ANOMALOUS VARIOMETER READINGS WHEN CIRCLING IN TILTED THERMALS

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Introduction

When one is correctly centered in a thermal it might be expected that the variometer reading would be constant. However, according to Brozel (1985) about 90% of thermals over flat terrain exhibit two maxima and two minima. When not correctly centered a variometer would be expected to indicate a purely sinusoidal variation of climb rate with one maximum and one minimum per circle. However, when not quite centered in a thermal, the author has often observed either two closely adjacent minima spaced by about $90°$ and separated by a minor maximum, or a single minimum that is relatively flat in comparison with a much sharper maximum. In the former case the two minima are not of equal magnitude, the second one being less marked than the first. This implies the presence of higher order sinusoidal components, in particular a pronounced second harmonic. The purpose of this paper is to explain the origin of anomalous harmonics in variometer readings, and to suggest alternative centering methods when they are present.

Thermal axis inclination in spherical bubbles

As a thermal in the form of a spherical bubble rises through the atmosphere it experiences a downward drag at its periphery. This drag tends to set up a pattern of flow within the bubble in the form of a ring vortex having upward flow along its vertical axis. If the bubble is rising through a wind shear, then it will experience a horizontal drag along the wind direction at its periphery. This drag tends to set up a ring vortex with flow against the wind along a horizontal axis. The combination of these two effects results in a vortex having upward flow along a central axis that is tilted against the wind shear.

Following Scorer (1978), the relation between the tilt angle ß and the vertical wind shear dU/dz is given by;

$$Tan(ß) = (Tw/(g • ß)) • dU/dz = (C^2 Tr/Tw) • dU/dz \qquad (1),$$

where Tr is the thermal radius and Tw its vertical velocity, and where C is a dimensionless constant with a value of about 1.2. It should be noted that this relation is only valid if Tr • dU/dz << Tw, that is the shear is fairly weak. From equation (1) some $30°$ of tilt can develop in the axis of a thermal, for which Tw is 3 m/s and Tr is 240 meters, with a vertical wind shear of only 5 m/s per km. Also, for a given vertical wind shear, the axes of weaker thermals will tilt more against the shear than those of stronger ones. Although this effect causes the thermal axis to tilt against weak wind shears, in

strong shears the bubble as a whole also experiences a significant torque acting in the opposite direction. This is because the wind speed and drag at the bottom of the bubble is less than that at its top. This causes the axis to tilt against strong shears, though by a lesser amount than would otherwise be the case.

Geometrical considerations

When a glider is circling in a horizontal plane at a radius r around a point located at a distance d from the center of a thermal with an upright axis, the perpendicular (shortest) distance R_1 of the glider from the vertical thermal axis varies according to the following formula;

$$R_1^2 = r^2 + d^2 - 2 \bullet r \bullet d \bullet \cos(\theta) \qquad (2),$$

where θ is the angular position co-ordinate of the glider relative to the distance vector $\underline{d}$, which is directed from the circling center to the thermal center. With an axially symmetric parabolic lift distribution this results in a sinusoidal variation of the lift that has one cycle per circle.

When a glider is circling in a horizontal plane at a radius r around the center of a thermal with an axis inclined at angle ß to the vertical, the perpendicular (shortest) distance R_2 of the glider from the tilted thermal axis varies according to the relation;

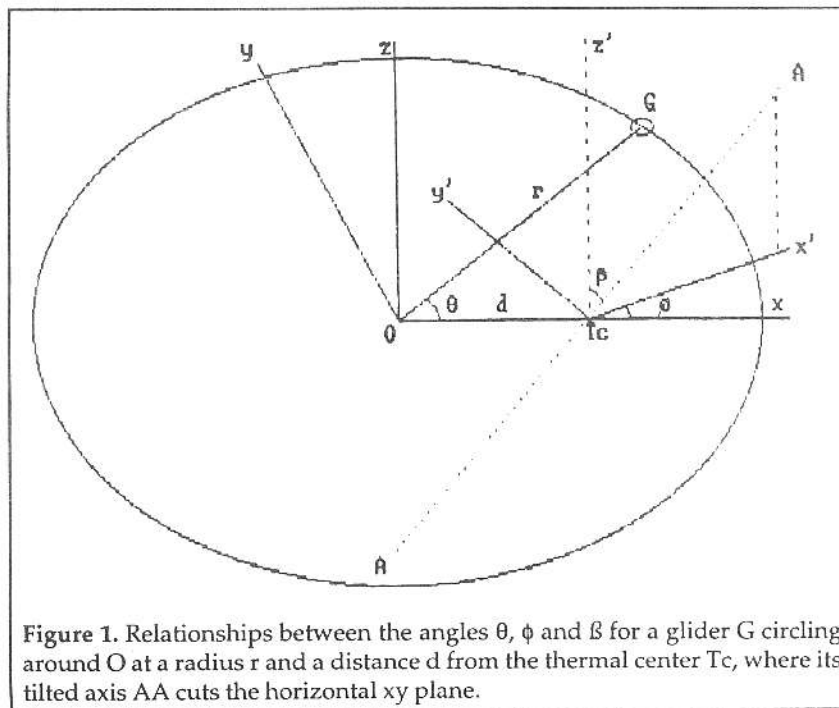$$R_2^2 = r^2(1-\sin^2(ß)/2) - r^2 \sin^2(ß)\cos(2\theta)/2 \qquad (3),$$

