q b_W^2$ is the induced drag when the tail load is zero; hence by subtraction the trim drag D_T is

$$D_T = \frac{L_T^2}{\pi q b_W^2}\left[\left(\frac{b_W}{b_T}\right)^2 - 1\right] \qquad \ldots 3$$

We next determine L_T. The complete expression for the wing and fuselage pitching moment M, about the aerodynamic center (A.C.), in steady gliding flight is

$$M = C_{M_0} q S_W c + W\cos\gamma\{(h-h_0)\cos\alpha + k\sin\alpha\}c - W\sin\gamma\{k\cos\alpha - (h-h_0)\sin\alpha\}c \qquad \ldots 4$$

where C_{M_0} is the zero-lift pitching moment coefficient, S_W is the wing area, c the reference chord for pitching moment coefficients, h and k the tangential and normal coordinates of the C.G. on c, h_0 the coordinate of the wing and fuselage aerodynamic center on c (h, k and h_0 being expressed as fractions of c), α is the angle of incidence of c, and γ is the gliding angle. $W\cos\gamma$ and $W\sin\gamma$ are equal to the lift and drag, respectively.

For gliders, k, $\sin\alpha$ and $\sin\gamma$ are usually small and may be neglected, and $\cos\gamma$ and $\cos\alpha$ may be taken as equal to unity, without much loss of accuracy. Except for very low lift coefficients (see paragraph 3 below). We may, therefore, rewrite 4 in its usual simpler form:

$$M = C_{M_0} q S_W c + W(h-h_0)c = \{C_{M_0} + C_L(h-h_0)\} q S_W c \qquad \ldots 5$$

where C_L is the (total) lift coefficient. Then for balance the tailplane lift is:

$$L_T = \{C_{M_0} + C_L(h-h_0)\} q S_W \frac{c}{l_T} \qquad \ldots 6$$

where l_T is the tail arm measured from the A.C. Substituting 6. in 3. and writing b^2_W / S_W as A, the aspect ratio, yields

$$D_T = \frac{\{C_{M_0} + C_L(h-h_0)\}^2}{\pi A} q S_W \left(\frac{c}{l_T}\right)^2 \left[\left(\frac{b_W}{b_T}\right)^2 - 1\right]$$

and if C_{D_T} is the trim drag coefficient referred to wing area, then

$$C_{D_T} = \frac{\{C_{M_0} + C_L(h-h_0)\}^2}{\pi A} \left(\frac{c}{l_T}\right)^2 \left[\left(\frac{b_W}{b_T}\right)^2 - 1\right] \qquad \ldots 7$$

Note that, in the above, C_L is the total of "ordinary" lift coefficient equal to W/qS_W; it is not necessary to determine the true wing lift coefficient $C_{LW} = L_W / q S_W$ or the tailplane lift coefficient L_T / qS_T, though of course, their values are implicit. The result can also be obtained by taking moments about the C.G. instead of the A.C.; in that case C_{LW} is required (though it need not appear in the answer) and the algebra is somewhat longer.

2.2 V-TAIL

V-tails can be treated in the same way as low tails by using an equivalent flat-tail span. Results of an analysis by Datwyler (Reference 8) as quoted by Hoerner (Reference 9) indicate that the effective or equivalent span may be taken as the actual span (the distance between the tips) multiplied by (sec Γ)$^{1/2}$ where Γ is the dihedral angle.

2.3TAILPLANE ON TOP OF FIN, OR T-TAIL

The simplicity of the low-tail method is not applicable here as σ is not equal to b_T/b_W. For the t-tail case, we introduce a factor F such that $\sigma = F . b_T/b_W$. We may call F the interference coefficient factor. Substituting this in 1. and putting L_W equal to $W - L_T$ as before gives

$$D_i = \frac{W^2}{\pi q b_W^2}\left(1 - 2(1-F)\frac{L_T}{W} + \left[\left(\frac{b_W}{b_T}\right)^2 - (2F-1)\right]\frac{L_T^2}{W^2}\right)$$

and

$$D_T = \frac{W^2}{\pi q b_W^2}\left(\left[\left(\frac{b_W}{b_T}\right)^2 - (2F-1)\right]\frac{L_T^2}{W^2} - 2(1-F)\frac{L_T}{W}\right) \qquad \ldots 8$$

