THE BUNGEE MODE IN TOWED SAILPLANE FLIGHT

by James E. Murray

Abstract

Simple mathematical modeling combined with ground-test characterization of a towrope predicts the existence of a low-frequency, lightly-damped oscillatory bungee mode during sailplane towing operations. Towrope tension and sailplane longitudinal accelerometer measurements made in a towed flight test program confirm the existence of the bungee mode. Low damping of the bungee mode combined with in-flight disturbances cause virtually continuous excitation of the mode while on tow. Dynamic tow tension excursions are typically large; peak tension values greater than 300 lb during takeoff and oscillatory components in steady flight of 50 lb peak-to-peak are not uncommon. Continuous excitation of the bungee mode makes accurate measurement of the static component of tension difficult. In-flight measurements of the static component of tow tension are somewhat larger than the tension values predicted by simple performance-based calculations.

Introduction

Towed aircraft flight has a history almost as old as powered flight itself. Towed flight operations were conducted at least as early as the 1920s (ref. 1), and over the succeeding decades a fantastic variety of aircraft configurations has been successfully towed, including piston and jet-powered fighter aircraft (ref. 2 and 3), a tailless rocket-powered fighter (ref. 4), and a lifting body (ref. 5). Today, safe sailplane towing operations are commonplace throughout the world (ref. 6).

Curiously, while the flight dynamics of powered and gliding flight have suffered exhaustive study, the study of the flight dynamics of towed flight is relatively immature and underreported. Theoretical models of towropes as tension members in the presence of inertial and aerodynamic forces have been developed (ref. 7), and a number of different towed objects has been tested in wind tunnels and in flight. Theoretical development of the equations of motion for the coupled aircraft and towrope system has followed. Recent researchers have concentrated generally on development of the dynamic system model (ref. 8, 9 and 10) and in particular on the problem of towplane upset (ref. 11). Yet the fact remains that very little of the theoretical modeling has been reported to be correlated with flight data. Flight results are typically qualitative and anecdotal; only minimal quantitative flight results have been reported (ref. 12). Consequently, validation of theoretical models with flight-experimental data has not proceeded apace as it has with powered and gliding flight.

This paper addresses the theory-to-flight correlation gap by presenting a simple model for a dynamic mode that is fundamental to the towing operation, and by correlating it with measurements made in a towed flight test program. A single-degree-of-freedom dynamic model of the towed sailplane configuration is developed. Ground test results are presented to quantify selected parametric values in the dynamic model and to predict the gross characteristics of in-flight response. The flight test configuration is described, including the instrumentation system and the maneuvers flown, and resulting flight data are presented. Salient characteristics of the static and the dynamic components of these data are compared with available preflight predictions. The potential sources of apparent discrepancies are also discussed. Finally, recommendations for further development and testing are presented.

Modeling and Test Equipment

Classical modeling of the longitudinal dynamics of an aircraft in free flight yields a system with two characteristic dynamic modes: the short-period mode and the phugoid mode (ref. 13). Modeling the longitudinal dynamics of two aircraft connected in towed flight by an elastic member (the towrope) yields a system with a large number of characteristic dynamic modes (ref. 9). Of interest to this paper is a dominant mode associated with the elastic characteristic of the towrope (originally called by de Matteis the elastic mode) known here as the bungee mode.

Dramatic simplification of the mathematical model yields an intuitive engineering model for predicting dominant characteristics of the bungee mode in flight. This single-degree-of-freedom model does not include any aerodynamic, gravity, or friction effects. Figure 1 shows a schematic representation of such a model. This model is a pair of masses connected by a spring-damper assembly. The masses, m_1 and m_2, represent the masses of the towplane and sailplane, respectively. Stiffness parameter (k), in lb/ft, represents the static force-elongation characteristic of the towrope, and damping parameter (c), in lb/(ft/sec), represents the energy absorption mechanism of the towrope.