Figure 2. Horizontal cross-section of a tilted thermal with glider circles for a double maximum (center) and a flat minimum (right).

where θ is the angular position co-ordinate of the glider relative to the vertical projection of the thermal axis vector onto the horizontal plane of circling. With an axially symmetric parabolic lift distribution an inclined thermal axis results in a sinusoidal variation of the lift that has two cycles per circle. However, the reading variations due to axial tilt only become large enough to be noticeable when the inclination to the vertical approaches 30°.

As the optimum bank angle depends upon the lift, a corresponding variation of the bank angle at two cycles per circle should improve the climb rate. However, as this variation is very small, 2° at the most, the improvement is negligible.

In general, as shown in Figure 1, when a glider is circling in a horizontal plane at a radius r around a point separated by a distance d from the center of a thermal with an axis inclined at angle ß to the vertical, the perpendicular (shortest) distance R_3 of the glider from the tilted thermal axis varies according to the relation;

$$\begin{aligned} R_3^2 &= r^2 + d^2 - 2 \bullet r \bullet d \bullet \cos(\theta) \\ &- r^2 \sin^2(ß)(1+\cos(2\theta - 2\phi))/2) \\ &- d^2 \sin^2(ß)\cos^2(\phi) \\ &+ 2 \bullet r \bullet d \bullet \sin^2(ß)\cos(\phi)\cos(\theta - \phi) \end{aligned} \qquad (4),$$

where, as for equation (2), θ is the angular position co-ordinate of the glider relative to the separation distance vector $\underline{d}$, and where ϕ is the slew angle between the

Figure 1. Relationships between the angles θ, ϕ and ß for a glider G circling around O at a radius r and a distance d from the thermal center Tc, where its tilted axis AA cuts the horizontal xy plane.
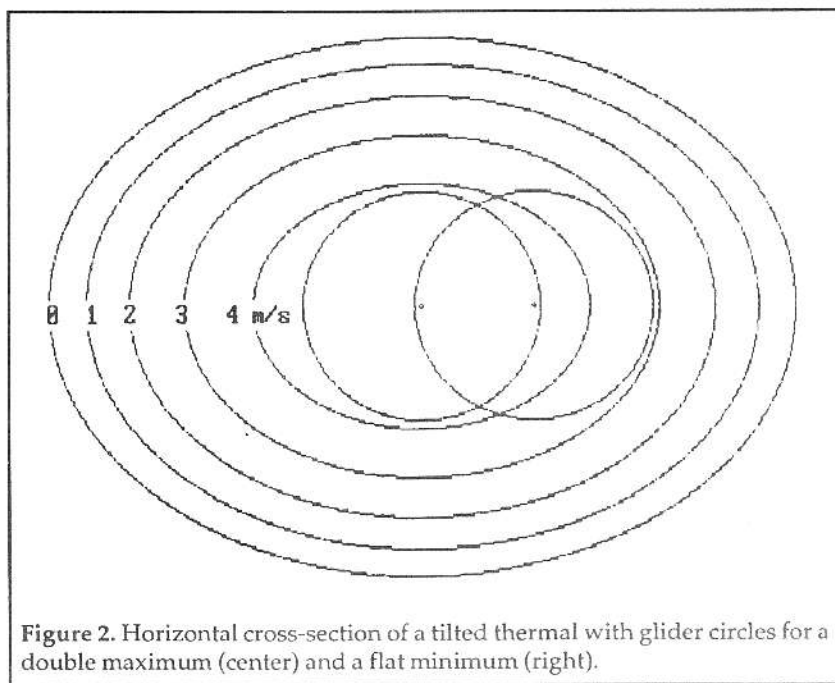
vector $\underline{d}$ and the vertical projection of the thermal axis vector onto the horizontal plane of circling.

