MINIMUM WEIGHT DESIGN OF COMPOSITE SAILPLANE STRUCTURES

by Stevan Maksimovic, Aeronautical Institute, Belgrade and Zdravko Gabrijel, University of Belgrade

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Introduction

The application of GFRP and CFRP for primary structures of sailplanes widely opens the possibilities for improving the aerodynamic and structural design and configuration. Specific stiffness and strength values of GFRP and CFRP is dramatically improved compared to the traditional materials [1], Figure 1.

This also allowed creation of new engineering in design that was not possible before. The new materials make possible flight performance improvements by using bigger wing spans and thinner airfoils, realization of variable geometry becomes easier, also increase water ballast and, what is very important, an increase of torsional rigidity and reduced flutter problems. Today, a wing span over 20 m with a thin airfoil is quite common. The designer's most important aim is to create an optimum design i.e. to achieve optimum strength with minimum possible weight of the sailplane structure. Some of the major factors which must be considered in the conceptual design stage are structural weight; costs associated with development, production, certification and operations; availability of special facilities; experience; and confidence, Figure 2. To fulfill this structural optimization algorithm with the specified design constraints, such as material strength, buckling, frequencies, displacement and flutter constraints were utilized. Since the imposed constraints are nonlinear and the structural model is generally indeterminate, the algorithms are always iterative in nature. The algorithm for optimization consist of two main steps. The first step is to analyze the structure in order to find its response to the applied loads, and the second step is to redistribute the material. When the structure is discretized into a number of elements, analysis is performed by finite element methods. The redistribution of the material is achieved by using a recurrence relation of a search formula, with the objective that after each iteration the weight of the structure is reduced and all constraints are satisfied. Recursion formulas for resizing the design variables based on Kuhn-Tucker necessary condition for each type constraints incorporated a design algorithm which

FIGURE 1. Specific strength vs specific modulus.

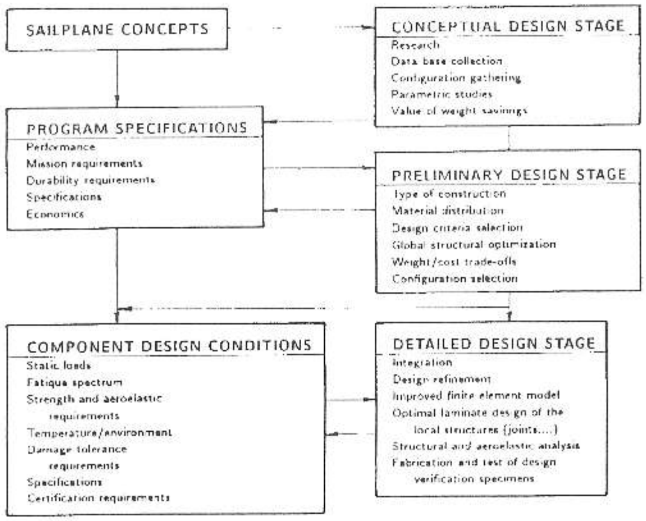

FIGURE 2. Simplified composite structure design process.

exploits the concept of a most critical constraint. The main advantage of the algorithm is that the computational efforts of resizing does not increase very sharply with increasing of the design variables. The algorithm was settled in its final form after different problems were solved and the results compared with those of other researchs.

Definition of the Problem and the Optimality Criterion

In the last two decades algorithms were developed in conjuction with finite element analysis (FEA) based on the nonlinear mathematical programming methods (MP) and the optimality method. In this paper we will primarily use algorithms based on the optimality criterion method [2,3]. Methods for the optimum design of structures have progressed rapidly in recent years. In particular, optimality criteria approaches have significantly advanced the state-of-the-art of the minimum weight design of structures involving large finite element assemblies, and for the optimization of large practical structures with static, dynamic and stability requirements [4-6]. The potential strength of the method is that the number of iterations needed to convergence to an optimum is virtually independent of the number of structural members. This property makes this method well suited for the optimum sizing of large practical structures. The optimality criterion for the generalized constraints is derived here first, then is specialized for the displacement constraint problem. Consider a structure which is discretized into N finite elements. For this structure, the load displacement relation is written as:

$$Ku = F \qquad (1)$$

where F is the applied load vector, u is the displacement vector, and K is the total stiffness matrix of the structure given by:

$$K = \sum_{i=1}^{N} a_i^T k_i a_i \qquad (2)$$

In Equation (2), k_i is the stiffness matrix of the i-th element, a is the compatibility matrix of the i-th element, and a_i is the transpose of a_i. The weight of the sailplane structure $W(X_i)$ is given by

$$W(X_i) = \sum_{i=1}^{N} \rho_i X_i l_i \qquad (3)$$

where p_i, is the mass density and $A_i l_i$ is the volume of the element. The design variable is X_i and l_i is a constant that depends on the geometry of the element. The generalized constraints $g_j(X_i)$ imposed on the structure can be written as

$$g_j(X_i) = C_j(X_i) - \bar{C}_j \leq 0, \quad j = 1, \ldots, p \qquad (4)$$

where $C_j(X_i)$ is the actual value of the j-th constraint and C_j is its limiting value of desired value. The total number of constraints on the structure is p. The objective is to minimize $W(X_i)$ subject to the constraints given in Equation (4). Using Equations (3) and (4), the Langrange function $L(X_i, \lambda_i)$ can be written as

$$L(X_i, \lambda_i) = \sum_{i=1}^{N} \rho_i l_i X_i + \sum_{j=1}^{p} \lambda_i g_j(X_i) \qquad (5)$$

where λ_j are the Lagrangian parameters. The necessary conditions for the local constrained optimum are obtained by differentiating Equation (5) with respect to the design variables X_i. This gives

$$\rho_i l_i + \sum_{j=1}^{p} \lambda_i \frac{\partial}{\partial X_i} g_j(X_i) = 0 \quad i = 1, \ldots, N \qquad (6)$$

where $\lambda j \geq 0$ and $\lambda_j g_i = 0$. Equation (6) is the optimality criterion. In the case of the displacement constraint problem, Equation (4) can be written as

$$g_i(X_i) = \sum_{i=1}^{N} \frac{E_{ij}}{X_i} - \bar{C}_j \leq 0 \quad j = 1, \ldots, p \qquad (7)$$

where E_{ij} is the flexibility coefficient given by

$$E_{ij} = u_i^T k_i s_i^j X_i \qquad (8)$$

where u_i and s^j_i are the displacement vectors associated with the i-th element due to applied load factor and the virtual load vector S^j corresponding to the j-th constraint. For the bar structure

$$E_{ij} = F_i U_i^j l_i / E_i \qquad (9)$$

where F_i is the force in the i-th bar due to applied load, U^j_i is the force in the i-th bar due to the virtual load vector S^j and E_i is the elastic modulus of the i-th bar. The coefficients E_{ij} are constant for statically determinate structures, and for determinate structures they depend on the design variables X_i. However, they may be assumed to be constant for small changes in X_i. Using Equations (6) and (7), the optimality condition can be written as

$$1 = \sum_{j=1}^{p} \lambda_j \frac{E_{ij}}{\rho_i l_i X_i^2} \quad i = 1, \ldots, N \tag{10}$$

where

$$\lambda_j \geq 0 \tag{11}$$

$$\lambda_j g_j = 0 \tag{12}$$

The optimum structure has to satisfy Equations (10) to (12) and the constraint, Equation (7). These are the Kuhn-Tucker conditions or optimality conditions. In Equation (10) the Lagrange multipliers λ_j are positive for active inequality constraints and zero for nonactive constraints. These are $(N+p)$ nonlinear equations corresponding to the N design variables X_i and the p Lagrange multipliers λ_j.

In the OC methods the criterion is derived for the dominant type of constraint imposed on the structure, and that criterion is used to develop the algorithm. In the case of most structures it is likely that one can predict the type of constraint which will be most active at the optimum and use the algorithm based on the constraint. Then one can treat all other constraints as being passive. In this paper a design algorithm exploits concept of a single most critical constraint.