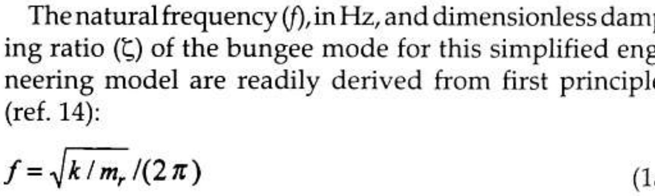

Figure 1. Schematic representation of the mass-spring-damper model.

The natural frequency (f), in Hz, and dimensionless damping ratio (ζ) of the bungee mode for this simplified engineering model are readily derived from first principles (ref. 14):

$$f = \sqrt{k / m_r} / (2\pi) \tag{1a}$$

$$\zeta = c / (2\sqrt{k m_r}) \tag{1b}$$

where m_r is the reduced mass

$$m_r = m_1 m_2 / (m_1 + m_2) \tag{1c}$$

Prediction of the frequency and damping ratio of the bungee mode in-flight requires values of towrope stiffness parameter (k) and damping parameter (c). These parameters were not readily available for the towrope used in flight, a 134 ft length of 5/16-inch-diameter twisted polypropylene rope. Therefore, the parameters were measured in a free-oscillation ground test experiment using a short section (approximately 55 ft) of the towrope. Results from this short test specimen were then extrapolated to the full length of the flight test rope based on the assumption that the material properties of the rope are uniform throughout its length.

The rope was suspended from a hangar roof truss. At the lower end of this rope a weight pallet was attached through an instrumented load link. The pallet was loaded with lead shot, raised above equilibrium position, and released. The load link tension measurement was recorded at 100 samples per second. This test was conducted with five different loads which spanned the anticipated flight tensions, and each load case was repeated three times. Figure 2 shows a representative time history of tension for one test and reveals the characteristics of a lightly-damped second-order system.

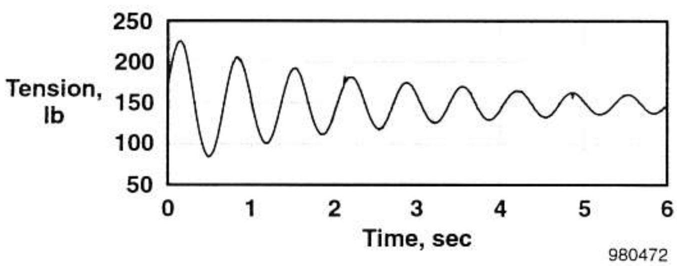

Figure 2. Time history of tension from ground test of towrope.

The frequency (f) and damping ratio (ζ) for each load mass (m_{load}) were estimated with curve-fitting software. In the ground test setup, the mass on the upper end of the towrope is essentially infinite, so equation (1c) reduces to

$$m_r = m_{load} \tag{2a}$$

Values of the towrope stiffness parameter (k_{gnd}) and damping parameter (c_{gnd}) for the length tested were calculated by inverting Equations (1a) and (1b) and replacing reduced mass m_r with the test load mass (m_{load}).

$$k_{gnd} = m_{load}(2\pi f)^2 \tag{2b}$$

$$c_{gnd} = 2\zeta\sqrt{k_{gnd}m_{load}} \tag{2c}$$

Results of the ground tests are presented in figure 3.

The frequency (f) decreases with increasing load, while the damping ratio shows some scatter. The stiffness parameter (k_{gnd}) increases with increasing load, demonstrating the nonlinear behavior characteristic of a stiffening spring; the damping parameter (c_{gnd}) also increases with increasing load.

