

State of the Natural Parameter Method for Chaotic Data Analysis and Modeling

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Abstract

The paper summarizes the state of development of the Natural Parameter Method (NAPAM). The NAPAM can be used, for example, to assess and analyze the seemingly random but basically deterministic load inputs on gliders by atmospheric turbulence and ground unevenness, respectively, and for analysis and modeling of atmospheric flow fields. The NAPAM is the chaotic upgrading of the stationary and ergodic stochastic process model. The starting was based on the Kovászny theorem. The initial success encouraged the general adoption of chaotic methods and procedures. The mode and depth of the analysis is limited by the character of the record, i.e. by the accuracy and frequency of sampling. The particularities of the method are treated in definitions, preprocessing, statistical analysis, correlation calculus, frequency analysis and in rating of similarity.

Nomenclature

f	frequency	s^{-1}
f()	function of...	
g()	function of	
h	sampling interval	
n	wave number	m^{-1}
p()	probability density function	
t	independent variable resp. time	
x	dependent (registered) variable	
x_0	lower limit	
x_m	upper limit	
A	constant	
C	constant	
$G_x()$	one-sided power spectral density function resp. amplitude spectrum	
$H_{yx}()$	frequency response function	
L	integral scale	m
L_t	time scale	s
P()	probability distribution function	
$R_x()$	one-sided autocovariance function	
T	time base (length) of the analysed record	
V	vehicle speed	ms^{-1}
X()	potential function	
α	exponent	
β	deviation parameter	
δ_h	neighbourhood number	
ε	phase angle	
ζ	spatial displacement	m
λ	Taylor's scale	m
λ_t	Taylor's time scale	s
μ	mean value	
σ_x	standard deviation	
τ	time displacement	s
ω	circular frequency	rad s^{-1}
Φ	scale of similarity	

Ψ scale of difference/distortion

Historical background and basic perceptions

Originally the system and methods of natural sciences were based on the presumed strictly deterministic character of the laws of nature. Later on this ideal concept could not be upheld and (at first in thermodynamics) the stochastic methods as well as the calculus of probability were admitted¹. The theory of turbulence, too, followed this trend but research for more exact relations continues.

The mathematical tool for this research was found in the form of correlation functions. Strictly speaking, the true random data sequences are statistically independent, consequently their correlation functions are zero. Strangely enough, the mathematicians did not object to this ambiguity and the traditional model of stationary and ergodic stochastic processes includes not only correlation functions but frequency analysis, as well.

Seemingly the turbulence theory chose the same way, however it did not give up the essence of determinism even in details. Knowing the differential equation and the solutions seemed quite normal. Introduction of the integral scale parameter L seemed to be a turbulence particularity at first. Then, some decades ago the Kovászny theorem² based on the interconnection

$$R_x(\tau)_{\tau=0} = \sigma_x^2 \quad (1)$$

$$G_x(\omega)_{\omega=0} = \frac{2}{\pi} L \sigma_x^2 \quad \text{resp.} \quad G_x(f)_{f=0} = 4L \sigma_x^2 \quad (2)$$

declared the scale parameter L to be a natural parameter equivalent to and complementary with the standard deviation σ_x for all stationary and ergodic stochastic processes.

This perception opened the way for the adaptation of turbulence concepts and methods to other lines, too. Moreover, it caused the stochastic process model to be a chaotic one in real-

ity. We believe thirty years ago the model was used to analyze and model road/terrain unevenness data for off-road vehicles. The initial success encouraged further work in this line^{3,4}.

At present, mathematics offers a wide choice of procedures for handling nonlinear differential equations with provinces of bifurcations. Some lines, e.g. meteorology, used these efficiently but our participation seemed to be limited to the boundary layer control because of limitations in funds and in time span for the identification and solution of the differential equations. Against this, the development of chaotic record analysis methods seemed to offer numerous new possibilities for aeronautical research and design.

The general trend in flight vehicle dynamics and operating load record assessment seems to distinguish three classes of functions (movements, forms): deterministic, stochastic and chaotic ones (Fig. 1). Due to the possibility of bifurcation the third one is supposed to be even less predictable than the simple random cases.

We do not share this understanding. The first reason is that we never get the measured function $x(t)$ directly. We have only the record of the function. More or less part of the true information is lost due to the original errors of the registration and due to the finite frequency of sampling. Moreover, errors may also accumulate in the recording (Fig. 2). Mode and possibility of the data analysis is limited by the character of the record. As it will be shown later, the degree of record to function correlation is determined by the measurement errors and by the sampling interval h . Therefore, the classification of the original data may be only a more or less reliable extrapolation.

