CRUISING FLIGHT OF LIGHT AIRCRAFT AND MOTORGLIDERS

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V = AVG. SPEED
f = FUEL CONSUMPTION

1. SUMMARY

After recalling the traditional cruising conditions for maximum endurance and range, the concept of cruise for maximum V/f or v^n/f is introduced.

A different type of cruise, alternating climbs at full power with glides with engine off or at low engine rating, is then taken into consideration.

A simple mathematical model, assuming constant engine specific fuel consumption and propeller efficiency, yields simple mathematical equations. Numerical solutions for minimum f and maximum V/f show that a substantial gain in performance is obtained whenever the aircraft aerodynamic polar for the gliding configuration is better than for the climbing configuration. This is typically the case for the motorgliders retracting the engine and the propeller during the glide.

Preliminary evaluations show that a sensible advantage should be obtained also in the case of aircraft having the same aerodynamic polar in both climbing and gliding configurations, provided that the variation of engine specific fuel consumption and propeller efficiency are accounted for.

2. INTRODUCTION

Cruising flight conditions traditionally defined and considered are the cruise for:

(a) maximum endurance,

(b) maximum range.

These are relatively low speed flight conditions.

To increase the speed of a vehicle has always been a primary objective in human history. Speed costs money, however, in terms of fuel burned or, in general, energy consumed.

It seems of interest, therefore, to study flight conditions aimed at the

(c) maximum speed/consumption ratio, V/F, where V is the average speed achieved along a given distance, and f is the fuel consumption over the same distance or the fuel consumption referred to a standard distance (for instance, 100 km).

Different aircraft may achieve the same ratio V/f with different airspeeds. To prize also the speed itself, it may be interesting to study the flight condition aimed at the

(d) maximum V^n/f,

where the exponent n is greater than one.

3. LEVEL CRUISING FLIGHT

It is well known that the aircraft equipped with a piston engine and propeller attain the cruising conditions (a) (max. endurance) and (b) (max range) at two different steady level flight conditions.

In the simplifying assumption that the propeller efficiency (n) and the specific fuel consumption (c, kg/HP•h) are constant over a sufficiently large range of operating conditions, the condition (a) is attained when the aircraft is flown at the maximum value of:

$$H = C_L^{3/2}/C_D$$

and condition (b) at the maximum value of

$$E = C_L/C_D$$

where C_L and C_D are the aircraft lift and drag coefficient, respectively.

Very rarely are these conditions adopted in cruising flight. The main reasons: (i) they correspond to rather low airspeeds, (ii) the operating conditions of the propulsion unit are too far from the optimum for which engine and propeller are designed and adjusted.

It is normal practice to cruise in the conditions recommended by the manufacturer (and specified in the flight manual) referring to given values of the engine rotational speed and manifold pressure. They usually correspond to engine ratings ranging from 60% to 70% of the engine maximum continuous power.

In these conditions, however, the endurance and range are far from being optimized.

If the objective is to achieve condition (c) (max. V/f), the maximum value of

$$U = C_L^{1/2}/C_D$$

should be attained.

In fact, f (kg/100 km) is given by

$$f = \frac{100}{3 \cdot 6 \times 75} \; \frac{c}{n} \; \frac{W}{E} = 0.37 \; \frac{c}{n} \; \frac{W}{E} \qquad (1)$$

(w = aircraft weight (kg), c = engine specific fuel consumption (kg/HP•h.))

The aircraft speed V is:

$$V = \frac{3 \cdot 6 \times 4}{\delta^{1/2}} \left(\frac{W}{S}\right)^{1/2} C_L^{1/2} \qquad (\text{km/h}) \qquad (2)$$

the ratio of (2) to (1):

$$\frac{V}{f} = \frac{38 \cdot 9}{\delta^{1/2}} \; \frac{W/S}{W} \; \frac{n}{c} \; \frac{E}{C_L^{1/2}} \qquad (3)$$

shows that $V/f = (V/f)_{max}$ when

$$\left(\frac{E}{C_L^{1/2}}\right)_{max} = \left(\frac{C_L^{1/2}}{C_D}\right)_{max} = U_{max}$$

Similarly, if the objective is to achieve condition (d) (max. V^n/f), the maximum value of

$$Y = \frac{C_L^{1-n/2}}{C_D}$$

should be attained.

