CENTER-OF-GRAVITY AND LIFT COEFFICIENT LIMITS OF A GLIDING PARACHUTE: CASE STUDY

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Abstract

Longitudinal static stability and control of a gliding parachute are analyzed both analytically and numerically. The origins of center-of-gravity and speed limits are discussed.

1. Introduction

Typical gliding parachute is shown in Figure 1. Longitudinal and lateral control of the vessel is commonly executed through lines attached to the trailing edge of the canopy - a pull on these lines causes the trailing part of the canopy to flex downward, serving much as a pair of conventional elevons. In parachute's jargon, the trailing edge lines (and the part of the trailing edge attached to them) are known as 'brakes'. In some designs, a pilot can also control the lengths of the lines attached to the forward half of the canopy so as to move him or her self (and the vessel's center-of- gravity) forward and backward relative to the canopy. In parachute's jargon the system of pulleys enabling a pilot to do so is known as a 'speed system'.

In Reference 1 a standard static stability analysis was used to demonstrate a peculiar nature of longitudinal center-of-gravity limits of a gliding parachute. It was shown that the *forward* center-of- gravity position of the vessel is limited by a loss of control power whereas the *aft* center-of-gravity position is limited by the stall of the canopy. It was also shown that that a loss of longitudinal static stability and a loss of control power limit the top speed of the vessel.

In this exposition we remove the most conspicuous simplifying assumptions of Reference 1 regarding the behavior of control derivatives, and deliniate the center-of-gravity and top-speed limits numerically.

The static stability analysis of Reference 1, without the pertinent simplifying assumptions, is recapitulated in the next three sections. It will be followed by an analysis of a model parachute.

2. Recapitulation - static stability

Consider a gliding parachute in a symmetric unaccelerated flight. Following conventional definitions of aerodynamic coefficients, let the projected wing area and its mean aerodynamic chord serve as the respective references.

Select a Cartesian coordinate system, with the x-, y- and z- axes pointing forward, right and downward, respectively. For the sake of being specific, it will be assumed that the x-axis connects the trailing and the

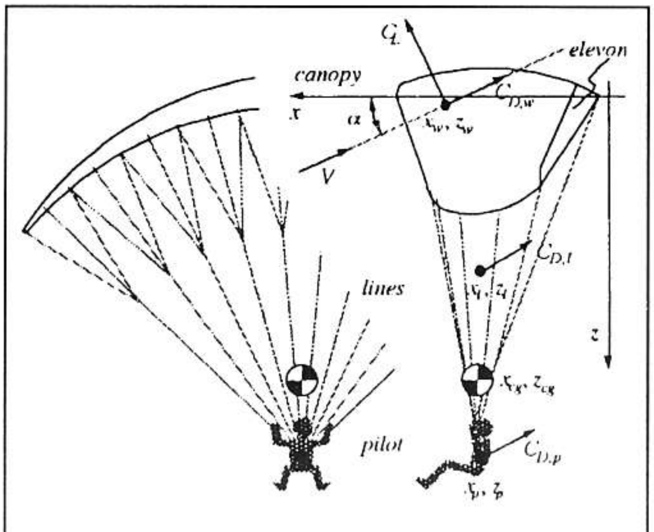

Figure 1. Notation.

leading edges of the mid-section of the wing.

Using wing's mean aerodynamic chord as a unit of length, let (x_w, z_w). (x_{cg}, z_{cg}), (x_l, z_l) and (x_p, z_p) be the respective dimensionless coordinates of the wing aerodynamic center, vessel's center of gravity, lines' center of pressure, and pilot's center of pressure - see Figure 1. Also, let α be the angle of attack, measured between the direction of the flow and "hex-axis. It will be assumed that

(i) α is small as compared with unity;

(ii) lift coefficient C_L is linear with the angle of attack, viz.

$$C_L = C_{L,0} + a\alpha, \tag{1}$$

where α is the lift slope coefficient, and $C_{L,0}$ is the lift coefficient at zero angle of attack;

(iii) drag coefficient $C_{D,w}$ is given by the parabolic polar

$$C_{D,w} = C_{D,w0} + KC_L^2, \tag{2}$$

where $C_{D,w0}$ and K are independent of C_L;

(iv) drag coefficients of the lines $C_{D,l}$ and of the pilot $C_{D,p}$ are independent of the lift coefficient;

(v) there are no elastic deformations of the canopy and of the lines with the change in lift coefficient - elevons held constant.