For gliders, F is unlikely to be less than 0.8, and for this value the factor in square brackets, for a typical b_W/b_T of 5, is equal to 24.4, only very slightly more than its value for the low tail of 24.0. The difference is insignificant and may be ignored. We can, therefore, express D_T as the value appropriate to the low tail plus an increment ΔD_T arising from placing the tail in the high or T position. Then

$$\Delta D_T = -2(1-F)\frac{W L_T}{\pi q b_W^2} = -2(1-F)\frac{C_L}{\pi A}\{C_{M_0} + C_L(h-h_0)\} q S_W \frac{c}{l_T}$$

and

$$\Delta C_{D_T} = -2(1-F)\frac{C_L}{\pi A}\{C_{M_0} + C_L(h-h_0)\}\frac{c}{l_T} \qquad \ldots 9$$

Note that ΔC_{D_T} takes the opposite sign to $C_{M_0} + C_L(h-h_0)$ and therefore to the tailplane lift, and hence is positive when the latter is negative. In this case, the middle term in 1. is negative, and the reduction in σ causes an increase in drag. Note also that ΔC_{D_T} is directly proportional to C_L and is only appreciable in high-lift conditions.

Values of σ are given in References 6 and 7 for span ratios down to 0.6 only—appropriate for the two wings of biplanes. The tailplane/wing span relationship for gliders is rather difference (b_T/b_W) $\approx$ 0.2 maximum) and interpolation of the data for these values shows that F is given approximately by

$$F = 1 - 0.8 \times 2Z_T/(b_W + b_T) \qquad \ldots 10$$

To determine Z_T and hence F, a side-view drawing is needed on which are marked the position of the wing-root chord and the sailplane. The zero-lift angle and lift-curve slope must be known; the ordinary wing (untrimmed) values may be used, the effect of tailplane lift (trimming) being ignored as for this purpose its effect is of second order. From these the angle of incidence corresponding to any particular C_L is calculated and the free-stream (flight) direction drawn through the wing-root trailing edge in the down-stream direction. Then at the longitudinal position of the tailplane, the wing vortex sheet may be taken to be displaced below that line by:

General flying	($C_L = 0.5$)	l_T / 50
Climbing	($C_L = 1.0$)	l_T / 30

using the rule of thumb derived in Appendix II. If desired, more accurate calculations can be made as also described there, but will rarely be needed. Indeed, since ΔC_{D_T} is only appreciable at high C_L it will in most cases only be worth taking it into account for the climb.

3. RESULTS AND DISCUSSION

C_{D_T} has been determined for C_{M_0} values of -0.05, -0.1 and -0.15, representative of wing sections having low, medium and high camber respectively, for C.G. positions $h - h_0$ of 0, 0.05, 0.1, 0.15 and 0.2 over the C_L range 0 to 1.2. The results are given in Figure 1. Figure 2 shows an alternative presentation. They apply to wing aspect ratio 20, b_W/b_T of 5, and l_T/c of 4. They may be scaled for other values where applicable, or simply be used universally for a rough approximation.

It will be seen that C_{D_T} is of the form neither of a constant increment to profile drag coefficient (except when the C.G. coincides with the aerodynamic center) nor of a simple addition to induced drag factor. It just has to be treated as a separate variable.

ΔC_{D_T} has been determined for a (constant) tail height $2Z_T/(b_W+b_T)$ of 0.12, giving $F = 0.9$, and the same values of other parameters as before. The results are given in Figure 3. For other values of F, ΔC_{D_T} is proportional to 1-F. Figure 4 gives total trim drag coefficient $C_{D_T} + \Delta C_{D_T}$ for this case, in both presentations, for $C_{M_0} = -0.1$ only.

The object of the paper was to devise a method for determining trim drag and to give examples which could serve as data sheets. However, some observations on the numerical results may be useful.

Trim rag can be significant in some circumstances. The following table gives an idea of trim drag coefficients as percentages of minimum drag coefficient, taken here as 0.010 (it can be appreciably lower), for the low-tail case in the following conditions:

Cruise, meaning likely inter-thermal speed or a little more, $C_L = 0.3$.

Climb, meaning thermaling at 30-35 degrees bank, C_L = 0.9 to 1.1 depending on airfoil section camber.

C.G. forward, taken as $h - h_o = 0$ to 0.05.

C.G. mid, taken as $h - h_o = 0.1$.

C.G. aft, taken as $h - h_o = 0.15$ to 0.2.