On the curve of P_R (power required for steady horizontal flight at a given altitude) as a function of airspeed V (Figure 1), points A,B,C,D correspond to the cruising conditions defined above:

A max. endurance	$(C_L^{3/2}/C_D)_{max}$
B max. range	$(C_L/C_D)_{max}$
C max. V/f	$(C_L^{1/2}/C_D)_{max}$
D max. V^n/f	$(C_L^{1-n/2}/C_D)_{max}$

whereas E may represent the cruising condition as currently specified (for instance, with 75% engine power).

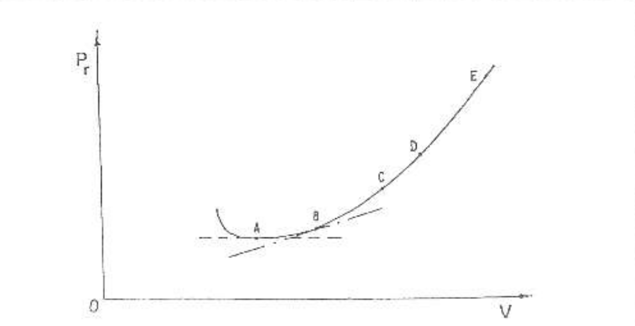


FIGURE 1.

If T_R (thrust required for steady level flight at a given altitude) is plotted versus V, point C (max. V/f) lies on the tangent to the curve drawn from the origin (Figure 2).

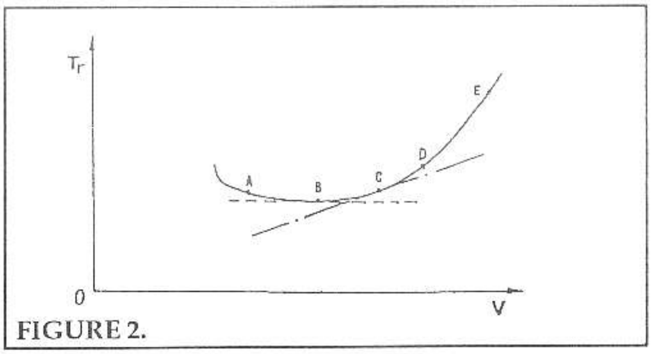


FIGURE 2.

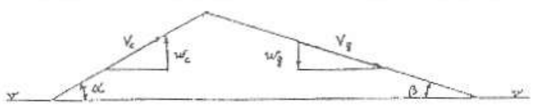
4. THE "SAW-TOOTH" CRUISING FLIGHT

Another cruising technique can be imagined, by which climbs at full power alternate with glides with engine off or at low engine rating (e.g. zero thrust).

The simplest mathematical model for this type of flight assumes constant c and n not only at different engine ratings but also with varying flight altitude.

If we also assume that: (a) the engine power output be independent of the altitude (or, if we assume an average value for it); (b) the climb angle α and the glide angle ß be small, the mathematical expressions shown in Figure 3 are easily derived, for the average speed along the course (V) the fuel consumption per 100 km distance (f), the ratios V/f and $V^n/f \cdot V/_c$, w_c, v_g, w_g, are all expressed in m/s; c in kg/HP•h, and P = engine power in HP.

For a given aircraft at a given configuration and total weight V_g and W_g are correlated by the speed polar, V_c and W_c by the climbing performance curve.



average speed along the course:

$$V = 3.6 \frac{w_g V_c + w_c V_g}{w_g + w_c} \quad [km/h] \qquad (4)$$

fuel consumption per 100 km distance:

$$f = \frac{27.8\, c\, P_m}{V_c + \frac{w_c}{w_g} V_g} \quad [kg/100\,km] \qquad (5)$$

in steady level flight: $f = 0.37 \frac{c}{\eta} \frac{W}{E}$

speed over fuel consumption:

$$\frac{V}{f} = \frac{0.1296}{c \cdot P_m} \frac{(w_g V_c + w_c V_g)^2}{w_g (w_c + w_g)} \quad \left[\frac{km/h}{kg/100\,km}\right] \qquad (6)$$

in steady level flight: $\frac{V}{f} = 38.9 \frac{\eta}{c} \frac{\sqrt{\frac{W}{S}}}{W} \frac{E}{\sqrt{C_L}}$

and $\frac{V^n}{f} = \frac{R}{c \cdot P_m} \frac{(w_g V_c + w_c V_g)^{n+1}}{w_g (w_c + w_g)^n} \quad \left[\frac{(km/h)^n}{kg/100\,km}\right]$

FIGURE 3.