Under these assumptions, the pitching moment coefficient C_M of the vessel about its center of gravity takes on the form

$$C_M = C_{M0} + [(x_w - x_{cg}) - \tfrac{1}{a}C_{L,0}(z_w - z_{cg})]C_L + \tfrac{1}{a}(1 - aK)(z_w - z_{cg})C_L^2, \tag{3}$$

where

$$C_{M,0} = C_{M,w0} - C_{D,w0}(z_w - z_{cg}) - C_{D,l}(z_l - z_{cg}) - C_{D,p}(z_p - z_{cg}). \tag{4}$$

Here, $C_{M,w0}$ is the pitching moment coefficient of the canopy about its aerodynamic center. By interpretation, $C_{M,0}$ is the hypothetical pitching moment coefficient of the parachute about its center-of-gravity when the canopy produces no lift.

If a vessel is to hold its attitude, then C_M needs to be zero. With (3), this requirement yields quadratic equation

$$aC_{M,0} + [a(x_w - x_{cg}) - C_{L,0}(z_w - z_{cg})]C_{L,trim} + (1 - aK)(z_w - z_{cg})C_{L,trim}^2 = 0 \tag{5}$$

for the lift coefficient $C_{L,trim}$ at equilibrium (trim). Among two possible solutions of (5), if exist, we shall choose the one at which the vessel is statically stable; i.e. we shall choose the solution for which the derivative

$$\partial C_M/\partial C_L = (x_w - x_{cg}) - \tfrac{1}{a}C_{L,0}(z_w - z_{cg}) + \tfrac{2}{a}(1 - aK)(z_w - z_{cg})C_L, \tag{6}$$

is negative at $C_L = C_{L,trim}$. It may be easily verified by direct substitution that such a solution is

$$C_{L,trim} = \frac{1}{2(1 - aK)(z_w - z_{cg})}\{-a(x_w - x_{cg}) + C_{L,0}(z_w - z_{cg}) - [[a(x_w - x_{cg}) - C_{L,0}(z_w - z_{cg})]^2 - 4aC_{M,0}(1 - aK)(z_w - z_{cg})]^{1/2}\}. \tag{7}$$

Since both $C_{M,0}$ and $C_{L,0}$ are functions of the (generalized) elevons deflection δ_e, equation (7) defines $C_{L,trim} = C_{L,trim}(x_{cg}, \delta_e)$.

It is clear that (7) exists only if the expression under the square-root is non-negative – that is, it exists only if

$$[a(x_w - x_{cg}) - C_{L,0}(z_w - z_{cg})]^2 \geq 4aC_{M,0}(1 - aK)(z_w - z_{cg}) \tag{8}$$

In (14), the left-hand side is nonnegative, whereas the right-hand side is either positive or negative, depending on signs of the respective multipliers. Under normal circumstances $aK < 1$, whereas $z_w - z_{cg}$ is negative by the choice of the coordinate system - see Figure 1. Hence, if $C_{M,0}$ is nonnegative, then (8) holds unconditionally. If, on the other hand, $C_{M,0}$ is negative, then there exist

$$x_{\pm} = x_w - \tfrac{1}{a}C_{L,0}(z_w - z_{cg}) \pm \tfrac{1}{a}\sqrt{4aC_{M,0}(1 - aK)(z_w - z_{cg})}, \tag{9}$$