Equation (4) includes one cycle, $\cos(\theta)$, and two cycle, $\cos(2\theta)$, per circle components that are in phase when ϕ is zero or $180°$.

When ϕ is set to $0°$ or $180°$ in equation (4) the relation becomes;

$$R_4{}^2 = r^2 + d^2 - 2 \bullet r \bullet d \bullet \cos(\theta) - r^2 \sin^2(\text{ß})(1+\cos(2\theta))/2) - d^2 \sin^2(\text{ß}) + 2 \bullet r \bullet d \bullet \sin^2(\text{ß})\cos(\theta) \qquad (5).$$

In this case the vertical projection of the tilted axis onto the horizontal plane of circling is along the line between the thermal center and the circling center.

When ϕ is set to $90°$ or $270°$ in equation (4) the relation becomes;

$$R_4{}^2 = r^2 + d^2 - 2 \bullet r \bullet d \bullet \cos(\theta) - r^2 \sin^2(\text{ß})(1 - \cos(2\theta))/2) \quad (6).$$

In this case the line defined by the vertical projection of the tilted axis onto the horizontal plane of circling is at right angles to the line extending between the thermal center and the circling center.

The variation of vertical flow in tilted thermals

The variation of the vertical flow in an upright thermal with a parabolic lift distribution is given by;

$$W = W_0 \left(1 - R^2/R_0{}^2 \right) \qquad (7)$$

where W_0 is the wind along the thermal axis, R_0 is the thermal radius at which the lift is zero, and for a tilted thermal R is R_3.

When a circular thermal core is tilted, the lines of constant flow in a horizontal plane become elliptical, as shown in Figure 2, and a glider following a circular path will experience a second harmonic component of lift in phase with the direction of tilt. From equations (5) and (7), assuming a tilt angle ß of 30° and a slew angle ϕ of $180°$, the variations of the vertical flow, $W_v = W\cos(\text{ß})$, for three different off-center distances d have been calculated, and are shown as the curves of Figure 3.

The effect of horizontal flow in tilted thermals

The horizontal wind, $W_h = W\sin(\text{ß})$, in a tilted thermal is everywhere proportional to the vertical wind, $W_v = W\cos(\text{ß})$, and always flows parallel to the vertical projection of the thermal axis vector onto the horizontal plane of circling. Owing to inertia, a glider is unable to adjust immediately to any variations of the vertical and horizontal winds experienced when circling, and this results in differences, $d(W_h)$ and $d(W_v)$, between the instantaneous values of these winds and the corresponding movements of the glider. For the vertical wind this only means that there is a slight delay between the variations of vertical flow and the corresponding vertical glider movement. For variations in the horizontal flow the situation is rather more complicated, since a glider with a total energy variometer circling at a constant speed in a tilted thermal experiences both real and apparent changes in lift due to the variations of the radial (W_r) and tangential components (W_t) of the horizontal wind respectively.

From equation (7) the effective horizontal wind $d(W_h)$ acting on the glider is given by;

$$d(W_h) = W_h(\theta) - W_h(\theta-k) = W_0 \sin(\text{ß})(R(\theta - k)^2 - R(\theta)^2)/R_0{}^2 \quad (8),$$

where k is the phase difference between the variations of the vertical and horizontal winds and the corresponding glider movement.

In general, the effect of a time invariant horizontal wind field on the indications of a total energy variometer, James (1993), is given by;

$$- (\underline{U} - (\underline{V} \bullet \text{grad})\underline{W})/g \qquad (9),$$

where $\underline{W}$ is the effective horizontal wind, $\underline{U}$ is the glider velocity relative to the local air, and $\underline{V}$ $(= \underline{U} + \underline{W})$ is the glider circling speed. $\underline{U}$, $\underline{V}$ and $\underline{W}$ are vectors in the horizontal plane, and here the dots after $\underline{U}$ and $\underline{V}$ indicates the scalar product.