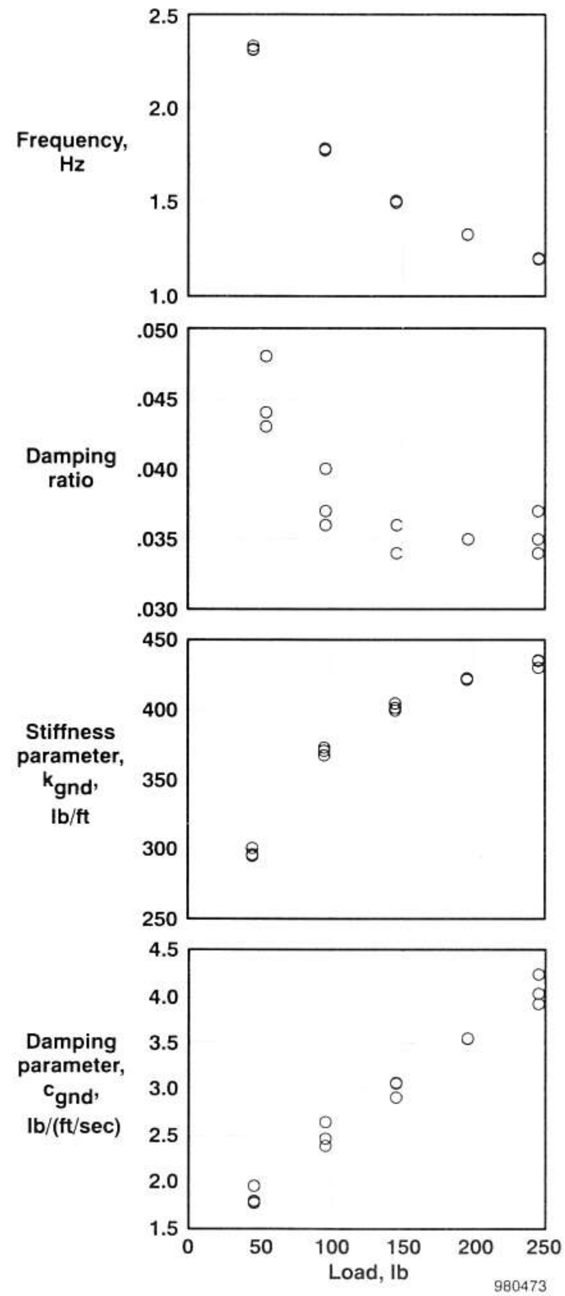

Figure 3. Towrope characterization ground test results.

A series of towed flight operations was flown at a local glider port using rental aircraft. Test-specific instrumentation systems were installed on both test aircraft to collect flight test data. The towplane was a Piper Pawnee (Piper Aircraft Corporation, Lock Haven, Pennsylvania) with a single crew member and a gross weight of approximately 1500 lb. The sailplane was a Grob 103 (Grob-Werke GMbH & Co. KG, Germany) with two crew members and a gross weight of approximately 1200 lb. The towrope was approximately 134 ft of 5/16-inch-diameter twisted polypropylene rope. Each aircraft was equipped with a data recorder based on a commercial single-board computer with integral 12-bit analog-to-digital converter and data storage memory. Custom signal-conditioning electronics were used to condition and filter the raw sensor inputs prior to digital conversion and storage of the digital data in memory. All flight data were recorded onboard at 100 samples per second and off-loaded to a laptop computer after each flight for permanent storage and analysis. Data from recorders on both aircraft were time-synchronized post-flight by identifying equivalent events (such as initial takeoff tension peak) on both time histories.

Towrope tension was measured on the towplane with an instrumented load link connected between the tow release mechanism and the forward ring of the towrope (fig. 4). The electrical cable from the load link was fed through the fuselage to the data recorder, which was mounted on the aft deck behind the pilot's seat. An in-line electrical quick disconnect near the load link allowed for emergency release of the load link and towrope assembly if required.