such that (8) holds if either

$$x_{cg} \leq x_- \tag{10}$$

or

$$x_{cg} \geq x_+. \tag{11}$$

For $x_{cg} = x_{\pm}$, the absolute value of $C_{L,trim}$ is

$$C_{L,1} = \sqrt{\frac{aC_{M,0}}{(1-aK)(z_w - z_{cg})}}, \tag{12}$$

whereas

$$C_{L,trim} \geq C_{L,1} \quad \text{for each } x_{cg} \leq x_-, \tag{13}$$

$$C_{L,trim} \leq -C_{L,1} \quad \text{for each } x_{cg} \geq x_+. \tag{14}$$

by (7) and (9). Since negative values of $C_{L,trim}$ are irrelevant in the present discussion, (11) is ruled out by (14); in which case the center-of-gravity should be positioned behind x_-. From (13) it thus follows that there exist a lower bound on the lift coefficient possibly attainable at trim.

Note that a gliding parachute with $C_{M,0} < 0$ is neutrally stable at $x_{cg} = x_-$, by (6), (9) and (12). Since, by assumption, the vessel is supposed to be statically stable if (6) exists, moving the center of gravity backward seems to have a stabilizing effect.

Under normal circumstances, the sum of drag contributions to $C_{M,0}$ is usually positive. Hence, at least in principle, by flattening the profile one can design a vessel with positive $C_{M,0}$. As already cited above, such a vessel will have no apparent limitations on its longitudinal center-of- gravity position, and therefore it could be designed to fly at any desired lift coefficient below stall [see (13)]. At the same time, it seems improbable that one can design an elevons-controlled gliding parachute in such a way that it will have a reasonable range of accessible lift coefficients on the one hand, and non-negative $C_{M,0}$. for all possible elevons deflections on the other. But in order to trim the vessel with negative $C_{M,0}$.,the most forward center-of-gravity position should be limited by (10). Thus, given the range $\Delta = (\delta_{min}, \delta_{max})$ of usable elevons deflections, the requirement that a trim condition should exist for each δe in Δ, limits the forward center-of-gravity position by

$$x_{cg,1} = \inf\{x_-(\delta_e) \mid \delta_e \in \Delta \text{ and } C_{M,0}(\delta_e) \leq 0\}. \tag{15}$$

3. Recapitulation - control

Consider the derivatives $\partial C_{L,trim}/\partial\delta_e$ and $\partial C_{L,trim}/\partial x_{cg}$. Differentiate, in turn, on both sides of (7) with respect to δ_e and x_{cg}, and use (7) in the resulting expressions so as to obtain

$$\frac{\partial C_{L,trim}}{\partial x_{cg}} = \frac{aC_{L,trim}^2}{H}, \tag{16}$$

$$\frac{\partial C_{L,trim}}{\partial \delta_e} = \frac{(z_w - z_{cg})C_{L,trim}^2}{H}\frac{\partial C_{L,0}}{\partial \delta_e} - \frac{aC_{L,trim}}{H}\frac{\partial C_{M,0}}{\partial \delta_e}. \tag{17}$$

where

$$H = -aC_{M,0} + (1-aK)(z_w - z_{cg})C_{L,trim}^2. \tag{18}$$

From (18), (13) and (8) it follows that H is negative for each $x_{cg} < x_{cg,1}$. Hence,

$$\partial C_{L,trim}/\partial x_{cg} < 0 \quad \text{for each } x_{cg} < x_{cg,1}; \tag{19}$$

that is, moving the center-of-gravity forward reduces the lift coefficient at trim - as in a conventional vessel.

Predictable longitudinal control of a gliding parachute dictates that *the lift coefficient at trim should be an increasing function of controls deflection*, viz.

$$\partial C_{L,trim}/\partial \delta_e > 0 \quad \text{for each } \delta_e \text{ in } \Delta. \tag{20}$$