System Stability Constraints

The linear stability of structure is defined by the eigenvalue problem defined as

$$(K - \zeta K_G)q = 0 \tag{13}$$

where K is the linear total stiffness matrix, K_G is the geometric stiffness matrix and q is the eigenvector associated with the eigenvalue ζ. The critical eigenvalue is the first eigenvalue ζ_1, if the eigenvalues are arranged in ascending order. The linear buckling load of structure is given as

$$F_{cr} = \lambda_1 F \tag{14}$$

Aranging Equation (14) such that, multiplying this equation by q gives

$$q_j^T K q_j - \zeta_j q_j^T K_G q_j = 0 \tag{15}$$

Thus, the eigenvalue ζ_j can be written as

$$\zeta_j = \frac{q_j^T K q_j}{q_j^T K_G q_j} \tag{16}$$

The constraint equation for the linear static buckling of a structure can be written in reference (4) by substitutin

$$g_j = \zeta_j - \bar{\zeta}_j \geq 0 \tag{17}$$

where ζ_j is the lowest critical load factor (the lowest eigenvalue) and ζ_j is given by Equation (16). The Lagrange function of the above problem is

$$L(X, \lambda) = \sum_{i=1}^{N} \rho_i l_i X_i - \sum_{j=1}^{p} (\zeta_j - \bar{\zeta}) \tag{18}$$

Applying Kuhn-Tucker conditions we get

$$\rho_i l_i - \sum_{j=1}^{p} \lambda_j \frac{\partial \zeta_j}{\partial X_i} = 0 \tag{19}$$

The gradient of the eigenvalue ζ_j can be obtained by differentiating Equation (13) with respect to the design variable X_i and multiplying both sides by q^T.

$$q_j^T \left[\frac{\partial k_i}{\partial X_i} - \left(\frac{\partial \zeta_j}{\partial X_i} K_G - \zeta_j \frac{\partial K_G}{\partial X_i}\right)\right] q_j = 0 \tag{20}$$

The second term between brackets must be equal zero, therefore it gives us

$$\frac{\partial \zeta_j}{\partial X_i} = \frac{1}{X_i} \frac{q_{ji}^T k_i q_{ji}}{q_j^T K_G q_j} \tag{21}$$

where k_i is the stiffness matrix of the i-th element and q_{ji} is the component of the buckling mode associated with i-th element. If the buckling modes are normalized so that denominator of Equation (21), $(q^T{}_j K_G q_j) = W$, is equal unity then Equations (16) and (21) can be written as

$$\zeta_j = \tilde{q}_j^T K \tilde{q}_j \tag{22}$$

and

$$\frac{\partial \zeta_j}{\partial X_i} = \frac{1}{X_i} \tilde{q}_{ij}^T k_i \tilde{q}_{ij} \tag{23}$$

where

$$\tilde{q}_j = \frac{1}{\sqrt{W}} q_j \tag{24}$$

$$1 = \sum_{j=1}^{p} \frac{\lambda_j}{\rho_i l_i} \frac{\partial \zeta_j}{\partial X_i} \quad i = 1, \cdots, N \tag{25}$$

Let

$$\kappa_{ji} = X_i \tilde{q}_{ji}^T k_i \tilde{q}_{ji} \tag{26}$$

Equations (22) and (23) can also be written in terms of Equation (26) as

$$\zeta_i = \sum_{i=1}^{N} \frac{\kappa_{ij}}{X_i} \tag{27}$$

$$\frac{\partial \zeta_j}{\partial X_i} = \frac{\kappa_{ij}}{X_i^2} \tag{28}$$

Substitute from Equations (28) into Equation (25) we get

$$1 = \sum_{j=1}^{n} \lambda_j \frac{\kappa_{ij}}{\rho_i X_i^2 l_i} \qquad (29)$$

Equation (29) shows the optimality criteria for the system stability constraint. If we multiply Equation (29) both sides by X^{β} and take the β-th root we get the expression to form the recursion relation for modifying the design variables

$$X_i^{\nu+1} = X_i^{\nu} \Big(\sum_{j=1}^{n} \lambda_j \frac{\kappa_{ji}}{\rho_i X_i^2 l_i} \Big)^{\frac{1}{\beta}} \qquad (30)$$

where $(\nu + 1)$ and ν are the iterations numbers and $1/\beta$ is the step size parameters which actuates on the convergence.