Note that the expressions (4), (5) and (6) can be applied or easily extended to aircraft other than light airplanes and motorgliders, provided that the assumption of small climb angle (α) and glide angle (ß) are reasonable. Of course, the idea of a "saw-tooth" cruising technique for a large transport aircraft may be risible.

5. NUMERICAL SOLUTIONS

For both functions V_g - W_g and V_c - W_c it is reasonable to assume polynomial quadratic expressions such as:

$$W_g = k_1 + k_2 V_g + k_3 V_g^2 \qquad (7)$$

$$W_c = k_4 - k_5 V_c - k_6 V_c^2 \qquad (8)$$

Numerical solutions have been determined for three typical aircraft: a light airplane (Cessna 172 P), a high performance motorglider with retractable engine and propeller (ASW-22 BE) and a motorglider with fixed propeller (Dimona H36).

Figures 4 to 9 show the results of these calculations as plots of V_c versus V_g, on which isocurves V/f = const. or f = const. appear.

It can be seen that the aircraft for which the same aerodynamic polar has been assumed for both climbing and gliding configurations reach the optimum when $V_c = V_g$, whereas the ASW-22 BE which has engine and propeller retracted in the gliding configuration, $V_c \neq V_g$.

In the former case this unstationary "saw-tooth" type of cruising technique is not advantageous: the same values of (V/f) max and fmin are attained as in the stationary cruise. In the latter case, the advantage is remarkable: about 30% for the ASW-22 BE.

In all cases, moreover, the optimum conditions for climbing and for gliding correspond to Umax for (V/f) max and to Hmax for fmin, respectively. This is true if both the climb and glide angle are assumed to be small, which is certainly verified for the cases considered.

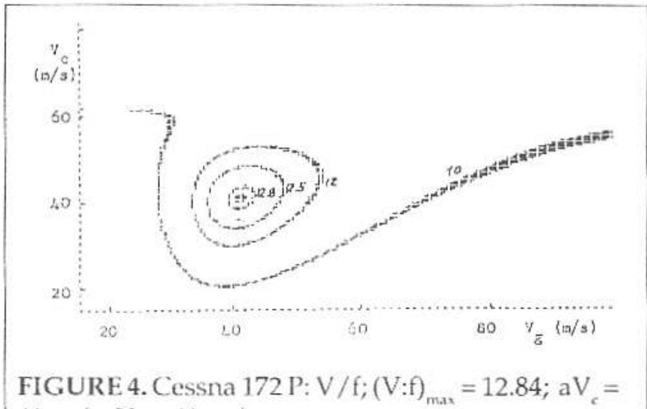


FIGURE 4. Cessna 172 P: V/f; $(V{:}f)_{max}$ = 12.84; aV_c = 41 m/s; V_g = 41 m/s

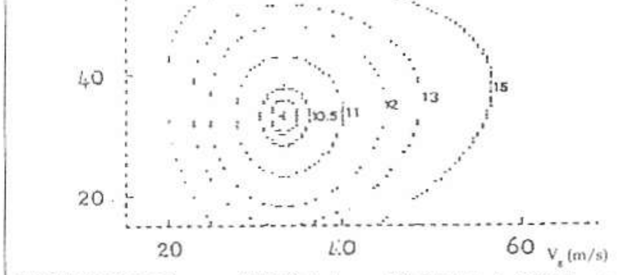


FIGURE 5. Cessna 172 P:f; f_{min} = 10.35 (litri/100 km); aV_c = 33.5 m/s; V_g = 33.5 m/s.