From (17), it immediately follows that in order to satisfy (20) for some $x_{cg} < x_{cg,1}$., one needs

$$(z_w - z_{cg})C_{L,trim}\frac{\partial C_{L,0}}{\partial \delta_e} - a\frac{\partial C_{M,0}}{\partial \delta_e} < 0 \tag{21}$$

for each δ_e in Δ. In (21), $\partial C_{L,0}/\partial\delta_e$ is normally positive, whereas $z_w - z_{cg}$ is negative. Hence, with

$$C_{L,2} = -\frac{a}{z_{cg} - z_w}\frac{\partial C_{M0}/\partial\delta_e}{\partial C_{L,0}/\partial\delta_e}, \tag{22}$$

equation (20) imposes a restriction

$$C_{L,trim}(x_{cg}, \delta_e) > C_{L,2}(\delta_e) \tag{23}$$

on the lift coefficient that is predictably controllable with elevons deflected at be. Concurrently, it also imposes an additional restriction on the most forward center-of-gravity position. With

$$\begin{aligned} x_{cg,trim}(C_L, \delta_e) &= x_w + C_{M,0}(\delta_e)/C_L \\ &+ (z_{cg} - z_w)\left[C_{L,0}(\delta_e) - (1-aK)C_L\right]/a, \end{aligned} \tag{24}$$

the longitudinal center-of-gravity position needed to trim a vessel at lift coefficient C_L with elevons deflected at δ_e, and

$$C_{L,12}(\delta_e) = \sup\{C_{L,1}(\delta_e), C_{L,2}(\delta_e)\}, \tag{25}$$

this restriction takes on the form

$$x_{cg} < x_{cg,2}, \tag{26}$$

where

$$x_{cg,2} = \inf\{x_{cg,trim}(C_{L,2}(\delta_e), \delta_e) \mid \delta_e \in \Delta\}. \tag{27}$$

From; (19) and (20), the minimal lift coefficient $C_{L,min}$ possibly attainable at trim corresponds to the most forward center-ofgravity position, i.e. $x_{cg} = x_{cg,2}$, and elevons released, i.e. $\delta_e = \delta_{min}$; explicitly,

$$C_{L,min} = C_{L,trim}(x_{cg,2}, \delta_{min}). \tag{28}$$

4. Forward center-of-gravity and minimal lift coefficient limits

The results of the preceding two sections suggest that the existence of the minimal lift coefficient limit, and, concurrently, of the forward center-of-gravity limit, is a direct consequence of requiring a predictably controllable trim condition for all possible elevons positions. A parachute with $C_{M,0}(\delta_{min}) > 0$ utilizing center-of-gravity movement for longitudinal control (elevons released and 'fixed') can, in principle, be predictably trimmed at any desired lift coefficient below stall [see (12) and (19)].

The minimal lift coefficient of an elevonscontrolled parachute depends on several design parameters, of which the most noticeable are CM,0 and the ratio of the derivatives $\partial C_{M,0}/\partial\delta_e$ and $\partial C_{L,0}/\partial\delta_e$.

Unless the canopy is stalled, the last derivative is positive. The first derivative,

$$\frac{\partial C_{M0}}{\partial\delta_e} = \frac{\partial C_{M,w0}}{\partial\delta_e} - \frac{\partial C_{D,w0}}{\partial\delta_e}(z_w - z_{cg}), \tag{29}$$

[see (4)] is either positive or negative, depending on the relation between the drag (positive) and pitching moment (negative) contributions. If it could have been made positive for the entire range of elevons deflections, the parachute with $C_{M,0}(\delta_{min}) > 0$ would have had no restrictions on the minimal lift coefficient and on the most forward centerof- gravity position.

Typically, the derivative $\partial C_{M,0}/\partial\delta_e$ is negative for small elevons deflections and positive for large ones. Since for small deflections $C_{M,0}$ can be designed positive, the predictability of elevons control at small deflections is, perhaps, the most significant design criterion that determines the minimal lift coefficient and the forward center-of-gravity limit. Numerous numerical simulations that have been done during this study seem to support this conclusion.

For small elevons deflections one can usually assume that $C_{L,2}$ is a decreasing function of δ_e. In this case, (20) and (23) imply that

$$C_{L,trim}(x_{cg,2}, \delta_{min}) > C_{L,2}(\delta_{min}); \tag{30}$$

viz., $C_{L,2}(\delta_{min})$ is the lower bound on the lift coefficient possibly attainable at trim. The concurrent center-of-gravity limit is $x_{cg,trim}(C_{L,2}(\delta_{min})\delta_{min})$.