Flight condition C.G. position Camber	C_{Mo}	Cruise For'd	Cruise Mid	Cruise Aft	Climb For'd	Climb Mid	Climb Aft
Low	-0.05	Neg	Neg	Neg	Neg	½%	2-4%
Medium	-0.1	2%	1%	½-1%	½-2%	Neg	½-2½%
High	-0.15	5%	3½%	2-2½%	2-5%	½%	0-1%

Neg = negligible (less than ½%)

For the cruise, low camber gives lowest trim drag; C.G. position is unimportant. For climb, typical (medium) camber is about right; mid C.G. position is about the optimum.

For a fixed wing section (no flaps), high camber is generally bad; for the cruise, the C.G. needs to be well aft and even then the trim drag is appreciable. For this case, taking into account that all flights will include both cruise and climb, and bearing in mind other considerations such as maximum lift coefficient and good handling characteristics, the optimum is probably typical medium camber with C.G. mid or a little aft; this accords with the conclusion of Reference 2.

When camber-changing flaps are used, resulting in low (numerical) C_{Mo} for the cruise and a higher one for the climb (values of -0.05 and -0.15 respectively may be considered representative) the optimum C.G. again seems to be mid or a little aft. It is interesting to note that, except in the climb with C.G. forward, the flaps almost eliminate trim drag — a part of their benefit which may not have been fully realized hitherto.

Comparison of Figure 4 with Figures 1 and 2 shows that the t-tail location has a negligible effect in the cruise. In the climb it is slightly adverse for C.G. forward and slightly favorable for C.G. aft, but the changes are insufficient to have a noticeable effect on the optimum C.G. position.

4. ADDITIONAL REMARKS

Two other points need to be mentioned. First, in the numerical examples the $C_L = 0$ values have been included to facilitate drawing the curves but strictly they are false. This is because at low lift coefficients, below 0.04 or so, the simplified moment equation 5, and consequently equations 6, 7 and 9 rise, are no longer valid, as the assumptions regarding sin γ and cos γ become increasingly inaccurate. $C_L = 0$ itself corresponds to a vertical dive, so that sin γ equals unity and matter cos g is zero, instead of the other way round. The matter could be covered by replacing $C_L(h - h_o)$ by $-C_D k$ just for that point, but there would be little object in doing so, since the condition is not normally a practical one.

Secondly, it has been said elsewhere (Reference 1) that minimum induced drag (that is zero trim drag) occurs when the tailplane lift is zero. This is true for the case being considered there — low-tail position — and nearly but not absolutely true for the t-tail case. Sharp-eyed readers will see that in Figure 4 $C_{Dt} + \Delta C_{Dt}$ is just negative (-0.00001) for the case $C_{Mo} = -0.1$, $F = 0.9$, $h - h_o = 0.1$ and $C_L = 1.2$. This is not because the approximation of ignoring the difference between $2F$-1 and unity in equation 8 has gone wrong for this case; it is quite in order. What has happened is that the t-tail is just nudging the genuine reduction in induced drag of the biplane compared with the monoplane.

This result is given for interest only; the numerical values are so small that it is of no significance. Kerlin (Reference 4) shows a similar result for aircraft of similar proportions. Incidentally, he also finds substantially larger negative total C_{Dt} values, but only for aircraft proportions more appropriate to airplanes.

For all practical purposes, we can take the zero-tailplane-lift condition for minimum induced drag as being universal for gliders. A simple physical explanation, which I have not seen stated before, is that all the weight is then carried by the