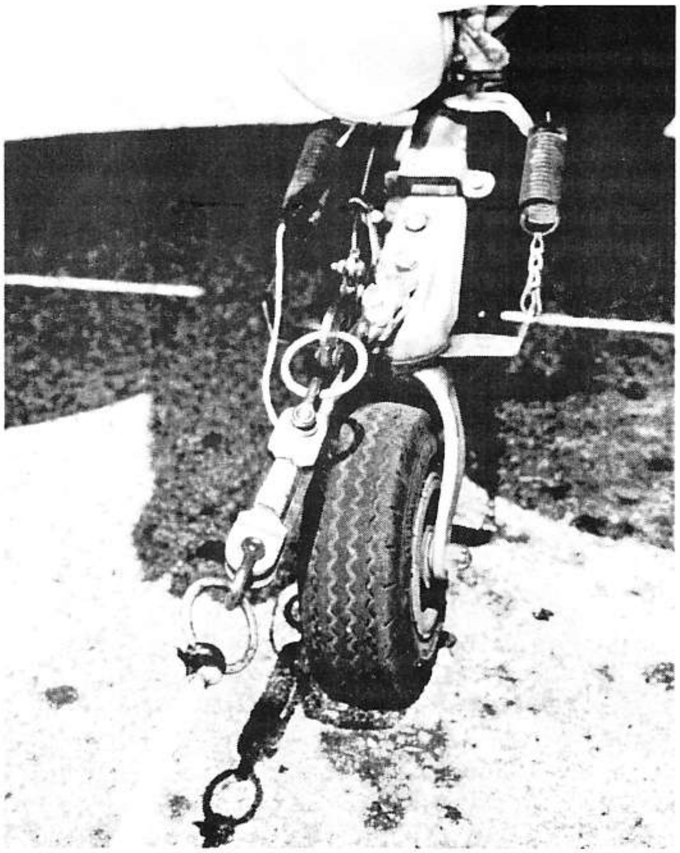

Figure 4. Installation of load link on Pawnee towplane (EC 98-44630-2).

Sailplane longitudinal accelerometer measurements were made with an accelerometer package integrated into the sailplane data recorder. The integrated assembly was mounted in the aft sailplane cockpit adjacent to the center console (Figure 5).

The stiffness and damping parameters of the ground test specimen (k_{gnd} and c_{gnd}) were extrapolated from the test specimen rope length (l_{gnd}) to the actual flight test rope length (l_{flt}), using the assumption that the rope material properties are uniform throughout its length and that the

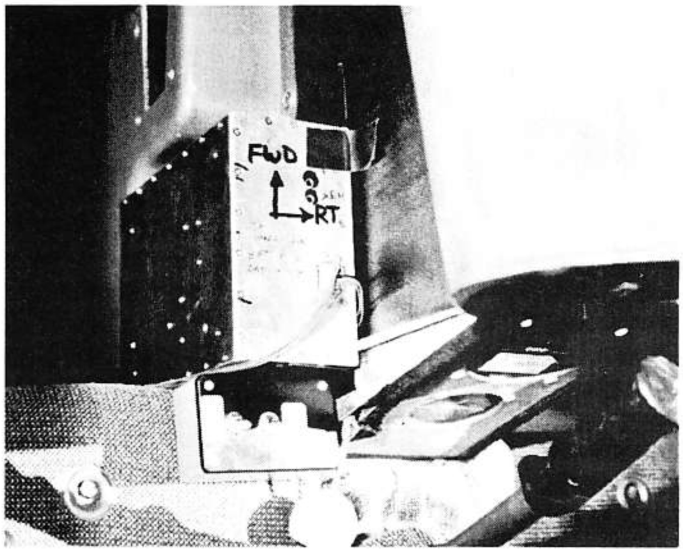

Figure 5. Installation of accelerometer and data recorder system on Grob 103 sailplane (EC 98-44630-3).

following relationships hold:

$$k_{flt} = k_{gnd} l_{gnd} / l_{flt} \tag{3a}$$

$$c_{flt} = c_{gnd} l_{gnd} / l_{flt} \tag{3b}$$

Using equations (3a) and (3b) to extrapolate from the ground test rope length to the flight test rope length yields the flight stiffness parameter (k_{flt}) and flight damping parameter (c_{flt}) shown in figures 6(a) and 6(b). Inserting k_{flt}, c_{flt}, and measurements of the aircraft masses into equations 1(a)–1(c) yields the expected in-flight frequency and damping values shown in figures 6(c) and 6(d).