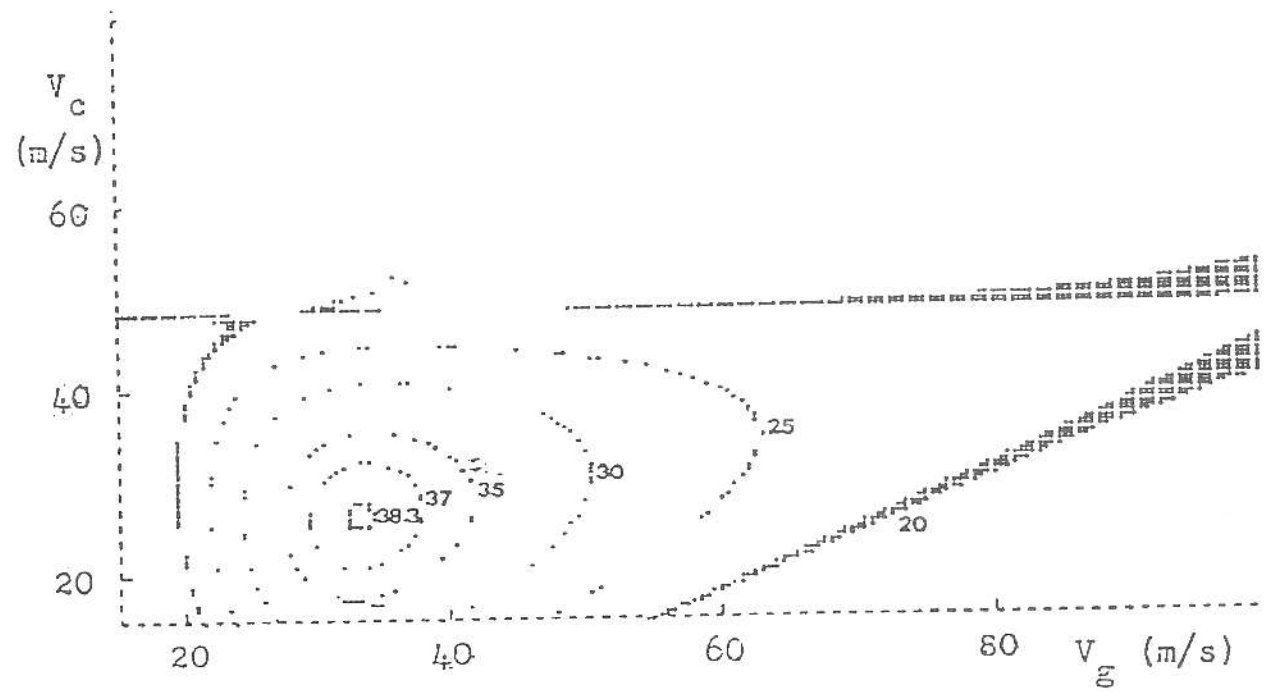


FIGURE 6. ASW 22 BE : V/f; $(V/f)_{max}$ = 38.4; aV_c = 26.5 m/s; V_g = 33.5 m/s

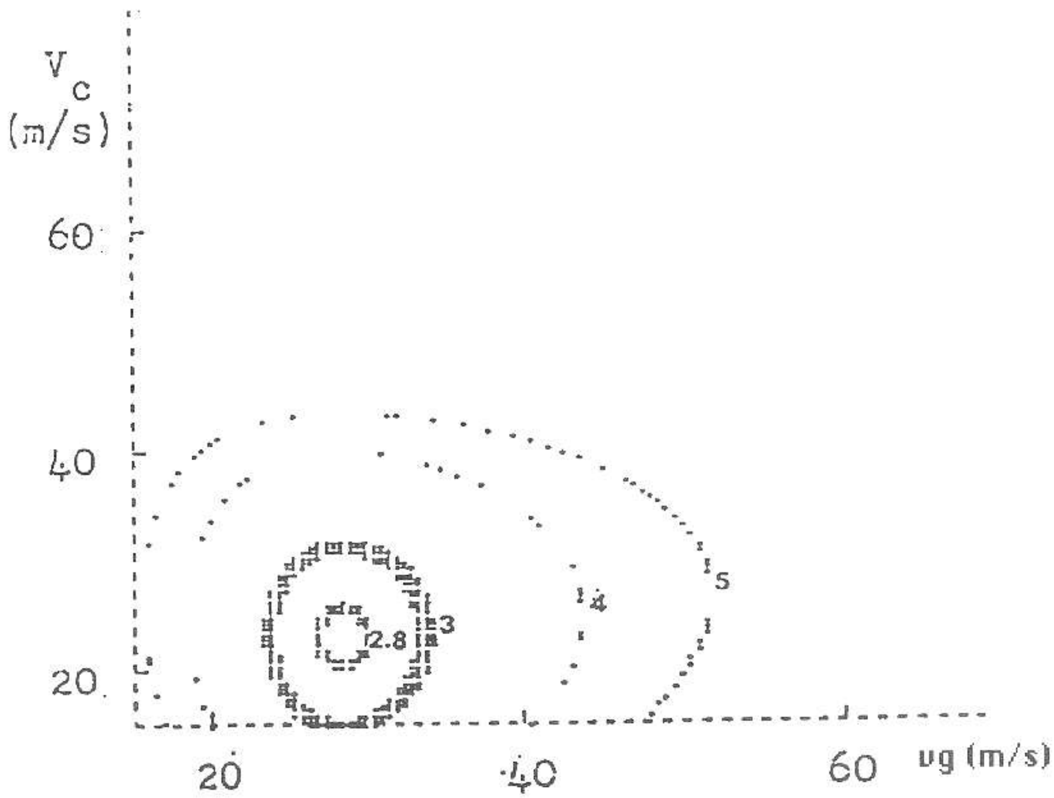


FIGURE 7. ASW 22 BE : f; f_{min} = 2.8 (litri/100 km; aV_c = 23.5 m/s; V_g = 28.5 m/s