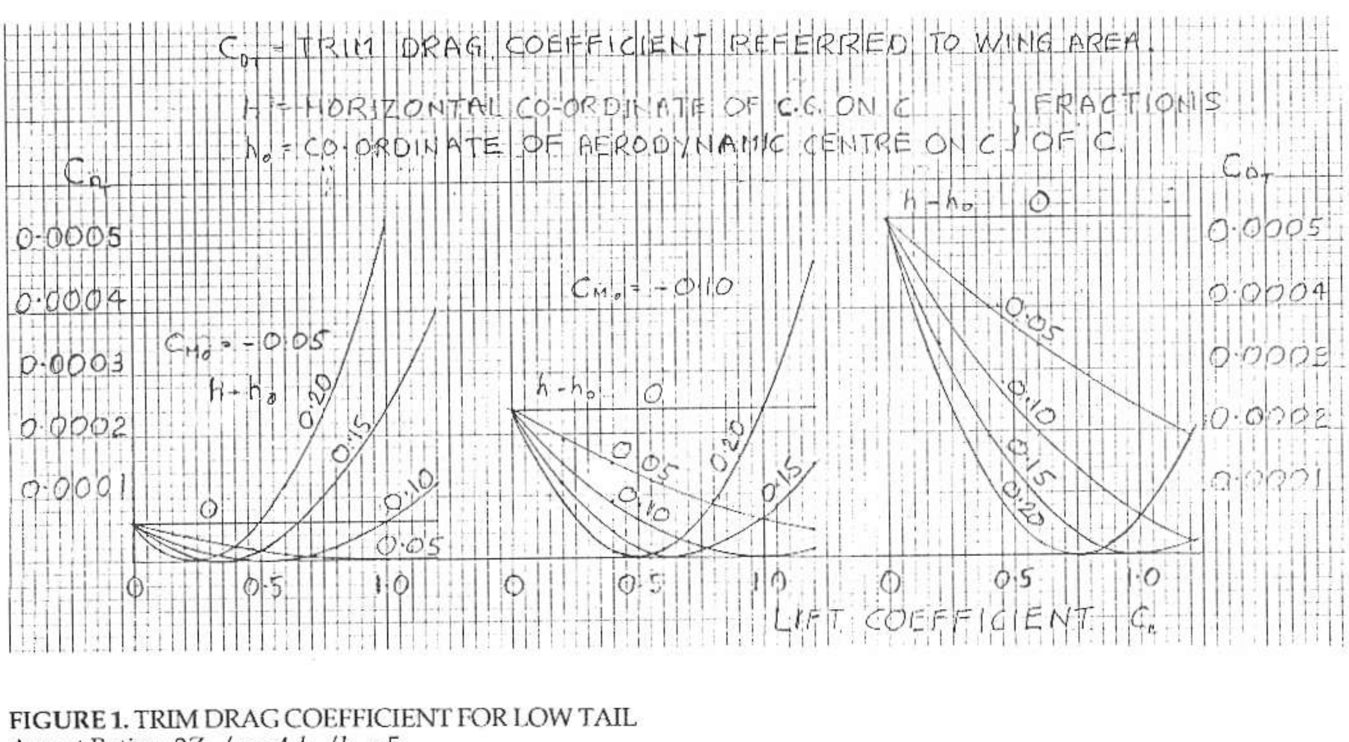

FIGURE 1. TRIM DRAG COEFFICIENT FOR LOW TAIL
Aspect Ratio = $2Z_T$ / c = 4 $b_W/b_T = 5$

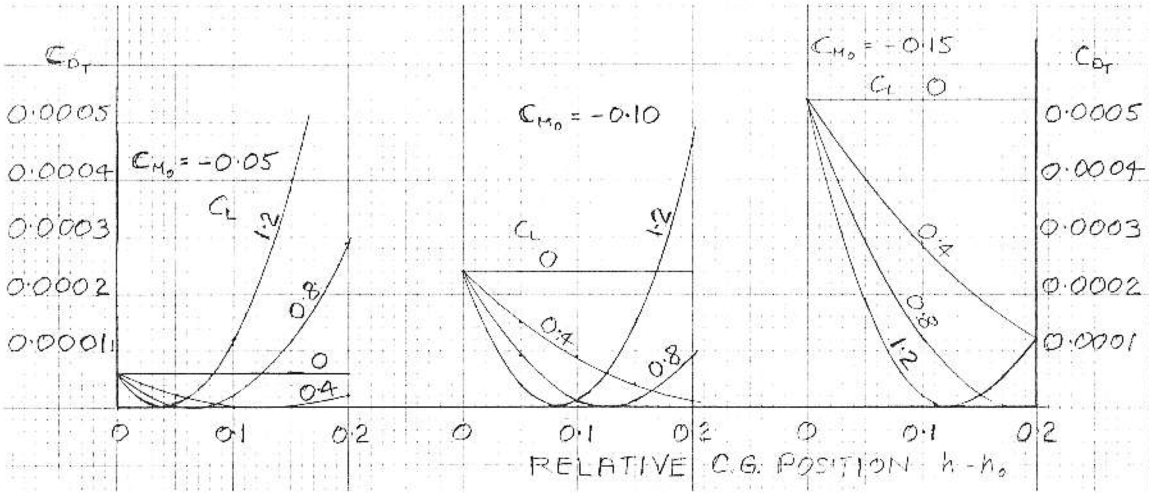

FIGURE 2. TRIM DRAG COEFFICIENT FOR LOW TAIL
Alternative Presentation
Conditions as on Figure 1