The expected in-flight frequency (f_{flt}) ranges from nearly 0.38 Hz for low-tension values to almost 0.46 Hz for high-tension values; the expected damping ratio (z_{flt}) is near 0.01. The rope is expected to act as a preloaded spring and the two aircraft to bounce back and forth along a line connecting them. This line is approximately aligned with the flightpath. The bungee mode is expected to be observed as the primary oscillatory component superimposed on the static component of the towrope tension and the longitudinal accelerometer measurements. Furthermore, the magnitude of the static component of towrope tension in steady level flight is expected to be approximately equal to the quotient of the weight and the lift-drag ratio of the sailplane; close to 40 lb.

The amplitude of the oscillations is more difficult to predict than their frequency. From the low damping ratio of the bungee mode, predictions indicate that the oscillations are readily excited and require many cycles to damp out. The amplitude of the oscillations is strongly dependent on the magnitude of disturbances encountered in flight.

Results and Discussion

Three test flights were flown on a single day and approximately 10 minutes of flight data were collected. Flight

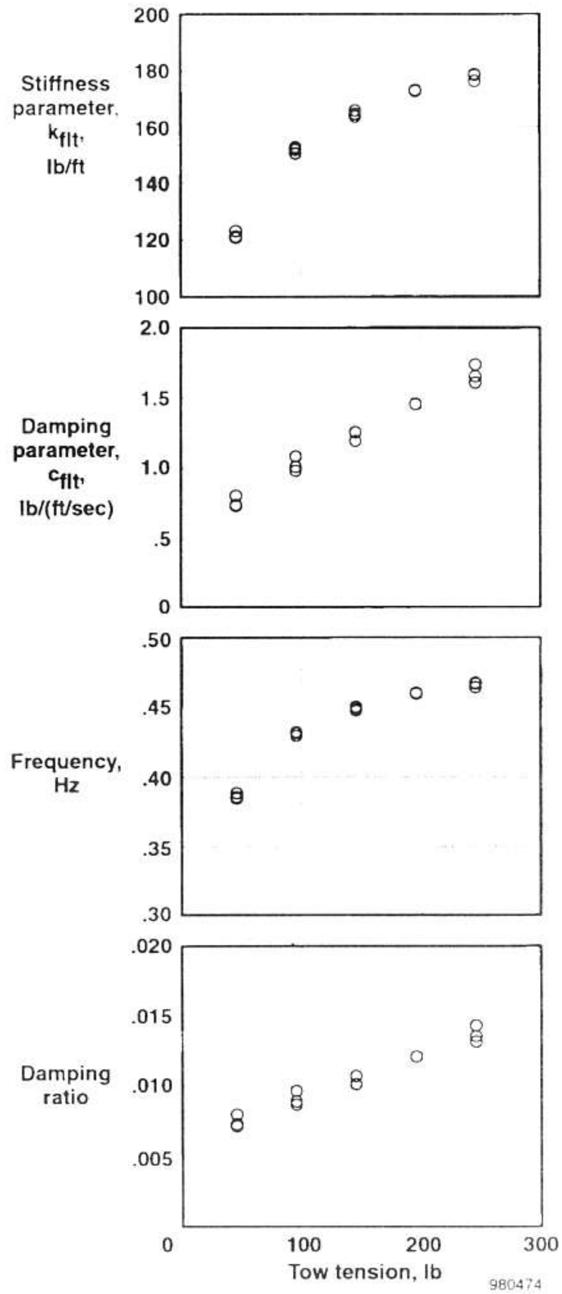

Figure 6. Expected flight test dynamic characteristics.

test data showed that the bungee mode was continuously excited during towed flight operations. Although all flights were flown in the morning and smooth air was actively sought on tow, the bungee mode was clearly evident in the tow tension and the sailplane longitudinal accelerometer measurements. No specific test maneuvers were executed; data were collected at a number of representative flight conditions which included takeoff, climbout, level flight, high tow, low tow, and various airspeeds. Production cockpit instruments (altimeter, airspeed indicator, variometer) were used to identify stabilized test conditions, but these data were not electronically recorded. A discussion of representative flight test data follows.