Now, $-(\underline{U} \bullet (\underline{V} \bullet \text{grad})\underline{W})/g$ equals $-\underline{U} \bullet (d\underline{W}/dt)/g$, and can be rewritten as;

$$W \bullet (dV/dt)/g + U \bullet (dU/dt)/g - V \bullet (dV/dt)/g \quad (10).$$

Here the first term includes the lift due to the radial wind component W_r, the second term is the total energy compensation and the third term is the "stick" lift when the circling speed is varied. If the circling speed is kept constant, then the third term equals zero and the first term
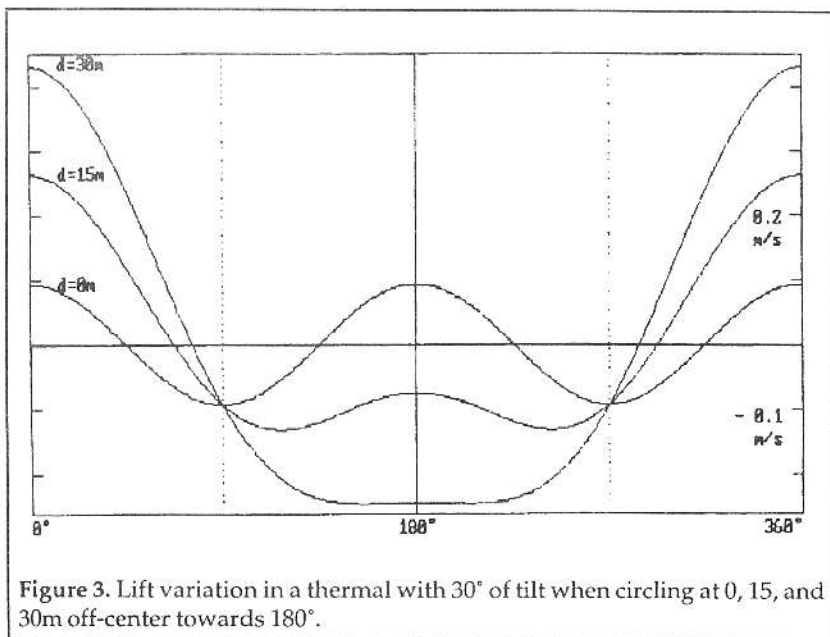

Figure 3. Lift variation in a thermal with 30° of tilt when circling at 0, 15, and 30m off-center towards 180°.

equals the lift due to the radial wind component, and can be rewritten as $Wr \cdot \tan(b)$, where b is the glider bank angle, in agreement with Gorisch (1985).

From equations (4), (8) and (9), provided that $d(Wh) \ll V$, it can be shown that the additional indicated lift Wte, due to the effective horizontal wind component, is given by;

$$Wte = - (W_o V^2 \sin(\beta)/gR_o^2)$$
$$x \left\{ \begin{array}{l} (2 \cdot d \cdot \cos(\phi)\cos^2(\beta)\sin(k)\sin(2\theta-2\phi-k) \\ + 2 \cdot d \cdot \sin(\theta)\sin(k)\cos(2\theta-2\phi-k) \\ + r \cdot \sin^2(\beta)\sin(3k/2)\sin(3\theta-3\phi-3k/2) \\ - r \cdot \sin^2(\beta)\sin(k/2)\sin(\theta-\phi-k/2)) \end{array} \right\} \quad (11).$$

The circling radius r is related to the circling speed V and the bank angle b by the relation $r = V^2 \cot(b)/g$. Thus, when d is zero, ϕ is $180°$ and b is 45, Wte of equation (11) becomes;

$$W_o V^4 \sin^3(\beta)/g^2 R_o^2$$
$$x \left\{ \begin{array}{l} \sin(3\theta-3k/2)\sin(3k/2) \\ -\sin(\theta-k/2)\sin(k/2) \end{array} \right\} \quad (12).$$

Thus, even when one is correctly centered in a tilted thermal a total energy variometer will indicate θ, 2θ and 3θ components, the first of which leads to an apparent thermal center that is slightly displaced from the true center. Although 2θ and 3θ components remain to confuse the pilot, the θ component is cancelled when a glider is circled about the point $(d, (270+k/2)°)$, where to first order;

$$d = V^2 \sin^2(\beta)\tan(\beta)\sin(k/2)/2g \quad (13),$$

which is typically a meter or so from the true center.

The lift indicated by a total energy variometer due the effect of glider inertia, while not negligible, is much less than the variation of the vertical lift due thermal inclination, being multiplied by an additional factor $2 \cdot$ $\tan(\beta)\sin(3k/2)$.

The variometer reading will also indicate the vertical movement of the glider, and a total energy variometer will therefore indicate;

$$Wvar = Wv(\theta-k) + Wte + Ws \quad (14).$$

where Ws is -ve, being the glider sink rate while circling, and k is about $15°$ (3/4 second lag) when the bank angle is $45°$.

The variation of the readings of a total energy variometer for two off-center distances and $180°$ slew angle have been calculated for a Nimbus 3 with a 45 kg/m² wing loading that is circling with $45°$ of bank at a velocity of 30 m/s, and are shown as the curves of Figures 4 and 5. One effect of inertia, as can be seen from Figure 5, is to make the first minimum more pronounced than the second, which is in agreement with the author's observations.

