

# COMPARISON OF CLASSICAL SPEED TO FLY THEORY USING SECOND, THIRD, FOURTH, AND FIFTH DEGREE POLYNOMIAL SPEED POLARS

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## I. ABSTRACT

With the introduction of natural laminar flow airfoils and more precise manufacturing methods, the modern sailplane has developed into a sophisticated aircraft. These advances have also led to the creation of more complex speed polars. Typical speed to fly models employing second degree polynomials are imprecise and sacrifices must be made, typically at the high speed end of the polar.

This paper will analyze and compare speed to fly models built upon second, third, fourth, and fifth degree polynomial polars. It will be shown that the lesser polynomial models are very inefficient, as much as under-estimating by 4 knots as the speed passes 100 knots, but are still good approximations, within 1 knot, in the lower speed end of the polar.

## II. NOMENCLATURE

Er Expected rate of climb in the next thermal  
P ( ) Speed polar function  
Sg Glider sink rate  
Si Interthermal sink rate  
Vg Glider speed

## III. INTRODUCTION

Since the advent of relatively efficient gliders with glide ratios better than 20/1, it has been the goal of aviators and theorists across the world to develop a best speed to fly theory that will enable the sailplane pilot to achieve the maximum cruise speed in cross country flight. By taking into account the rising air masses (thermals), and assuming a constant vertical air mass sink rate between thermals, a relatively simple and efficient model was created. The man largely responsible for this model, Dr. Paul MacCready, also devised a speed ring which would allow a practical application

of his theory. The model assumed that all height gains would be done in a thermal and the pilot would only make distance gains by flying between them. This type of assumption is considered classical speed to fly theory.

With the advent of more complex manufacturing methods and more precise control of laminar flow over the wing, the speed polar has developed into a very complex graph. Much of the MacCready models in use today are based on speed polars described by a second degree polynomial. With more complex polars, a higher degree polynomial is needed to define more accurately the best speed to fly. It is possible to use polars described by third, fourth, or even fifth degree polynomials to determine the best speed to fly values.

#### IV. SIMPLIFIED EXPLANATION OF MACCREADY SPEED TO FLY MODEL

In soaring, there exists a paradox, to travel forward, you have to sink downwards, and to gain altitude, you have to circle in a stationary spot. Realizing this problem, a theory was developed to extract the maximum average speed between thermals based on the rate of climb expected in the next thermal.

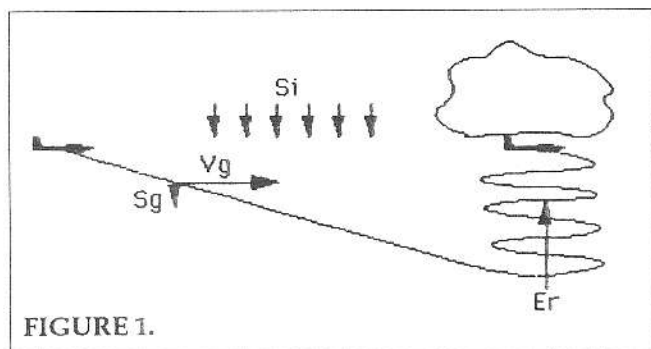


FIGURE 1.

Figure 1 displays a glider altitude that is traveling forwards towards the thermal marked by the cloud. In between the glider and the thermal, is an air mass that is sinking at a constant rate. This sink rate,  $S_i$ , is the interthermal sink rate. Upon reaching the thermal, the glider climbs to the beginning altitude at a rate of  $E_r$ . Based on this description of flight, a model was developed that would extract the minimum time necessary to fly from the top of one thermal and then up to the top of the next thermal, thereby creating the highest average speed.

Figure 2 describes this model graphically. The best speed to fly can be found by drawing a tangent, from a point on the  $S_g$  axis representing the expected climb rate plus the interthermal sink rate, to the glider's speed polar.