5. Numerical example

We turn now to implement the above theory for a typical recreational gliding parachute. The vessel to be discussed has a pseudo-elliptic canopy of 25.6m^2 [pertinent chord distribution is given by (A6) in Appendix A], with midchord of 3.2m and span of l0.5m; the canopy is curved into an 86° arc of radius 7m, centered about the pilot. The canopy has 15% thick cross section with 2% camber. 175 lines, each about 2.2m long and 1mm in diameter are attached to the canopy in 5 rows; these lines merge into 42 lines attached to the pilot, each about 4.8m long and 1.5mm in diameter. The vessel weighs 910N and its center of gravity is located approximately 0.5 meter above the center of the canopy's arc.

The parachute is equipped with full-span elevons, located between the trailing edge and the fourth row of lines; the latter being connected at the respective 80%-chord points. Since the shape that the canopy accepts with the pull on the trailing edge is difficult to predict, it was simulated using several simplified models. Two of these models will be discussed below. Model A assumes that the elevons behave as 20%-chord plain flaps of variable deflection angle; the latter changes between 0° at δ_{min} and 60° at δ_{max}. Model B assumes that the elevons behave as 45°-down plain flaps with variable chord; the latter changes between 0% at δ_{min} and 20% at δ_{max}.. For the sake of display, we shall set henceforth $\delta_{min} = 0$ and $\delta_{max} = 1$.

A small code was written to reduce this data to a form which is readily usable in equations displayed above. Pertinent formulae are listed in Appendix A. The resulting coefficients at $\delta_e = 0$ are following

$a = 3.1$,	$K = 0.1$,	$z_{cg} = 2.43$,
$z_l = 1.18$,	$z_w = 0.2$,	$z_p = 2.62$,
$C_{D,l} = 0.03$,	$C_{D,p} = 0.02$,	$C_{D,w0} = 0.03$,
$C_{L,0} = 0.16$,	$C_{M,w0} = -0.06$,	$C_{M,0} = 0.045$

Trim conditions as functions of elevons deflection are presented in figures 2 and 3 for the two models of elevons deflection. In each figure, the top two graphs represent equations (6) and (7). The third graph from the top represents flight velocity, as computed from $C_{L,trim}$ at standard sea level conditions. The bottom graph represents the glide ratio, viz.

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D,w0} + C_{D,l} + C_{D,p} + KC_L^2}. \tag{31}$$

Consider Figure 2 first. With the center-of-gravity positioned forward of $x_w + 0.02$, the static stability will be lost for some range of elevons deflections (centered at about 30% of maximal deflection) - see the top graph in Figure 2. Hence, $x_w + 0.02$ is the most forward center-of-gravity position that allows a steady flight without active stabilization. This limit corresponds to $x_{cg,1}$ - see section 2.

With the center-of-gravity positioned between x_w + 0.02 and x_w, the parachute is statically stable, but the elevons control is reversed,for small deflections - the speed increases with the pull on the lines - see the third graph in Figure 2. This behavior infers poor handling qualities, and therefore x_w is the most forward center-of-gravity position that allows a predictable con-