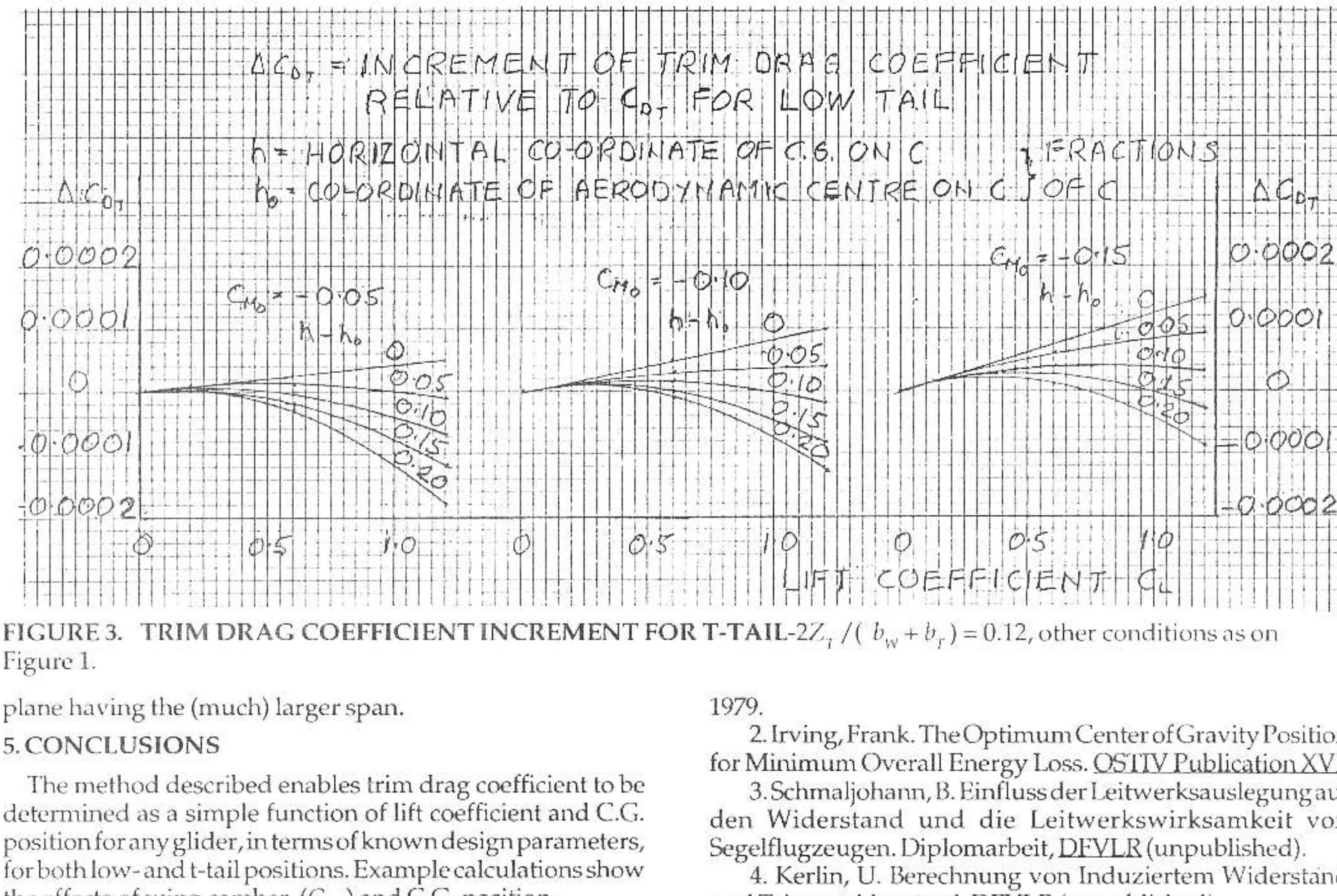

FIGURE 3. TRIM DRAG COEFFICIENT INCREMENT FOR T-TAIL-$2Z_T / (b_W + b_T) = 0.12$, other conditions as on Figure 1.

plane having the (much) larger span.

5. CONCLUSIONS

The method described enables trim drag coefficient to be determined as a simple function of lift coefficient and C.G. position for any glider, in terms of known design parameters, for both low- and t-tail positions. Example calculations show the effects of wing camber (C_{Mo}) and C.G. position.