For every glider there is a corresponding polar. The polar seen in figure 2 relates the horizontal velocity, speed, of the glider with the vertical velocity, sink. Mathematically, a polar can be simply represented by a polynomial equation where  $V_g$ , the speed of the glider is plotted on the x axis and  $S_g$ , the sink rate is plotted on the y axis as a function of  $V_g$ . Today, speed polars are getting more complex and more difficult to describe

with a second degree polynomial, but for the ease of this explanation we will use a second degree polynomial. The polar can be easily described with the following equation:

$$P(V_g) = S_g = aV_g^2 + bV_g + c \quad (1)$$

where  $a, b, c$  are arbitrary constants. Using the graphical description of the MacCready model it is possible to

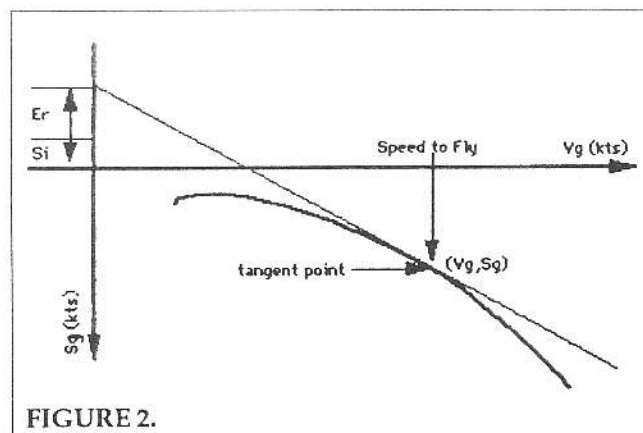