For a glider circling off-center at slew angles θ other than those close to $0°$ or $180°$, a total energy variometer displays a heavily distorted sinusoidal variation of the climb rate, and for slew angles ϕ slightly less than $180°$ or $360°$ the second minimum is enhanced relative to the first and may even be the larger of the two. Whenever a minor maximum occurs it is located close to $180°$, directly opposite the thermal center Tc.

Off-center drift in tilted thermals

If a glider circling in a tilted thermal had no drag, it would not sink relative to the air and would remain on the surface of a tilted cylinder having a horizontal cross section that was circular. Although its circling center would rise along the axis of this tilted cylinder in a non uniform way, a dragless glider would always remain at a constant horizontal distance from this axis, while its vertical position on the cylinder surface varied.

However, a real glider does experience drag, and as it circles in a horizontal plane it sinks vertically relative to the air. For an initially centered glider in a tilted thermal, this means that as it sinks within the rising air a separation between the circling and thermal centers develops along a line defined by the projection of the tilted thermal axis onto the horizontal circling plane. With a $30°$ tilt this separation increases by between 9 and 12 meters per turn. Thus a centering correction will be necessary during every circle, and the resulting in phase $\cos(2\theta)$ component then betrays its presence by giving a minimum that divided into two adjacent minima, as observed by the author. These effects are only observable when the tilt angle β approaches or exceeds $30°$, and the slew angle ϕ is close to $0°$ or $180°$. Even if initially this is not the case, after applying centering corrections,

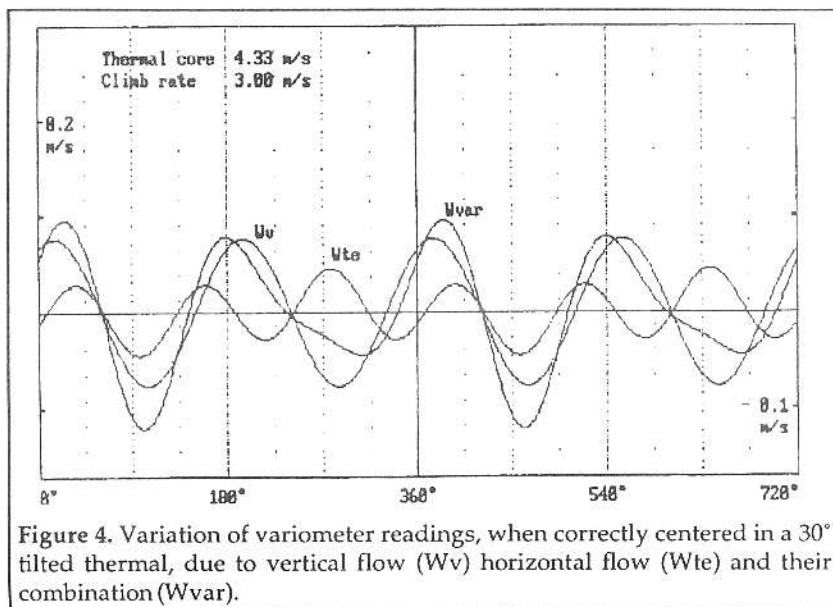Figure 4. Variation of variometer readings, when correctly centered in a 30° tilted thermal, due to vertical flow (Wv) horizontal flow (Wte) and their combination (Wvar).
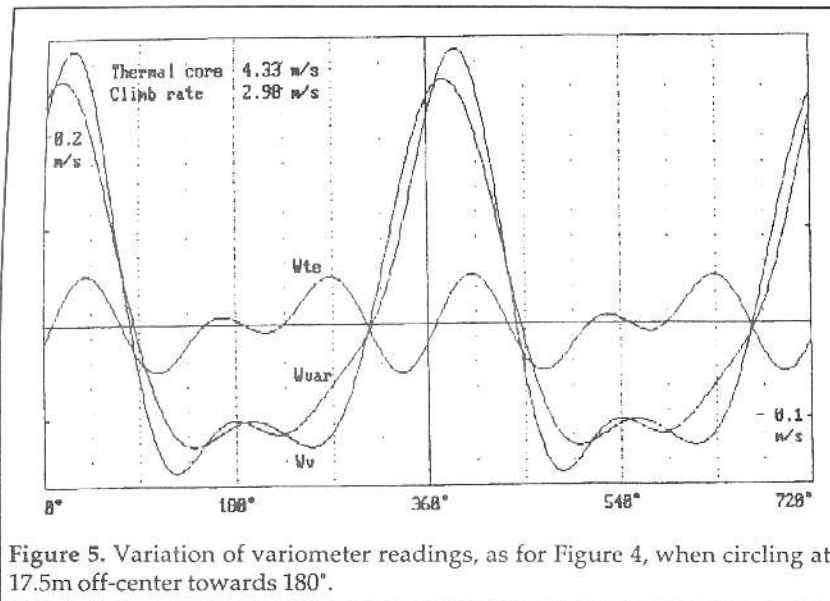
Figure 5. Variation of variometer readings, as for Figure 4, when circling at 17.5m off-center towards 180°.