Figure 7 shows time histories of towrope tension and sailplane longitudinal accelerometer measurements for a

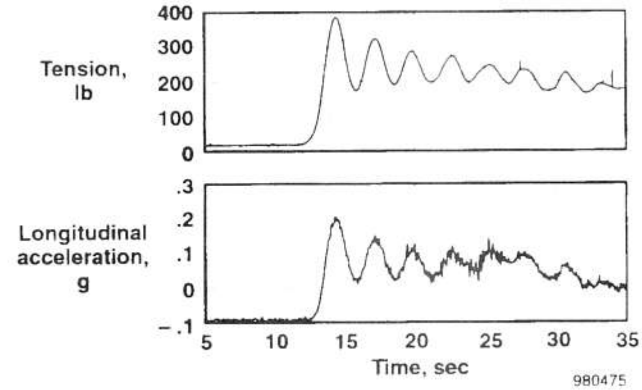

Figure 7. Takeoff time history of tow tension and sailplane longitudinal acceleration.

takeoff. Several features are noteworthy. The initial tension peak reaches nearly 400 lb. The static component of tension is about 200 lb; the dynamic component of tension is initially about 200 lb peak-to-peak and decays to about 70 lb peak-to-peak. Both the tension and the accelerometer signals show a strong dynamic component with a frequency of approximately 0.37 Hz. The accelerometer signal also contains a significant high-frequency component throughout; this is likely caused by runway vibration during the ground roll. The obvious bias in the accelerometer measurement is likely caused by a misalignment or calibration error. The tension signal also shows a bias, possibly a result of calibration error. Otherwise the two signals are strongly correlated in time and confirm the existence of the bungee mode near the expected frequency.

Figure 8 shows time histories of towrope tension during level flight in light turbulence at an airspeed of approximately 69 knots. The bungee mode is readily excited by light turbulence; the flat portions of the time history, which indicate approximately 20 lb of tension, demonstrate that a significant fraction of the time was spent with a slack or nearly-slack towrope. The tow tension is not periodic, but shows several discrete peaks that are separated by approximately 4 sec each at which the slack was aggressively removed by the towplane. In-flight visual observations showed that the towrope was seldom straight between the

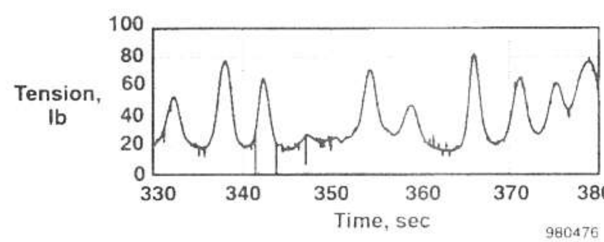

Figure 8. Time history of tow tension in level flight at 69 knots with light turbulence.

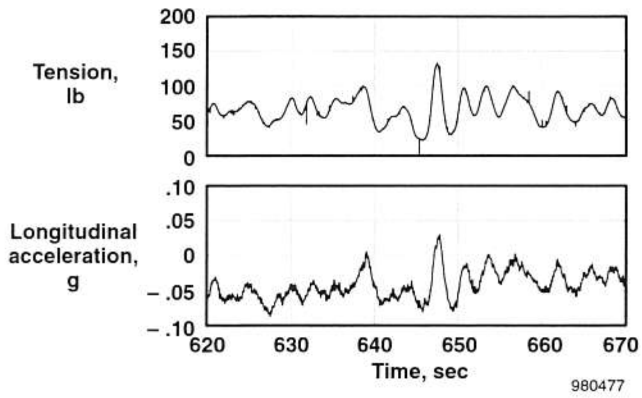

Figure 9. Time history of tow tension and sailplane longitudinal acceleration at 53 knots in smooth air.

two aircraft; during dips in the tow tension the towrope assumed noticeable curvature. Apparently even light turbulence provided enough excitation to drive the system into a region of nonlinearity and produce a non-sinusoidal tension response.