Updating the traditional stochastic process model (Fig. 3), we regard the stationary chaotic record as a transient one between the deterministic and the true random states (Fig. 4). This situation is similar to the theory of relativity without the concept of a single static inertia coordinate system. Giving up the concept of perfect data processing, we attempt to save the remaining part of the recorded function. The percentage of randomness depends on the magnitude of the measurement errors and on the sampling frequency.

Definitions

Reliable and productive research needs correct and unambiguous basic definitions. Before describing the characteristic features of the NAPAM concept and procedures, let us glance at our peculiar view of some of the basic definitions. We do not believe the formal definitions to be complete as yet. A clear physical sense, too, is aimed at as far as possible.

Deterministic, stochastic, chaotic

A function (movement, form) is deterministic if the initial conditions determine every one of the following values of the function exactly. Deterministic data can be represented by explicit mathematical relationships.

In the stationary and ergodic stochastic process model, a true random character of the measured data was presumed. If this would be true, neither correlation nor frequency analysis would be possible. In order to correct this, the attribute *sto-*

chastic is not used in our nomenclature. Correlation-free data sets are said to be *random* or *truly random*.

In the following we refer to the stochastic process as a *chaotic process*. It implies that the record of the measured function is neither exactly deterministic nor truly random.

Stationarity

Stationarity is defined and checked individually on each of the function parameters by the traditional stochastic process model. The requirement is the invariance of the parameter to the value of the initial time t_1 .

Joining with the theory of Wigner⁵, the generalization formally not yet proven but obvious reads as follows. An $x(t)$ periodic or nonperiodic function (movement, form, etc.) is stationary if and only if it has an $X(t)$ (scalar) potential function and the value of this $X(t)$ function is constant. We expect a quadratic form of the potential function.

The so-called weakly stationary function (i.e. only the mean value and the standard deviation are constant) is instationary.

Natural parameters

The standard deviation σ_x and the integral scale L (Taylor's scale L_t), indicating the scales for the two dimensions, are named *natural parameters* of the recorded function. The analytical formulae for the probability-, correlation- and spectral functions are composed with use of these parameters.

Regular-instationarity

A nonstationary function $x=f(t)$ is regular-instationary if and only if the standard deviation is $\sigma_x=g_1(t)$ and/or $L=g_2(t)$, with respect to, $L_t=g_2(t)$ and the transformation

$$\sigma_x \rightarrow \frac{\sigma_x}{g_1(t)} \text{ and/or } L \rightarrow \frac{L}{g_2(t)}, \text{ respectively, } L_t \rightarrow \frac{L_t}{g_2(t)} \quad (3)$$

makes the transformed functions stationary. For example the road unevenness, stationary in space, gives a regular-instationary input time function to the wheels of an accelerating or braking car with $g_1(t)=1$ and $g_2(t)=V(t)$, see also Eqs. (20) and (23).

Spectrum

The series $\{G_x(\omega_i)\}_{i=0}^m$ is the spectrum of the function $x(t)$ if and only if for $0 \leq t \leq T$

$$\sum_{i=0}^m G_x(\omega_i) \cos(\omega_i t + \varepsilon_i) = x(t) \quad (4a)$$

or

$$\lim_{m \rightarrow \infty} \sum_{i=0}^m G_x(\omega_i) \cos(\omega_i t + \varepsilon_i) = x(t) \quad (4b)$$

Similarity

Similarity is a well known concept in everyday life. It is used in this interpretation in a scientific or technical sense too, but its traditional exact definition has a much narrower sense in geometry (similar triangles and other plane figures). The

chaotic way of assessment needs a much broader and, nevertheless, exact and measurable interpretation of the concept.

The similarity problem is current in several trades. Our treatise focuses on the geometric similarity; other cases may be traced back to it. In the rating of geometric similarity we take the following fundamentals into consideration.

Comparing is possible only with the same sort of things (e.g. triangle with triangle, living standards with living standards, etc.). Generally two forms can be declared to be similar if the characteristic features are identical while the subordinate ones may be different if the similarity is, nevertheless, obvious, that is, clearly demonstrable.

In this form the definition seems to be uncertain. On an impartial scale one of the parties is regarded to be the norm while the other one or the other ones is/are ranked to it.

The similarity calculation procedure consists of the following steps:

1. selection of the subordinate parameters (e.g. size of the triangles and mean value of the respective component directions in the central frame of reference);
2. converting the subordinate parameters into the reference values;
3. determination/calculation of the differences in the characteristic parameters;
4. calculation of the scale of difference/distortion and of the similarity.

The scale of perfect similarity in the Euclidean sense shall be $\Phi=1$ and the scale of zero difference/distortion reads $\Psi=0$. For the intermediate cases we declare

$$\Phi^2 + \Psi^2 = 1 \quad (5a)$$

The scale of difference determined at first gives the similarity readily

$$\Phi = (1 - \Psi^2)^{1/2} \quad (5b)$$

Preprocessing

The NAPAM system of data processing is a development of the time-honoured stochastic process model as used, for example, by Bendat and Piersol⁶. A sketch of the model is shown in Fig. 3. Preparation of the record for analysis is the usual way except the following statements, with respect to, operations.