the continual off-center drift towards 180° soon causes this requirement to be fulfilled.

Although a glider actually descends through the sloping side of a thermal, the pilot usually interprets this as being continually blown out of the thermal sideways. ßy way of consolation, it turns out that circling slightly off-center in tilted thermals reduces the overall climb rate less than it does in upright thermals. Another effect of drag, as can be seen from Figure 6, is to enhance the second minimum relative to the first, but not by enough to cancel out the opposite effect of inertia.

Anomalous yaw string deflections

It has been observed, Steckner (1993), that short periodic yaw string deflections occur when circling in tilted thermals. This effect is caused by inertia, whereby a glider is unable to adjust immediately to any variations of the thermal wind W that it experiences when circling, which leads to differences d(W) between the instantaneous values of this wind and the corresponding movements of the glider, as shown for a centered glider in Figure 7. This results in a varying cross-flow component across the glider canopy and corresponding deflections of the yaw string. The cosine of the angle w between the tilted thermal flow and the cross-flow over the canopy of a glider is given by;

$$\cos(w) = \sin(b)\cos(\beta) + \cos(b)\sin(\beta)\cos(\theta - \phi) \quad (15),$$

where b is the bank angle and ß is the thermal tilt angle. From equation (7) the cross-flow acting on the glider is given by;

$$d(W)\cos(w) = W_0\cos(w)\,(R^2(\theta-k)-R^2(\theta))/R_0{}^2 \quad (16),$$

This is augmented by the radial component of d(W), which produces lift, Wr • tan(b), and gives another, much smaller, cross-flow component equal to;

$$d(W)\sin(\beta)\tan(b)\sin(b)\cos(\theta - \phi) \quad (17).$$

If the pilot does not correct the resulting slip with the rudder, then the tangent of the slip angle a between the air flow and the fuselage axis will be given by;

$$\tan(a) = d(W)(\cos(w) + \sin(\beta)\tan(b)\sin(b)\cos(\theta - \phi))/U \quad (18),$$

where U is the airspeed along the fuselage axis. The variation of the slip angle a with the angular position, for a glider banked at 45° in a thermal tilted by 40°, is shown in Figure 8. It includes θ, 2θ and 3θ components. Although the variation of the slip angle is very small, just a few tenths of a degree, the actual deflection of the yaw string is magnified by the curvature of the canopy on which it is mounted. A central reference mark at the free end of the yaw string also helps. If a thermal core is tilted with the wind, then yaw string deflections in the downwind sector are much greater than those in the upwind sector, and are most pronounced when circling at a shallow bank angle in a well tilted thermal.

After one enters the downwind sector, the yaw string moves from the center towards the upper wing, then in the most downwind quadrant it moves back across the center towards the lower wing, and finally in the last part of the downwind sector it moves back to the center again. These anomalous yaw string movements can be used to judge the timing of the centering corrections
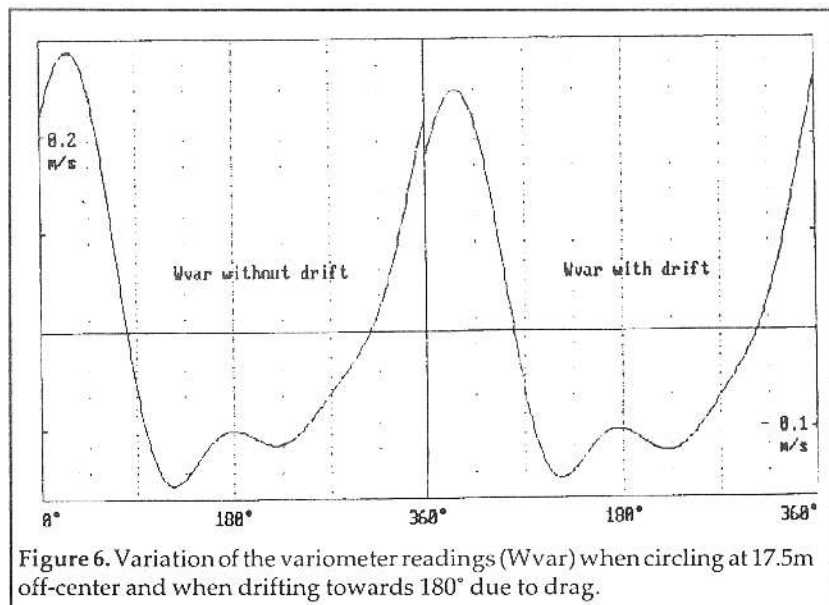