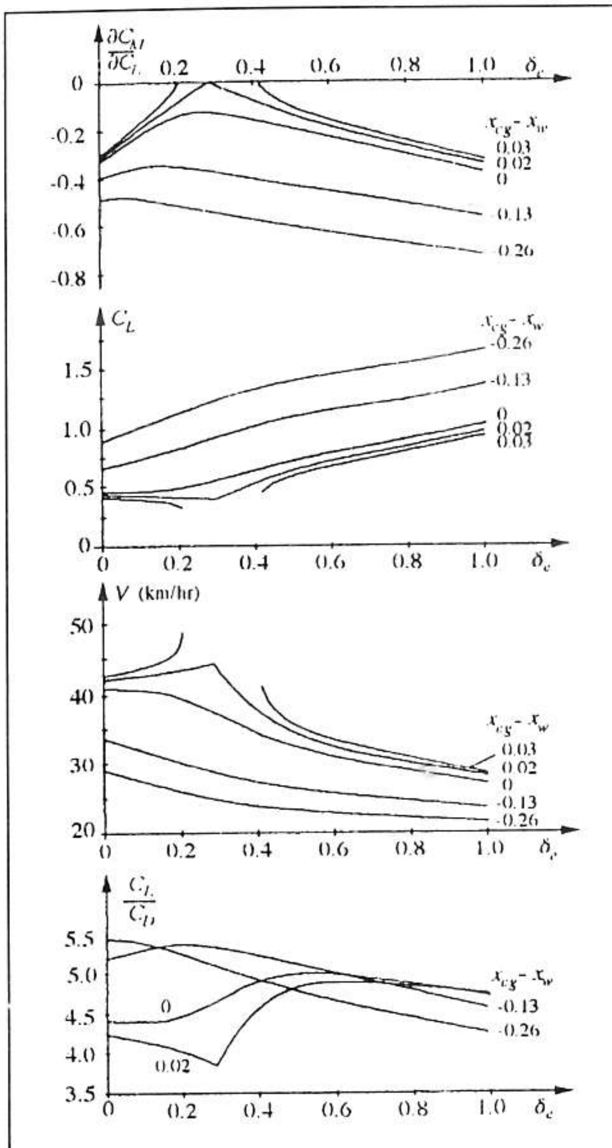

Figure 2. Trim conditions as functions of elevons deflection. Model A.

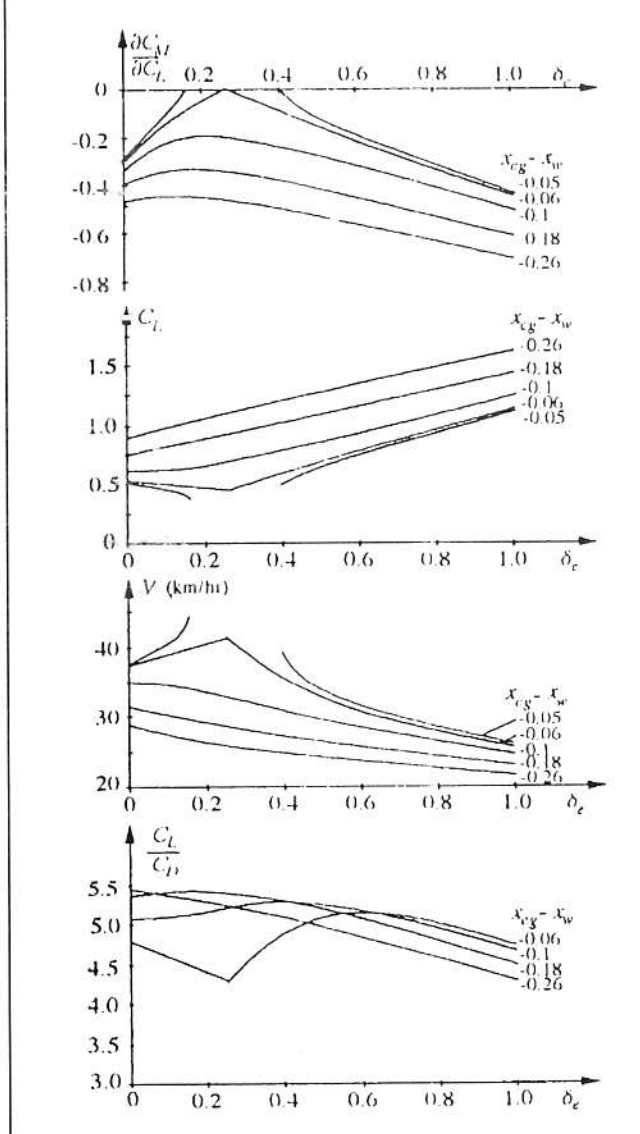

Figure 3. Trim conditions as functions of elevons deflection. Model B.

trol of the parachute. This limit corresponds to $x_{cg,2}$ - see Section 3.