FIGURE 2.

solve for the y intercept values if we know the function and its derivative.

Given  $P(V_g)$  and its derivative  $P'(V_g)$  and the equation of a line,  $y=mx+b$ , we can substitute in point slope form and solve for the y intercept, "b."

$$(y - P(V_g)) = P'(V_g)(x - V_g) \quad (2.1)$$

$$y = P'(V_g)x - P'(V_g)V_g + P(V_g) \quad (2.2)$$

$$b = P(V_g) - P'(V_g)V_g \quad (2.3)$$

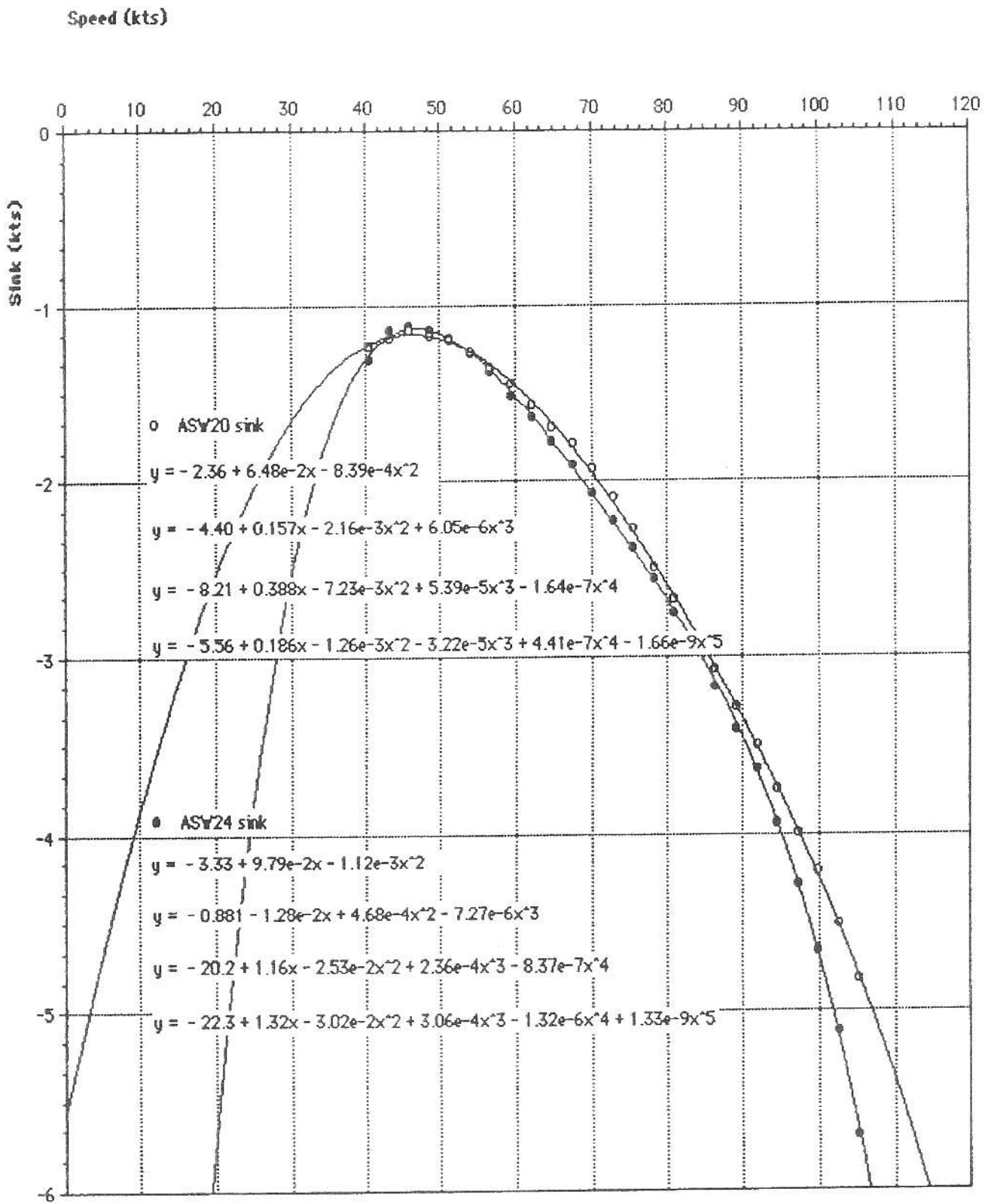
Clearly as the function to describe the speed polar increases in polynomial degree, a more accurate relationship between "b" and  $V_g$  will be obtained. The next step then is to use values obtained from a graphics program that fits varying degree polynomials to a graph and then analyze the differences between the values of "b" obtained from the relationship in equation 2.3.