Figure 9 shows time histories of tow tension and sailplane longitudinal accelerometer measurements during level cruise low-tow flight in smooth air at an indicated airspeed of 53 knots; near the airspeed for maximum lift-drag-ratio for this aircraft (ref. 15). Even without an obvious source of excitation the bungee mode is apparent in both measurements. The oscillatory tow tension component is approximately 50 lb peak-to-peak with a frequency in the range of 0.33 to 0.40 Hz. Estimating the static component of tension is difficult because the bungee mode is present, but it is approximately 70–80 lb. Although the tension excursions are larger than those seen in figure 7, the tension response is more sinusoidal; for this flight condition the system remains in a region of relative linearity.

The frequency of the bungee mode observed in this flight test ranged from approximately 0.33 Hz to 0.40 Hz, which is lower than the preflight prediction of 0.38 Hz to 0.46 Hz. The preflight prediction was based on a single-degree-of-freedom model; one in which the towrope is straight but extensible between the two aircraft. Casual in-flight observation showed this to be an oversimplification; during excursions to low tension the towrope assumes significant curvature. Despite the simplification, the single-degree-of-freedom engineering model does a reasonable job of predicting bungee-mode frequency. Measurement of the damping ratio in-flight is difficult given the limited flight data, but it is clear that the damping ratio is small.

In-flight excitation of the bungee mode makes measurement of the static component of tow tension difficult. However, the data collected indicate larger tow tension static components than those predicted by simple performance-based calculations. Clearly tow tension by itself is not a direct measurement of sailplane drag; the measurement is potentially contaminated by numerous error sources. The short list presented below is not meant to be all-inclusive.

The tow tension measurement appears to have an unexplained bias of about 20 lb. In this experiment tow tension was measured at the towing plane end of the towrope; the tow tension is increased by the additional drag of the towrope. The system may not always be in steady, level flight; positive longitudinal acceleration and positive climb rate would increase tow tension. The tow tension may apply a significant additional pitching moment and require retrimming of the sailplane; the additional trim drag could increase tow tension. The sailplane may be flying in the turbulence and downwash field of the towplane; the former would likely increase sailplane drag directly, and the latter would likely force retrimming the sailplane and could increase drag indirectly. As a result of the additional trim drag, the airspeed for maximum lift-drag ratio may be different on- than off-tow, and is likely to be dependent on the location of the tow hook and the tow position.

Recommendations for Further Work

While the single-degree-of-freedom model and a simple flight instrumentation suite showed some promising results, comprehensive validation of theoretical models with flight-experimental data requires more sophisticated modeling, flight test techniques, and instrumentation. Existing comparative high-order dynamic models of the towed system (ref. 8 and 9) are available to predict system response (tow tension as well as other aircraft variables) to generalized forcing functions. These high-order models typically include a large number of modes associated with the towrope itself. Selection of the model order that captures the essential characteristics of the towed system without oversophistication is the crux.

Flight test is the environment for collecting experimental data to make an informed selection of appropriate model order. Flight test maneuvers which excite and isolate the expected dynamic modes need to be defined and executed. Also, flight test instrumentation which can collect the experimental data sets needs to be selected and installed. In addition to making the conventional measurements of the rigid-body modes of both test aircraft, careful attention needs to be paid to making measurements of the modes most closely associated with the towrope. Only when the requisite maneuvers have been flown and the experimental data collected can the candidate models be evaluated and validated.

Concluding Remarks

A simple mathematical model and simple ground testing do well to predict the characteristics of a low-frequency, lightly-damped bungee mode inherent to towed sailplane operations, and in-flight measurements confirm its existence. The bungee mode is virtually continuously excited as a result of its low damping. Tension excursions during takeoff typically exceed 300 lb, and peak-to-peak oscillations of 50 lb during cruise flight are not uncommon. This dynamic component of tow tension makes accurate measurement of the static component of tow tension difficult. The static component of tow tension appears to be larger than predicted by simple performance-based calculations.

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