With the center-of-gravity limited by x_w, the minimal lift coefficient at which the parachute can be trimmed is about 0.45 - see the second graph in Figure 2. It corresponds to the top speed of about 40km/hr at standard sea-level conditions - see the third graph in Figure 2. This is the maximal speed at which the parachute can be predictably trimmed, i.e. the absolute top speed of the vessel - see Section 4.

The use of model B to simulate elevons deflection yields qualitatively similar results, although the limits $x_{cg,1}$ and $x_{cg,2}$ move aft by about 0.1, the minimal lift coefficient increases to about 0.6, and the top speed reduces to about 35km/hr. The sensitivity of the forward center-of-gravity limit to the modelling of the elevons deflection suggests that experimental study of the flexible elevons is indispensable.

It is commonly accepted with parachute designers that $C_{L,trim}$ with brakes released is set to be slightly less than the lift coefficient $C_{L,glide}$ giving the best glide ratio; this setting seems to be the most convenient for recreational soaring. Hence, a designer would, probably, set the center of gravity at (or somewhat forward of) x_w-0.26 - see bottom graph in Figures 2 and 3. In this case the maximal speed of the parachute (with brakes released) is only about 30km/hr - see third graph from the top in Figures 2 and 3. Hence, in order to utilize the maximal speed potential of the vessel, which was shown above to be somewhere between 35 and 40 km/hr, there should be an in-flight possibility to move the center-of-gravity forward. This is the purpose of the 'speed system'.

The analysis of section 2 implies that static stability imposes no limit on the aft center-ofgravity position. This result is clearly elucidated by the top graph in figures 2 and 3. Analysis of section 3 implies that $C_{L,trim}$ increases as center-of-gravity moves aft, elevons fixed. This result is supported by the second graph in Figures 2 and 3. Thus, the aft center-of-gravity position is limited by the canopy's stall only.

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Appendix A - Estimates of the pertinent aerodynamic coefficients

Profile

Figures 7 and 9 of Reference 2 infer that the parasite drag coefficient $C_{d,w0}$ of LS1-0417 profile varies from about 0.01 to about 0.05, depending on the size of the air intakes. The corresponding separation drag coefficient (the coefficient of the drag polar) varies from 0.01 to 0.1, and the lift-slope coefficient a_{2D} varies from 5.5 to 5. We assume that these figures are typical for any profile used in gliding parachutes.

With no data on size and location of the air intakes on the particular canopy, we choose representative values, viz. C_{dw0} = 0.03, k_s = 0.02 and a_{2D} =5. Note that by neglecting aerodynamic interference between the canopy and the lines, one has that $C_{D,w0} = C_{d,w0}$..

Assume that the wing's profile has a circular arc mean camber line. Then, the pitching moment coefficient $C_{m,ac}$ about the quarter chord point, and the lift coefficient $C_{l,0}$ at zero angle of attack, can be estimated from the thin airfoil theory (see Reference 3) to be

$$C_{m,ac} \approx -\pi\zeta, \tag{Al}$$

$$C_{l,0} \approx 4\pi\zeta, \tag{A2}$$

where ζ is the maximal relative camber. In the present case, $\zeta = 0.02$; hence $C_{m,ac} \approx -0.063$ and $C_{l,0} \approx 0.25$.

Elevons

Assume that the effect of elevons on the profile characteristics is similar to that produced by plain flaps. Let c_e and δ be the relative elevons chord and elevons deflection angle, respectively. Let, also, $\theta = \arccos(1 - 2c_e)$. Then, from Reference 4,

$$\Delta C_{l,0} \approx 2(\theta + \sin\theta)\eta\delta, \tag{A3}$$

$$\Delta C_{m,ac} \approx -0.25(2\theta - \sin 2\theta)\eta\delta, \tag{A4}$$

$$\Delta C_{d,w0} \approx 1.7 c_e^{1.38} \sin^2\delta, \tag{A5}$$

where η is an empirical correction factor found as a function of δ in Figure 3.33 of Reference 4. Its value varies from about 0.8 at δ ** 0 to about 0.4 as δ approaches unity.