#### V. EXPLANATION OF EXPERIMENTAL TECHNIQUES

The sailplanes chosen for this analysis are representative of the current generation sailplanes in both the 15-meter class, and the standard class. Chosen to represent the 15-meter class is the ASW-20C, and representing the standard class is the ASW-24. Using the graphing program "Cricket Graph," it is possible to obtain polynomial equations describing the curvature of data. Using speed polar data obtained from flight tests and manufacturers measured data, four equations describing each speed polar are obtained. See Appendix A. The constants from these equations are then entered onto a spreadsheet and the calculated values of "b" are created using the formula:

$$b = P(V_g) - P'(V_g)V_g \quad (2.3)$$

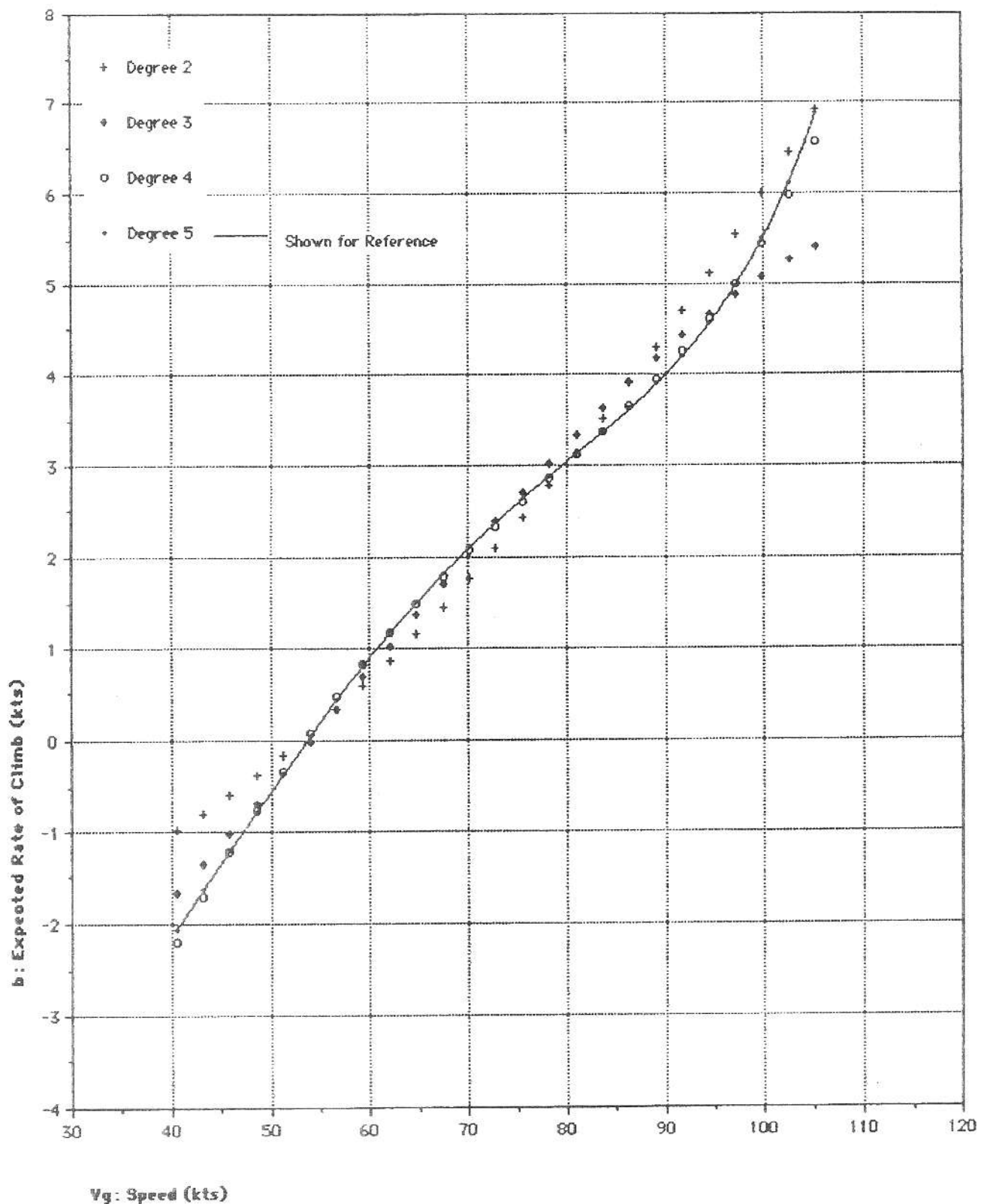
APPENDIX A: Speed polars and polynomial equations