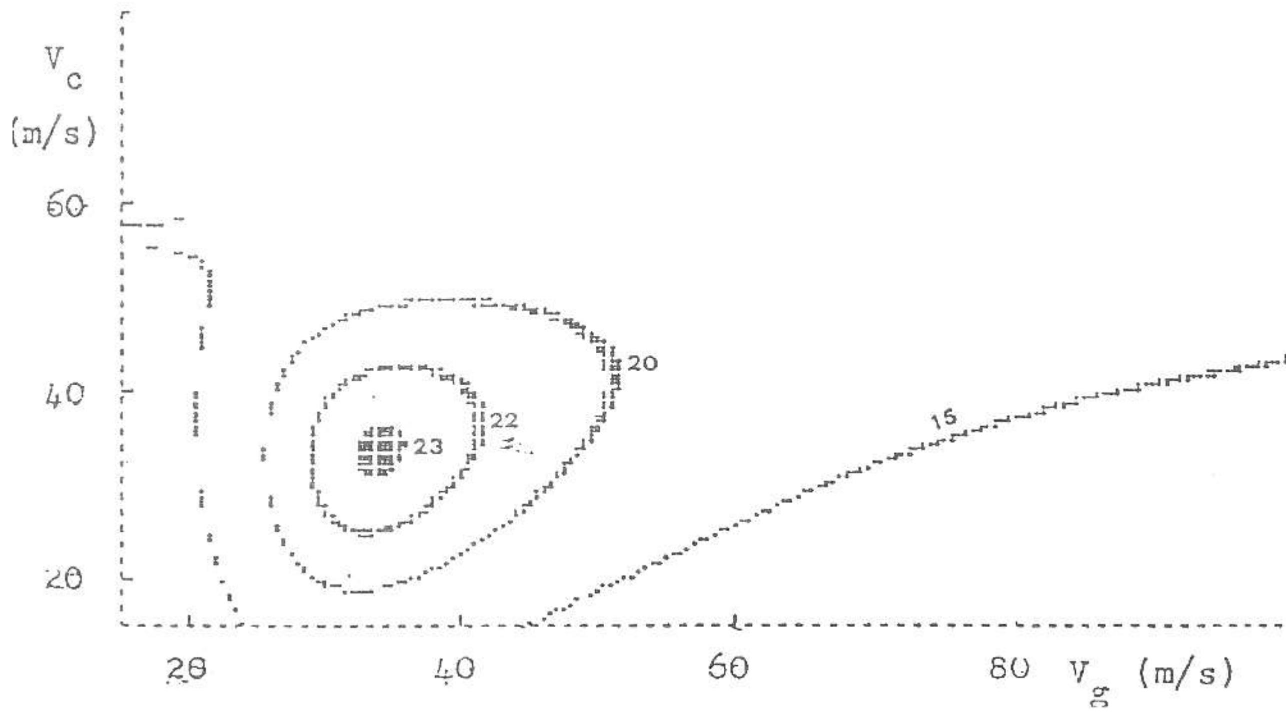


FIGURE 8. Dimona H 36 : V/f; $(V/f)_{max}$ = 23.0; aV_c = 34 m/s; V_g = 34 m/s

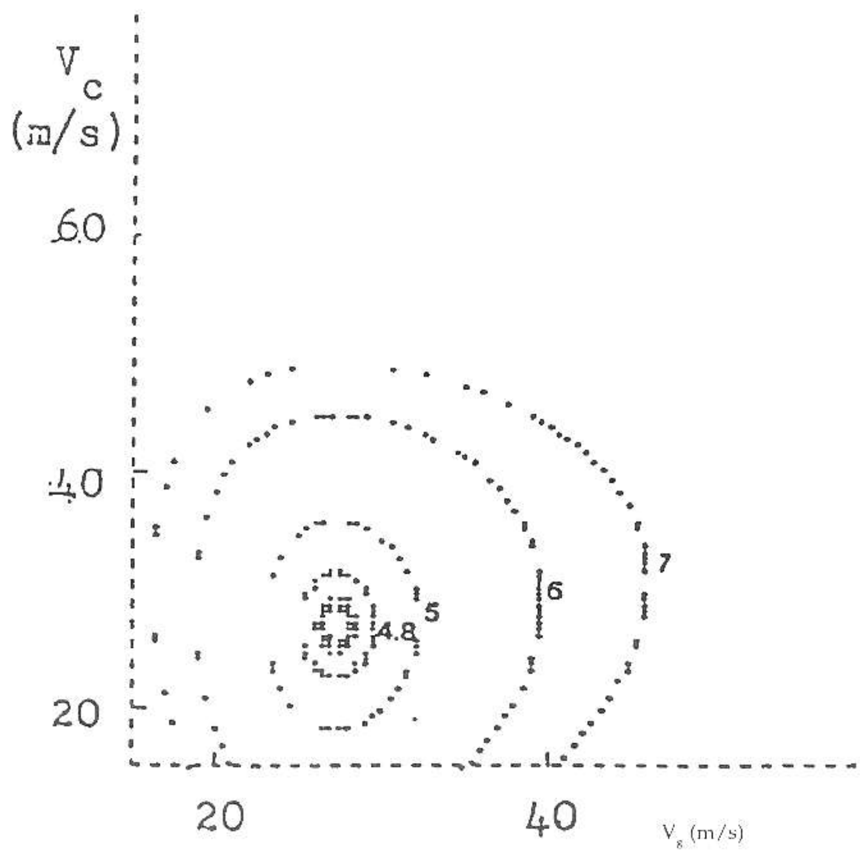


FIGURE 9. Dimona H 36 : f; f_{min} = 4.74 (litri/100 km); aV_c = 27.5 m/s; V_g = 27.5 m/s

6. AN ANALYTICAL APPROACH

With the help of a good mathematical mind, that of my friend and colleague Aldo Muggia, Professor of Aerodynamics (retired), Politecnico di Torino, the following analytical expressions and graphical interpretations have been derived.

Maximum range (f_{min}):

From expression (5), f is at a minimum when

$$Vc + Vg\ \frac{W_c}{W_g} \tag{9}$$

is maximum.

Differentiating the function (9) yields:

$$dV_c + dW_c\frac{V_g}{W_g} + W_c\frac{dV_g}{W_g} - \frac{W_c}{W_g^2}V_g dW_g = 0$$

and therefore:

$$\frac{dV_c}{dW_c} = -\frac{V_g}{W_g} \tag{10}$$

and:

$$\frac{dV_g}{dW_g} = \frac{V_g}{W_g} \tag{11}$$

If the curves W_c - V_c and W_g - V_g are available and plotted as shown in Figure 10, the condition (11) is satisfied if the tangent from the origin is drawn to the speed polar W_g - V_g: the abscissa of the tangent point A defines the optimum V_g.