Canopy

Let R and ϕ_o, be the radius and half the angle of the wing's arc. With ϕ in $(-\phi_o, \phi_o)$, let the local chord $c(\phi)$ of the wing be given by

$$c(\phi) = c_0 \frac{(\tan^2\frac{\phi_o}{2} - \tan^2\frac{\phi}{2})^{\frac{1}{2}}}{\tan\frac{\phi_o}{2}(1 + \tan^2\frac{\phi}{2})}, \tag{A6}$$

where $c_0 = c(0)$ is the chord length at the midsection of the wing.

Let $\tau = -\tan 2(\phi_o/4)$.. It is shown in equation (6.73) of Reference 5 that for such a wing, the mean aerodynamic chord can be approximated by

$$c_{av} = c_0 \frac{8}{3\pi}[1 + 0.6\tau + O(\tau^2)]. \tag{A7}$$

Under the present circumstances, where $\phi_o = 43°$ and $c_0 = 3.2m$, (A7) yields $c_{av} \approx 2.7m$.

With S being the wing area, let $A = 4R^2\sin^2(\phi_o/2)/S$ be the respective aspect ratio. We now use the lifting line theory developed in References 5 and 6 to obtain the following approximate formulae

$$C_L \approx \frac{G\rho A(1-\tau)^3[a_{2D}\alpha(1+4\tau) + C_{l,0}(1+2\tau)]}{\pi A(1-\tau)^3 + a_{2D}(1+\tau)^3} \tag{A8}$$

$$C_{d,i} \approx \frac{1+2\tau}{\pi A} C_L^2 \tag{A9}$$

and

$$C_{M,w0} \approx C_{m,ac}(1 + 0.625\tau), \tag{A10}$$

for the wing's lift, induced drag and pitching moment coefficients, as well as

$$z \approx R(1+2\tau) \tag{A11}$$

for the distance between the wing's aerodynamic center and the center of the wing's arc. The accuracy of the above formulae with respect to the exact formulae of the lifting line theory is estimated to be of the order of τ^2. Under present circumstances, (A8), (A10) and (A11) yield

$$C_L \approx 3.1\alpha + 0.67C_{l,0}, \tag{A12}$$

$$C_{M,w0} \approx -0.06, \tag{A13}$$

$$z_w \approx 0.2. \tag{A14}$$

Following equation (4.31) of Reference 4, (A9) implies that K can be approximated by

$$K = \frac{1+2\tau}{\pi A} + k_s, \tag{A15}$$

where k_s was already estimated above. Thus, $K \approx 0.1$.

Lines

Assume all lines to be almost perpendicular to the flow, and no aerodynamic interference between them. Given typical flight velocity of 10*m/sec*, the corresponding cross flow Reynolds number on a single line turns out to be of the order of 10^3; whence the drag coefficient of all lines, based on their frontal area, should be about 1.1, by figure 4.6 of Reference 4. Accordingly, $C_{D,l}$ = 0.03.

Assume that the density of the lines per unit angle of the wing's arc is constant (it is allowed, however, to vary with the distance from the center of the arc). In the present case, the density of the lines is about 30 per radian of the wing's arc at all distances which are less then 4.8m from the center of the arc, and about 117 per radian for all distances grater than that. For constant density lines located between radii R_0 and R_1, the distance z between their centerof-pressure and the center of the wing's arc is, simply,

$$z = \frac{R_0 + R_1}{2}\frac{\sin\phi_0}{\phi_0}. \tag{A16}$$

A detailed computation based on this formula yields $z_l = 1.18$.

Pilot

Assume that a pilot holds a sitting position. Then, the effective flat-plate area of the pilot should be about 0.5m^2, by Reference 7; whence $C_{D,p} \approx 0.02$. Since, by assumption, the pilot is located at the center of the wing's arc, therefore z_p = 2.62. Note that with the center-of-gravity located 0.5m above the center of the wing's arc, and c_{av} = 2.7m, one has that z_{cg} = 2.43.