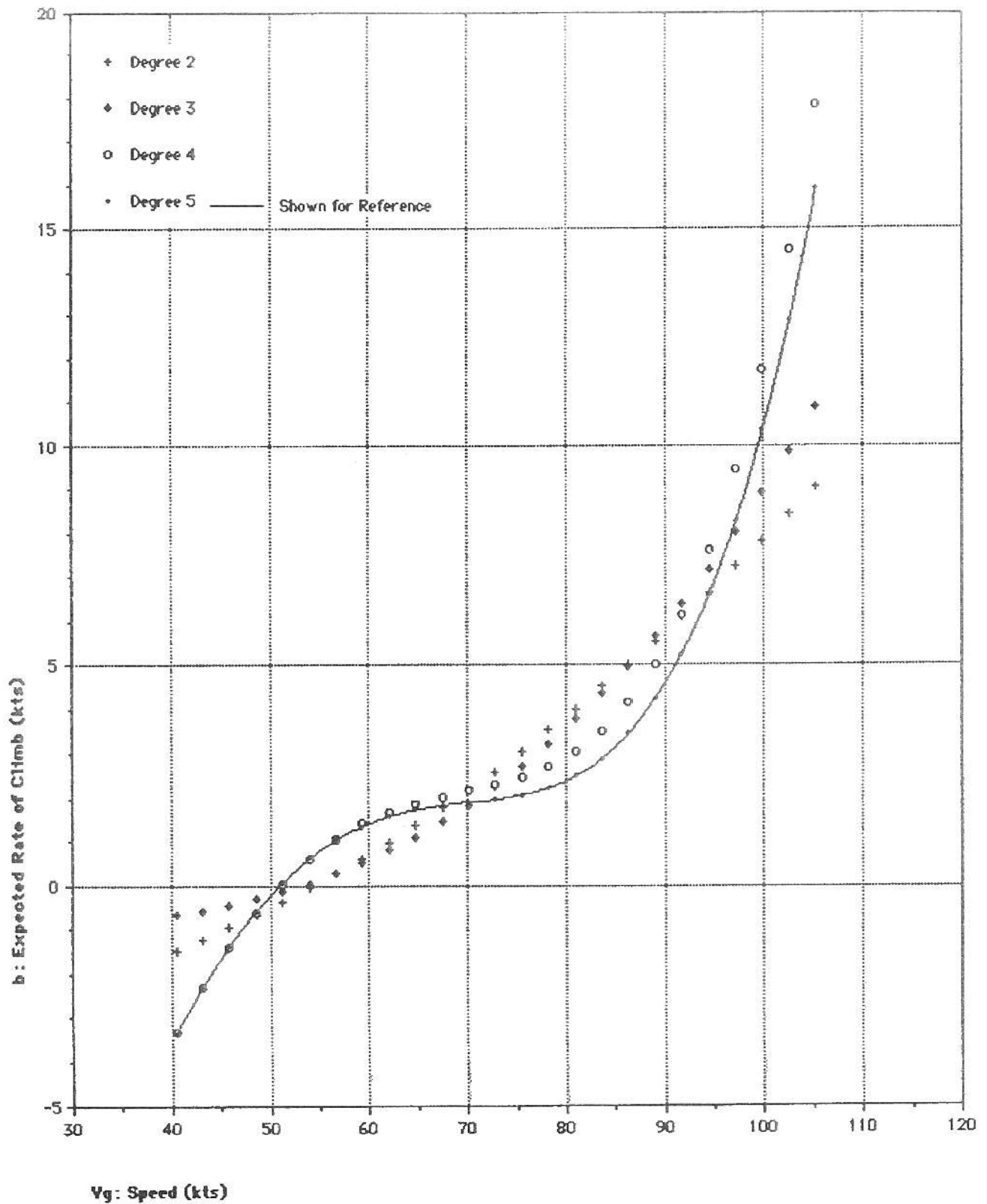
Fifth Degree Polynomial Curve Fit Shown for Reference

These values are then calculated using all four of the equations to describe the polar. These data are then graphed, again using cricket graph. See Appendix B.