Figure 6. Variation of the variometer readings (Wvar) when circling at 17.5m off-center and when drifting towards 180° due to drag.
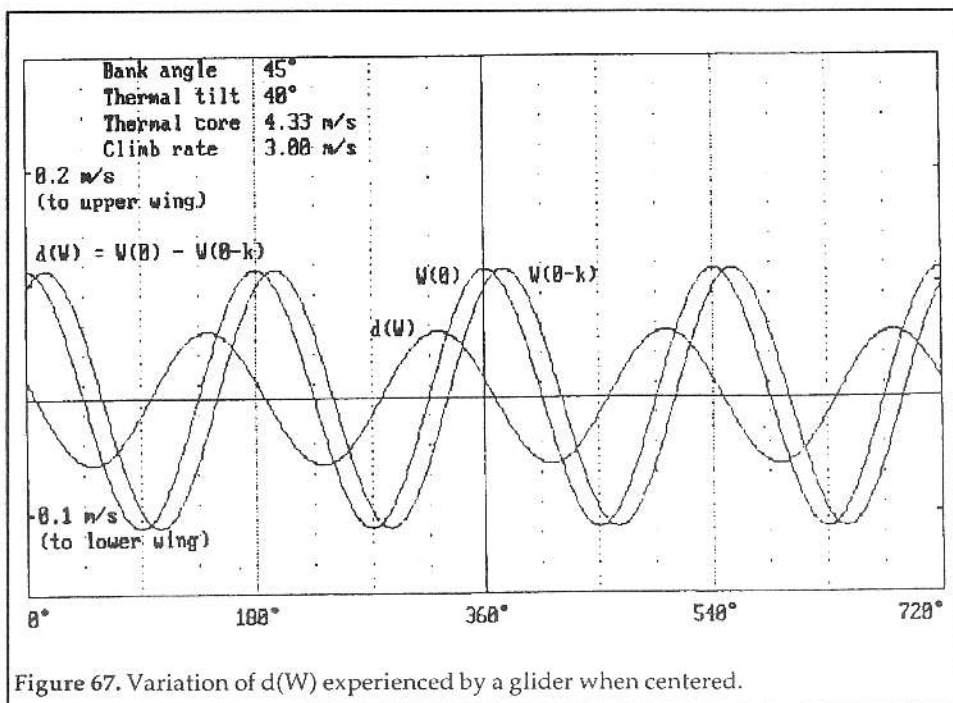
Figure 67. Variation of d(W) experienced by a glider when centered.

required to compensate for the off-center drift in tilted thermals. Since variometer readings no longer provide this information when a glider is correctly centered in a tilted thermal, this can be very useful.

Additional centering rules

The above results lead to simple modifications of the worst heading rule, whereby the glider is straightened up momentarily between 30° and 45°, depending upon the glider speed, bank angle and roll rate, after passing the worst heading. As the (major) maximum is sharpened by the presence of an in phase $\cos(2\theta)$ component, any best heading rules remain unchanged apart from a phase shift (circa 30°) of twice the lag due to inertia.

Rule 1: When the minimum is flat, the glider should be straightened up momentarily (1 second) as soon as the variometer reading starts to climb again.

Rule 2: When there are two closely spaced minima, the glider should be straightened up momentarily (1/2 second) as soon as the variometer reading starts to climb again after the second minimum.

Rule 3: When there are two widely spaced minima,

the glider should be straightened up momentarily (1/4 second) between 30° and 45°, depending upon the glider speed, bank angle and roll rate, after passing the minor maximum. This must now be done before the second minimum has been reached.

Rule 4: When the two maxima are equal, the glider wings should be straightened up momentarily between 30° and 45°, depending upon the glider speed, bank angle and roll rate, after the yaw string crosses the centerline while moving towards the lower wing.