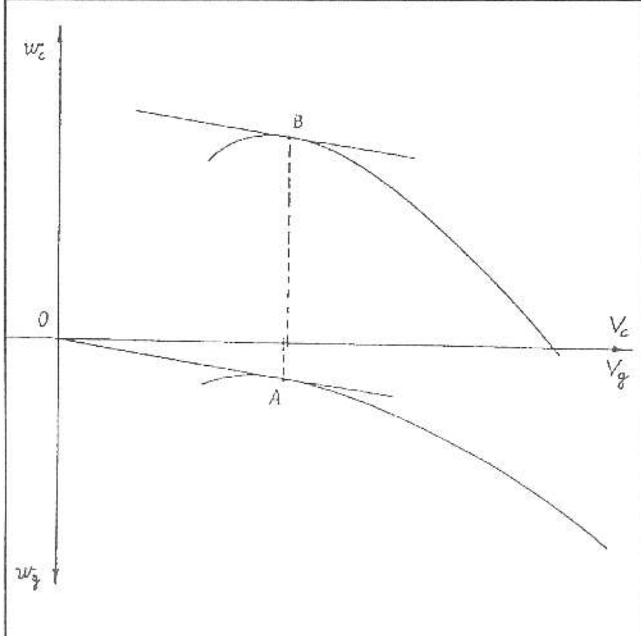


FIGURE 10. Conditions for f_{min}

Condition (10) is satisfied if a tangent to the climb curve W_c - V_c is drawn parallel to OA: the abscissa of the tangent point B defines the optimum V_c.

Maximum speed/fuel consumption (v/f):

From expression (6), V/f is at a maximum when

$$\frac{\left(\frac{V_c}{W_c}+\frac{V_g}{W_g}\right)^2}{\frac{1}{W_c}\left(\frac{1}{W_c}+\frac{1}{W_g}\right)} = \frac{(W_gV_c + W_cV_g)^2}{W_g(W_c + W_g)} \tag{12}$$

is maximum.

Differentiating the function (12) yields:

$$\frac{2}{W_gV_c + W_cV_g}(W_gdV_c + V_cdW_g - W_cdV_g + V_gdW_c) = \frac{dW_g}{W_g} + \frac{dW_c + dW_g}{W_c + W_g}$$

and, therefore:

$$-\frac{dV_c}{dW_c} = \frac{dV_g}{dW_g} = \frac{1}{2}\ \frac{V_g - V_c}{W_c + W_g} + \frac{1}{2}\ \frac{V_g}{W_g} \tag{13}$$

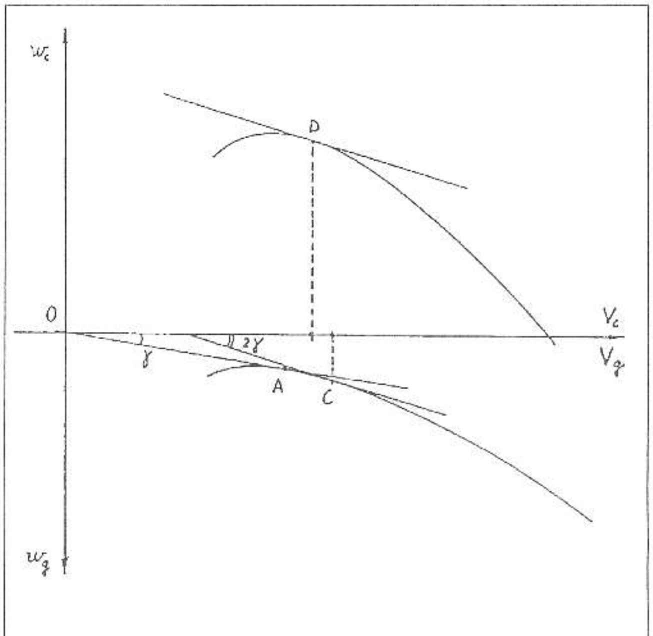


FIGURE 11. Condition for $(V/f)_{max}$

Whenever $(V_g-V_c)/(W_c + W_g)$ can be considered a small quantity with respect to V_g/W_g, (13) is simplified into

$$\frac{dV_c}{dW_c} \simeq -\frac{1}{2}\ \frac{V_g}{W_g} \tag{14}$$

and

$$\frac{dV_g}{dW_g} = \frac{1}{2}\ \frac{V_g}{W_g} \tag{15}$$

In Figure 11, according to (15) the tangent to the speed polar W_g - V_g in C corresponds to a glide angle double that in A. The abscissa of C yields the optimum V_g.