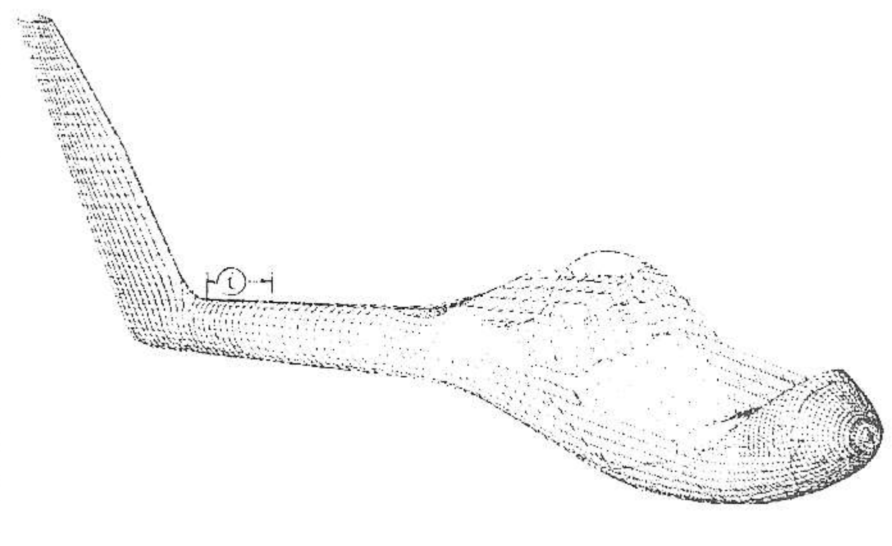

FIGURE 3. Finite element model of fuselage sailplane structure.

Reduction of the Problem Size

The main obstacles to the development of efficient structural optimization capabilities, based on the use of *MP* algorithms, were associated with the fact that the general formulation, defined by Equations (2) and (3), involved:

(1) large numbers of design variables

(2) large numbers of inequality constraints, and

(3) many inequality constraints that are computationally burdensome implicit functions of design variables.

The computational cost of the minimization problem described by Equation (2) and (3) becomes prohibitive when large structures are considered. Reduction of the problem of dimensionality has been achieved by replacing the basic problem statement, (1) and (2), with a sequence of relatively small, explicit problems that preserve the essential features of the original design optimization problem. This can be achieved through the coordinated use of approximation concepts which include:

(1) reduction of the number of independent design variables;

(2) temporary deletion of unimportant constraints; and

(3) construction of high-quality explicit approximations for retained constraint functions.

These explicit approximations of the constrains retained are used in place of the finite element analysis. It is worth pointing out that for a statically determinate structure, the behavioral constraints are linear with respect to the reciprocal of the design variables. It is then reasonable to assume that they remain nearly linear in the case of redundant structures.

For design of large structures, efficient design sensitivity analysis is particularly critical. For such structures, the substructuring concept can be effectively integrated into structural analysis and optimal design procedures. Each substructure may now be considered as a "hyper finite element" for the entire structure, whose nodal points are the boundary nodes for the substructure.

Numerical Examples

The feasibility of the optimality criterion approach to layered composites is established by initially focused attention on rear fuselage sailplane structures. Local buckling rather than system buckling is usually the main cause of elastic instability in fuselage structures under representative load cases. The skin of fuselage structures that carry compressive forces due to bending of the fuselage are most vulnerable to local buckling. In our case the rear fuselage is an unstiffened layer composite structure. Hence, system stability is the main cause of elastic instability.