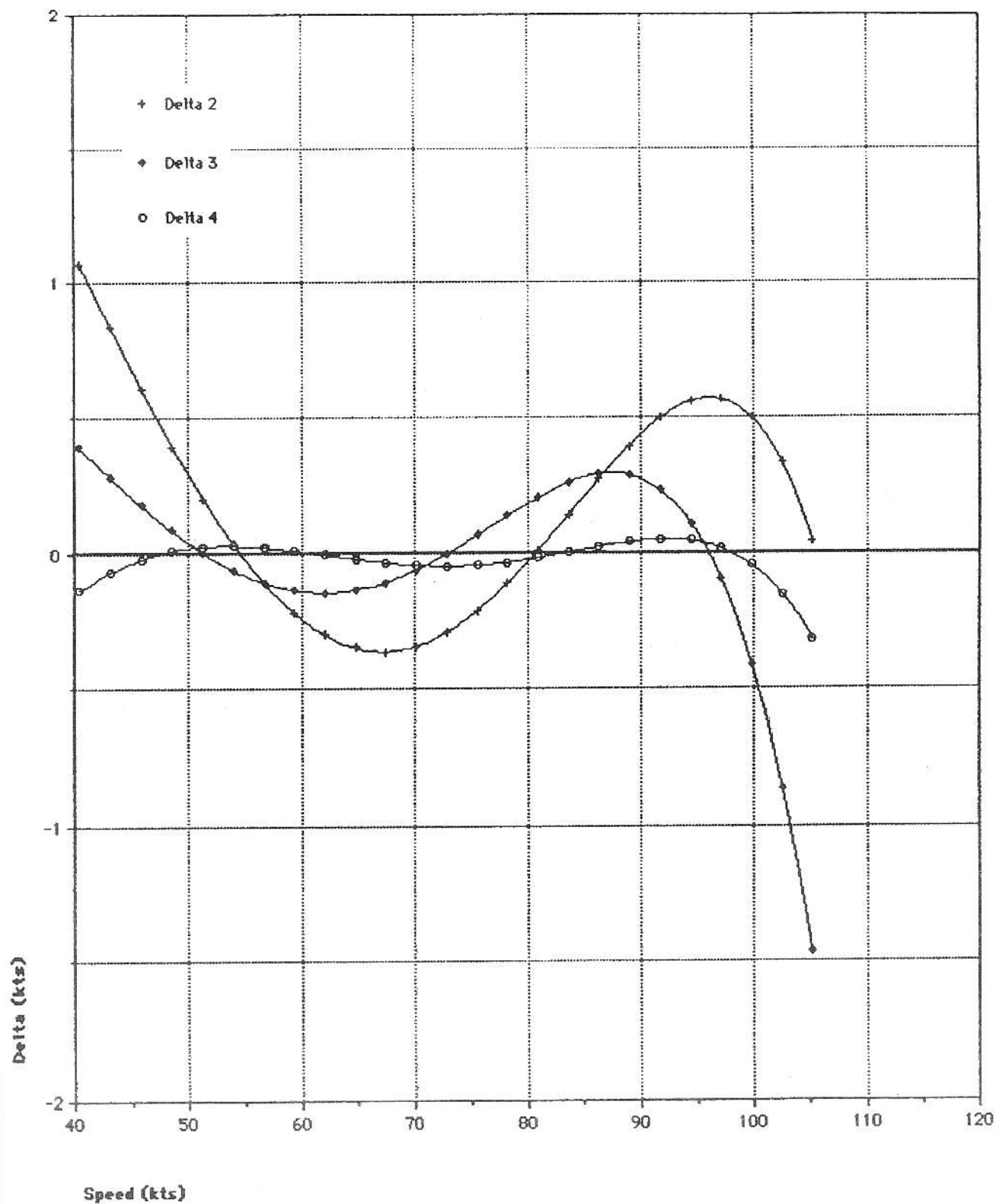
APPENDIX B: ASW 20C Speed to Fly



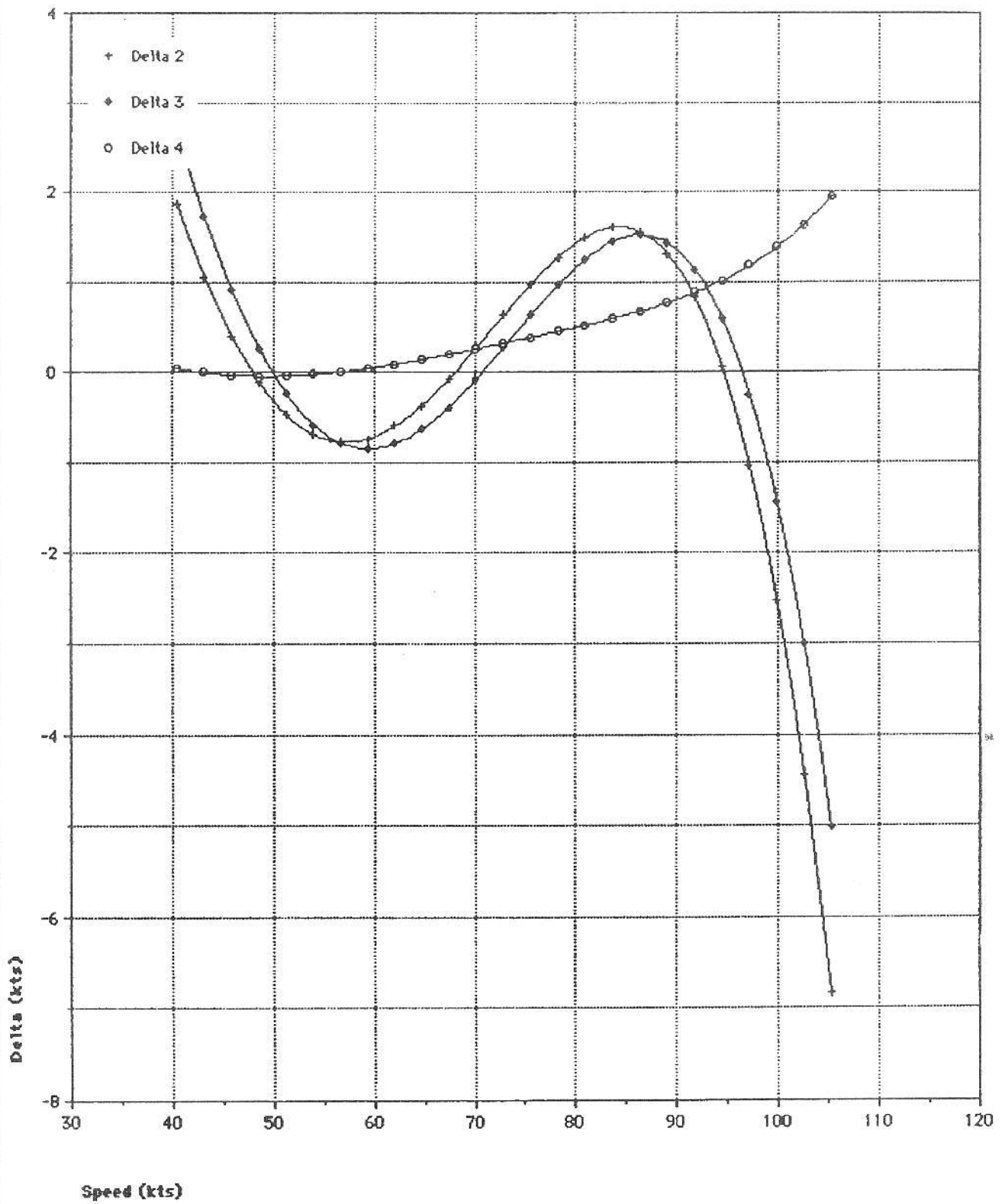
APPENDIX B: ASW 24 Speed to Fly



APPENDIX C: ASW 20C Errors



APPENDIX C: ASW 24 Errors



## VI. COMPARISON OF SECOND, THIRD, FOURTH, AND FIFTH DEGREE POLYNOMIAL SPEED POLARS

To compare the various speed polars, we will assume that the calculated values of "b," using the fifth degree polynomial, are correct. The values of the other speed polars will be subtracted from the values of the fifth degree and then graphed against one another. It should be noted that positive values reflect a condition where the pilot could be flying faster, and negative values reflect the condition where the pilot is flying too fast and thereby losing too much altitude. Also note that we need only concern ourselves with the differences that occur after positive values of "b." Negative values of "b" indicated an expected negative lift which is illogical.

Appendix C shows the results of these differences. In the "ASW-20C Errors" graph it is observed that the errors are mostly within the 1 knot difference. It also shows how more accurately the values of "b" are obtained when the degree of polynomial is increased. In the "ASW-24 Errors" graph it is observed that the values are close up to a speed of 100 knots, but that they fall sharply off afterwards. Also note that the Fourth Degree polynomial does very well until 100 knots where it starts to rise sharply upwards. Since negative values indicate traveling too fast, and thereby losing much altitude, significant decreases in cruising speed will be obtained. Especially if one chooses to fly an ASW-24 and is using second degree polynomials to model speed to fly theory.

## VII. CONCLUSION

It is interesting to note that a second degree polynomial does quite well in describing the ASW-20C, but is much worse when describing the ASW-24. Using higher degree polynomial speed polars do more accurately describe the best speed to fly and are not too difficult to use. Although pilots may over estimate or under estimate the current conditions and cause a greater error than the polar, competition pilots are not as likely to make gross errors. In competition where the difference between first and tenth place can be only a few minutes, it is essential that the pilots have the most accurate information to enable them to fly as fast as possible.

## VIII. BIBLIOGRAPHY

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