Conclusions

When the axis of a thermal is tilted at an angle ß from the vertical, the vertical flow experienced by a circling glider includes a sinusoidal component that has two cycles per circle, and whose amplitude is proportional to the thermal strength W_0, the square of the circling radius r^2, and to a geometric factor $\sin^2(\beta)\cos(\beta)$, and is inversely proportional to the square of the thermal radius R_0^2. Consequently, the lift variation due to thermal tilt is most pronounced for heavily ballasted gliders climbing at shallow bank angles in strong thermals. A glider circling not too far off-center in an inclined thermal will be subject to two lift maxima that are 180° apart and
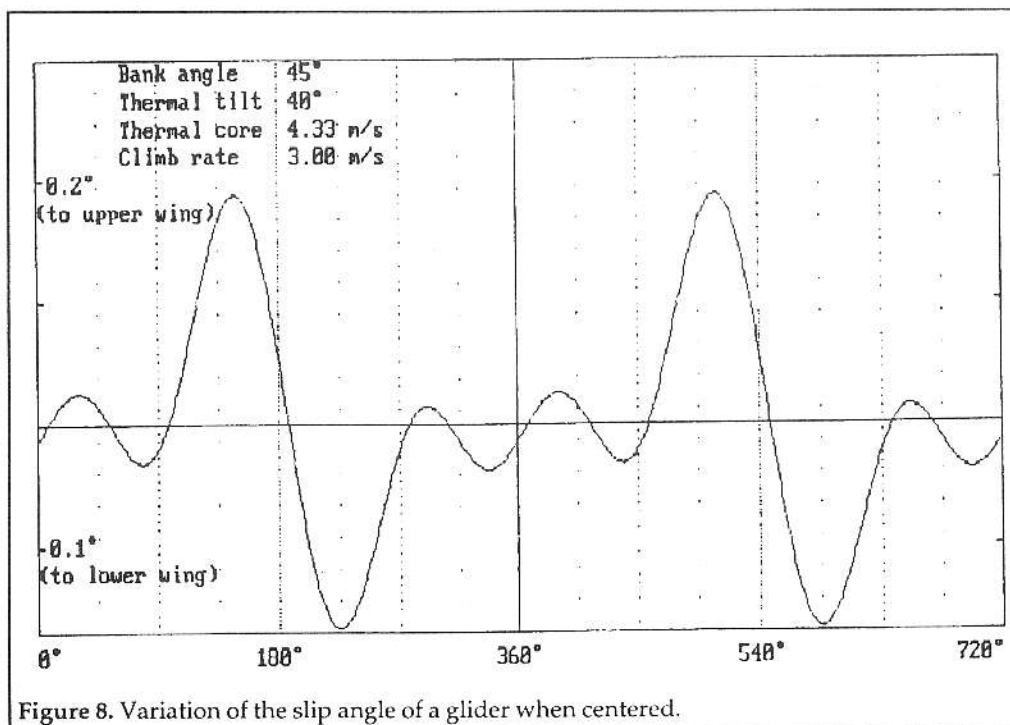

Figure 8. Variation of the slip angle of a glider when centered.

aligned along the wind shear.

The lift indicated by a total energy variometer also includes additional sinusoidal components that have one and three cycles per circle. This effect is due the horizontal wind component acting on a glider as a result of its inertia, but its amplitude is less than the variation of the vertical flow due to thermal inclination, being further multiplied by a geometric factor $2 \cdot \tan(\text{ß})\tan(b)$ and by $\sin(k/2)$ or $\sin(3k/2)$, as appropriate, where k is the phase lag due to inertia.

As a glider sinks vertically relative to the inclined axis of a thermal, the separation between the circling and thermal centers increases along, or parallel to, a line defined by the vertical projection of the tilted thermal axis onto the horizontal circling plane. This steady drift off-center makes the thermals "tricky," as repeated centering is necessary. If not adequately corrected, this drift may result in a variometer displaying a sharpened maximum near $30°$ and a minimum that is either flattened, or divided into two more or less closely adjacent minima separated by a minor maximum at $180°$ directly opposite the thermal center Tc.

When a glider is correctly centered in a tilted thermal the variometer readings no longer provide any useful information, although repeated centering is still necessary to compensate for the off-center drift. In this case anomalous yaw string movements in the downwind sector can be used to judge the timing of the required corrections. These deflections, which are also due to inertia, include sinusoidal components that have one, two and three cycles per circle. Their amplitude is much less than the variation of the vertical flow due to thermal inclination, being also inversely proportional to the airspeed U and further multiplied by a geometric factor $\sin(k)$, where k is the phase lag due to inertia.

References

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