6. ACKNOWLEDGEMENTS

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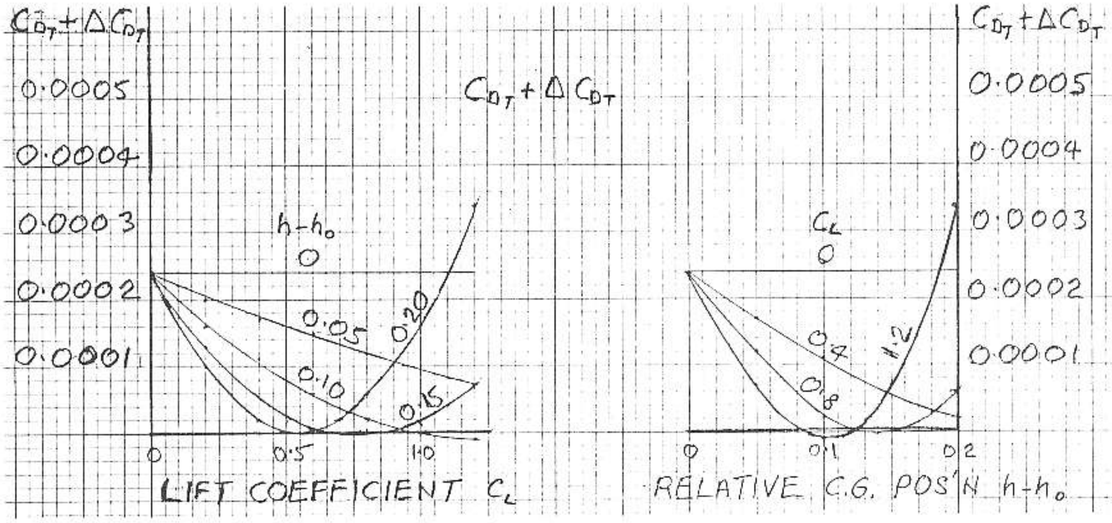


FIGURE 4. TRIM DRAG COEFFICIENT FOR T-TAIL
$2Z_T / (b_W + b_T) = 0.12$ $CM_O = -0.10$
Other conditions as on Figure 1.

APPENDIX I
LIST OF SYMBOLS

b_W	wing span
b_T	tailplane span
c	reference chord for pitching moment coefficients
C_{D_T}	trim drag coefficient
ΔC_{D_T}	increment of trim drag coefficient
C_L	lift coefficient (W/qS_W)
C_{L_W}	lift coefficient of wing (L_W/qS_W)
C_{L_T}	lift coefficient of tailplane (L_T/qS_T)
C_{M_0}	zero-lift pitching moment coefficient
D_i	induced drag
D_T	trim drag
ΔD_T	increment of trim drag
F	interference coefficient factor
h	tangential coordinate of C.G. on c) fractions
h_0	aerodynamic centre coordinate on c) of c
k	normal coordinate of C.G. above c)
L_W	wing lift
L_T	tailplane lift
l_T	tail arm measured from the aerodynamic centre
M	pitching moment
q	$\frac{1}{2}\rho V^2$
S_W	wing area
S_T	tailplane area
V	airspeed
W	total weight of glider
z_T	distance between tailplane and wing vortex sheet
α	angle of incidence
Γ	dihedral angle
ρ	air density
σ	interference coefficient
$\bar{c}$	mean chord of wing
c_0	root chord of wing
C_L	lift coefficient of wing
L	wing lift
l	distance aft of aerodynamic centre
l'	value of l at the tailplane position
s	wing semi-span
s'	half-distance between rolled-up trailing vortices
w	downward induced velocity
z	displacement of vortex sheet below trailing edge
z'	value of z at the tailplane position
Δz	rise of vortex sheet relative to tailplane in turning flight
ε	angle of downwash
ϕ	angle of bank