Condition (14) is satisfied if a tangent in D is drawn to the climb curve parallel to the tangent in C to the speed polar. The abscissa of D yields the optimum V_c.

Maximum ratio V^n/F:

V^n/f is at a maximum when

$$\frac{(W_g V_c + W_c V_g)^{n+1}}{W_g (W_c + W_g)^n} \tag{16}$$

is maximum.

Differentiating the function (16) yields:

$$\frac{dV_g}{dW_g} = -\frac{dV_c}{dW_c} = \frac{nW_g(V_g - V_c) + V_g(W_c + W_g)}{(n+1)\,W_g\,(W_c + W_g)} \tag{17}$$

if

$$\left| \frac{W_c + W_g}{V_g - V_c} \right| \gg \left| \frac{W_g}{V_g}\, n \right|$$

(17) becomes:

$$\frac{dV_c}{dW_c} = -\frac{1}{n+1}\,\frac{V_g}{W_g} \tag{18}$$

and

$$\frac{dV_g}{dW_g} = \frac{1}{n+1}\,\frac{V_g}{W_g} \tag{19}$$

In Figure 12, according to (19) the slope of the tangent to the speed polar in E is n + 1 times greater than that in A. The abscissa of E yields the optimum V.

Condition (18) is satisfied if a tangent in F is drawn to the climb curve parallel to the tangent in E to the speed polar. The abscissa of F yields the optimum V_c.

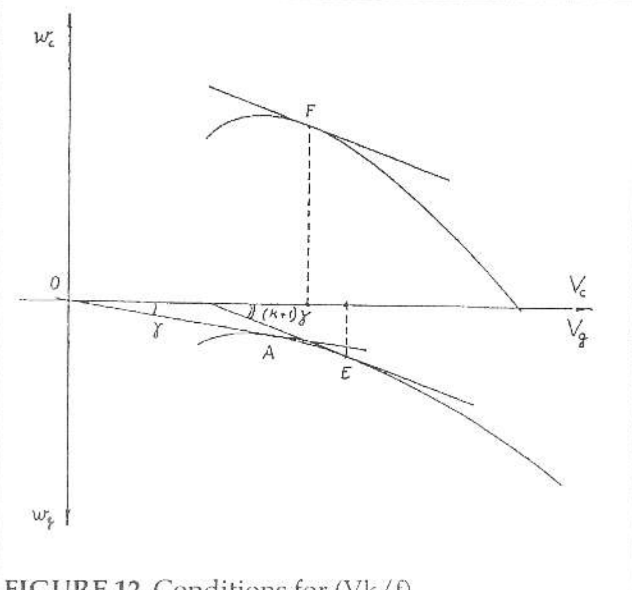


FIGURE 12. Conditions for $(Vk/f)_{max}$

7. FURTHER STEPS

In a further study of this problem, the different simplifying assumptions should be removed.

It is likely that, if the variations of the propeller efficiency and of the engine fuel consumption are accounted for, the "saw-tooth" type of flight would give an advantage also in the case of aircraft having the same aerodynamic polar in both climbing and gliding configurations (as the Cessna 172 and the Dimona exemplify here).

In fact, it is certainly true that the engine efficiency (and, therefore, the specific fuel consumption) improves, with respect to the "stationary" cruise condition, if full throttle is employed in climbing. It is also likely that the optimum gliding condition would be attained with a low engine rating (but not engine off), probably very close to the "zero thrust" condition.

The effect of the altitude, in both the climbing and gliding phases, should also be accounted for.

In particular, for the motorgliders with retractable propeller (and often engine as well) the transient phases at the end of the climb and of the glide should also be taken into account.

In a more realistic mathematical model, therefore, the performance of a "saw-tooth" cruising flight would be no more independent of the utilized change of altitude, as in the simplified model presented here.