In this example, minimum weight design of the representative part of rear composite fuselage is determined subject to stability constraint and the Hill-Tsai failure criterion:

$$T_t = \sqrt{\Big[\Big(\frac{\sigma_1}{F_1}\Big)^2 + \Big(\frac{\sigma_2}{F_2}\Big)^2 - \frac{\sigma_1 \sigma_2}{F_1^2} + \Big(\frac{\tau_{12}}{F_{12}}\Big)^2 \Big]} \leq 1 \qquad (31)$$

where σ_1, σ_2 and τ^{12} are the components of the stress vector σ; F_1, F_2 and F_{12} are the stresses of failure in uniaxial tension, compression and shear, respectively and T_1 is Tsai's number.

$E_{11} = E_{22} = 21$ GPa	$F_{11} = F_{22} = 185$ MPa
$G_{12} = 10$ GPa	$F_{12} = 74$ MPa
$\upsilon_{12} = 0.25$	$\rho = 1.63 \bullet 10^{-3}$ KG/m^3

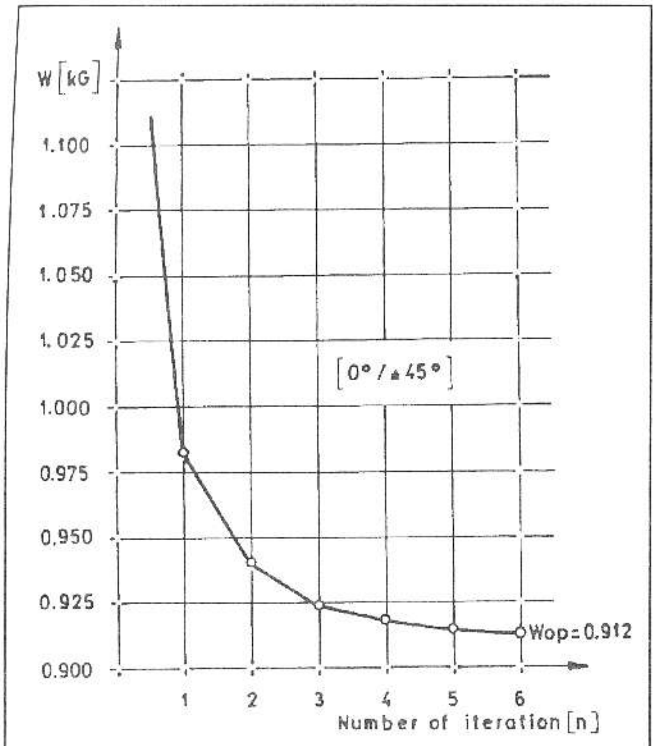

FIGURE 4. The optimization weight history for fuselage substructure.

Figure 3 shows finite element meshes for a fuselage structure. The fuselage is divided into a number of smaller substructures. Here is considered only one fuselage substructure as indicated at i. This substructure is L = 400 mm length. Stacking sequence for optimization was [0° / ± 45°]. Initial thickness of the fuselage skin was 2.25 mm. After optimization, final fuselage skin thickness is 1.65. The weight history for this substructure vs. number of iterations is shown in Figure 4, which shows that minimum weight is obtained after six iterations only. As expected in this case the stability constraints were critical. In this analysis the fuselage was under two load cases; symmetrical and unsymmetrical, respectively.

Conclusions

Advanced composite material has attractive potential for reducing the structural mass of modern sailplane components. To achieve this potential, minimum mass design must be provided that simultaneously satisfies a multitude of local and global sailplane design constraints, such as material strength, minimum-gage, buckling, displacement and flutter constraints.

An integrated structural design procedure was applied to produce the light weight KORUNDUM sailplane structure. Compared to a conventional design, a mass saving of approximately 6% of composite sailplane mass is estimated.

The present paper extends an efficient optimality criterion method, and combines system stability constraints with earlier developed [6,7] material strength, displacement, constraints, etc.

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