APPENDIX II

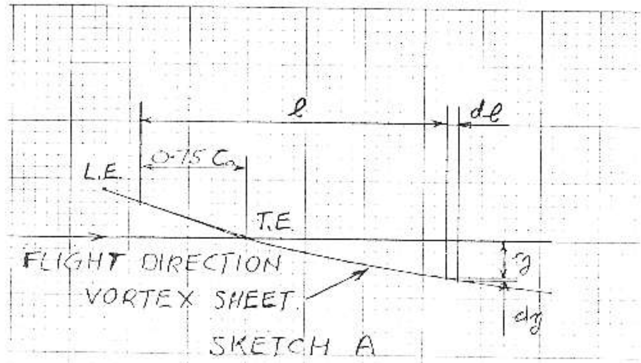
LOCATION OF WING VORTEX SHEET

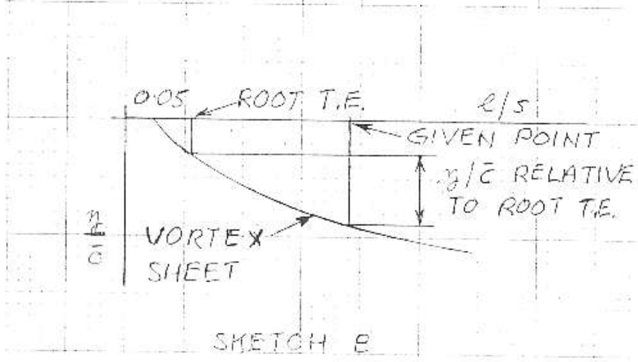
Vortex sheet location is not given directly by airfoil theory; it has to be obtained by integration of downwash. If at any point l is the distance downstream of the bound vortex (which we take to be the quarter-chord point) and z is the displacement of the vortex sheet below the wing trailing edge, the downwash angle ε is given by

$$\varepsilon = \frac{dz}{dl} \quad \text{whence} \quad dz = \varepsilon . dl$$

and, if C_0 is the root chord of the wing, z in the region behind the wing root (see Sketch A) is

$$z = \int_{0.75C_0}^{l} \varepsilon \, dl \qquad \dots \text{A}$$



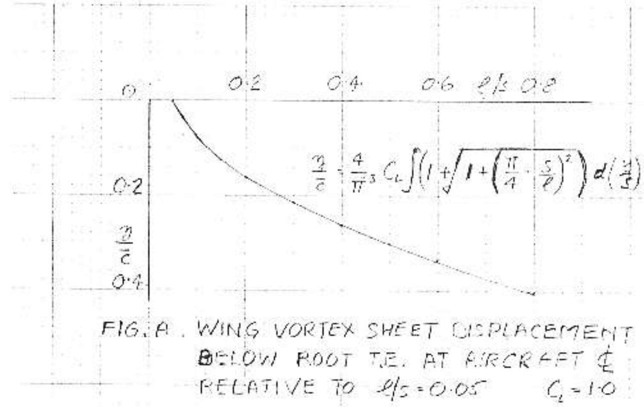


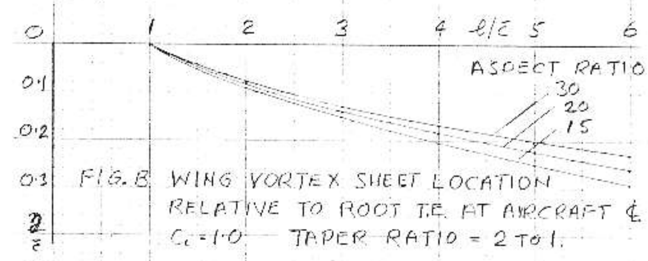
SKETCH A & B

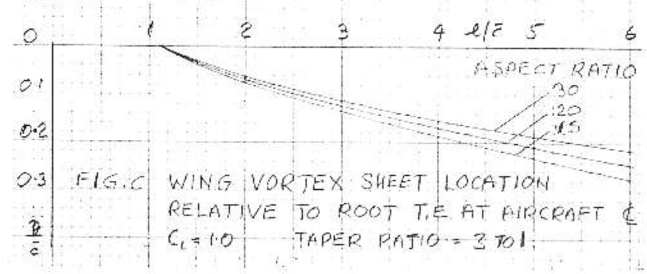
For gliders, the tailplane span is only around 0.2 of the wing span, so we can take center-line values to apply to the whole tailplane with sufficient accuracy. The effect of the local upwash mentioned in paragrpah 2 of the main paper can be ignored for the present purpose.

Now, if L is the lift, s is the wing semi-span and s' the half distance between the rolled-up trailing vortices, then from ref 6 the downward induced velocity w at the center-line is given by

$$w = \frac{L}{4\pi s'^2 \rho V}\left(1 + \sqrt{1 + \left(\frac{s'}{l}\right)^2}\right)$$







FIGURES A,B, AND C

and for elliptic loading $\frac{s'}{s} = \frac{\pi}{4}$.

Also $L = C_L \frac{1}{2} \rho V^2 S_W = C_L \rho V^2 s\bar{c}$ where $\bar{c}$ is the geometric mean chord.

Then

$$\varepsilon = \frac{w}{V} = \frac{4}{\pi^3} \cdot \frac{C_L \bar{c}}{s}\left(1 + \sqrt{1 + \left(\frac{\pi}{4} \cdot \frac{s}{l}\right)^2}\right) \qquad \dots \text{B}$$

Substituting B. in A., dividing both sides by $\bar{c}$ and taking s into the differential yields

$$\frac{z}{\bar{c}} = \frac{4}{\pi^3} C_L s \int_{0.75C_0}^{l} \left\{1 + \sqrt{1 + \left(\frac{\pi}{4} \cdot \frac{s}{l}\right)^2}\right\} d\left(\frac{l}{s}\right) \qquad \dots \text{C}$$

The above assumes that quarter-chord points of the mean chord and root chord are longitudinally coincident. This is not always the case, but any difference can normally be ignored for present purposes.

Only one integration is needed to cover all cases. Different aspect ratios can be accommodated via the relationship between $\bar{c}$ and l, and different taper ratios via that between $\bar{c}$ and C_0. The integration has been carried out for $C_L = 1.0$ from an arbitrary lower limit l/s of 0.05. The results are given in the following table and in Figure A.

l/s $z/\bar{c}$	0.07 0.028	0.1 0.059	0.15 0.096	0.2 0.123	0.3 0.164
l/s $z/\bar{c}$	0.4 0.196	0.6 0.248	0.8 0.292	1.0 0.332	1.2 0.369

In practice, $z/\bar{c}$ relative to the root T.E. for any point is the difference between the value for the point relative to $l/s = 0.05$ and the value for the root T.E. relative to $l/s = 0.05$ (see Sketch B).

The process has been performed for straight-tapered wings of aspect ratio 15, 20 and 30 and taper ratio 2 and 3 to 1. The results are shown in Figures B and C. For clarity the vertical scale is five times the horizontal scale. For other C_L values, $z/\bar{c}$ is C_L times the values on the curves.

Now let the values of l and z at the tailplane position be l' and z'; l' is then the ordinary tail arm, as used in the body of the paper, but measured from the wing quarter-chord instead of from the aerodynamic center. The latter, taking the fuselage into account, is usually at 0.20 to 0.22 of the mean chord. In the present context the difference can be ignored. For gliders, $l'/\bar{c}$ is usually in the region of 4 to 5.5, and for these values the curves can be represented to a good approximation by straight lines through the origin. The slopes of these lines thus measure z'/l' for $C_L = 1.0$, and the values are as follows:

Aspect ratio	15	20	30
Taper 2 to 1	0.050	0.045	0.040
Taper 3 to 1	0.046	0.042	0.038

In practice, z' can be found simply by multiplying the standard tail arm by the value of z'/l' from the table, and by C_L. However, further simplification is possible. The variation of z'/l' with aspect ratio and taper ratio is comparatively small, and a single mean value, say 0.043 C_L probably provides all the accuracy that is really justified. We now consider the two conditions cruise and climb as in the body of the paper.

For cruise we took $C_L = 0.3$, corresponding to inter-thermal speed or a little more. This gives $z'/l' = 0.013$. To cover a range of speeds an average C_L of, say, 0.5, giving $z'/l' = 0.020$, is suggested.

For climb we took $C_L = 1.0$ and the mean value of 0.043 applies. However, there is an additional effect to be considered. In turning flight the rate of turn has a component in the pitching plane, and this causes the vortex sheet to move upwards relative to the tail. If ϕ is the angle of bank, the relative rise Δz is given by:

$$\Delta z = \frac{\rho g}{4} . C_L . \frac{S_w}{W} . l'^2 \sin^2\phi \qquad \ldots \text{D}$$

For $C_L = 1.0$, a bank angle of 30° typical values of $\Delta z/l'$ for altitudes near sea level are given in the following table:

W/S_w	lb/ft² kg/m²	6 30	8 40
Single seaters	l' = 12.8 ft 3.9 m	0.010	0.008
Two seaters	l' = 14.8 ft 4.5 m	0.012	0.009

Again, for practical use a mean value of 0.010 can be adopted. Combining this with the mean value for $C_L = 1.0$ and straight flight gives a corrected value of 0.033 in turns.

From the above, we come finally to a simple rule of thumb for the displacement of the wing vortex sheet below the wing-root trailing edge at the tailplane position:

Cruise	$l'/50$
Climb	$l'/30$

where "below" means normal to the respective flight paths.