It is advisable to pre-estimate the character of the record before the processing. In our opinion it is not the presumed character of the recorded processes but that of the records which shall determine the choice of the analysis process.

Neighborhood figure and function

The neighborhood figure is a graphical tool to visualize the relations between the coherent pairs of values of the sampled record. The pairs of values are coherent in the sense that there is a constant displacement of kh between the sampling points of them given as a multiple of the sampling interval

$$\{x_i = x(ih)\}_{i=0}^m \rightarrow \{P_i[x_i, x_{i+k}]\}_{i=0}^{m-k} \quad (6)$$

The plot, as its name indicates, shows the relations between the neighboring points of the record when the displacement is only one sampling interval ($k=1$).

The neighborhood figure is suitable for the classification of the records, because its shape determined by the character of the record, varies between two extreme cases.

When the record comes from a continuous function, sampled with extremely high frequency ($h \rightarrow 0$) and the measurement error is practically zero, the graph approaches a straight line on the diagram.

If the continuous function is periodic or almost periodic, then, the increasing of the displacement k moves away the points from the straight and forms a fine polygon containing loops (Fig. 5a).

Against this, if the record is a true random series, the points form a symmetric cloud irrespective of the displacement kh (Fig. 5c).

Between these extremes, in a real case, the points form a narrow band around the straight line (Fig. 5b). The width of the band depends on the relation between the short term changing rate of the observed phenomenon and the sampling interval and on the magnitude of the measurement error. Therefore, the figure also is suitable for the fast preliminary check on the sampling conditions. The effect of increasing the sampling interval is shown in Fig. 6. Neighborhood figures of further types of records can be seen in Dóra⁷ Fig. 1.

The neighborhood number δ_h has been defined for the exact numerical characterization of the width of the cloud of points as the normalized RMS value of the differences of the sequential sampling readouts x_{i+1} and x_i

$$\delta_h = \frac{1}{\sigma_x} \left[\frac{1}{m} \sum_{i=0}^{m-1} (x_{i+1} - x_i)^2 \right]^{1/2} \quad (7)$$

The δ_h value of say $0.03 \div 0.2$ indicates a sufficiently fast sampling and negligible measurement error. True random sequences give $\delta_h = \sqrt{2}$ irrespective of the value of the sampling interval.

The variation of the sampling interval gives the neighborhood function (Fig. 7a-c)

$$\delta_h(kh) = \frac{1}{\sigma_x} \left[\frac{1}{m-k+1} \sum_{i=0}^{m-k} (x_{i+k} - x_i)^2 \right]^{1/2} \quad (8)$$

The neighborhood function is periodic for periodic functions (Fig. 7a), it is nonperiodic for almost periodic ones (Fig. 7b), while it is nearly constant for random number sequences (Fig. 7c). When the record comes from a continuous function sampled with extremely high frequency ($h \rightarrow 0$) without measurement error, the extrapolated value of the neighborhood function at $k=0$ is practically zero (Fig. 7a). In real cases a significantly nonzero extrapolated value appears which is characteristic for the magnitude of the random measurement error (Fig. 7b).

Running mean

The theory of stochastic processes declares the mean value of the continuous function $x=x(t)$ to be:

$$\mu = \lim_{t_1 \rightarrow \infty} \frac{1}{t_1} \int_{t_1}^{t_1} x(t) dt \quad (9a)$$

or in case of sampling at equally spaced intervals:

$$\mu = \lim_{m \rightarrow \infty} \frac{1}{m+1} \sum_{i=0}^m x_i \quad (9b)$$

The NAPAM system follows the aero/hydro-dynamic concept in analysing the mean flow and the seemingly random turbulence separately. It gives not a constant μ but a running mean function $\mu(t)$ (Fig. 8). In the following, we analyze only the seemingly random part, hence the mean values are left out from the formulae:

$$x(t) - \mu(t) \rightarrow x(t)$$

For prospectively stationary records the calculation of the running mean shall be done before the neighborhood calculations.

Statistical analysis

Calculation formulae follow the general stochastic trend except the following.

Standard deviation, probability distribution and density function

Having subtracted the mean value from the recorded samples, the formula for the standard deviation reads:

$$\sigma_x = \lim_{t_1 \rightarrow \infty} \left[\frac{1}{t_1} \int_0^{t_1} x^2(t) dt \right]^{1/2} \quad \text{resp.} \quad \sigma_x = \left[\frac{1}{m+1} \sum_{i=0}^m x_i^2 \right]^{1/2} \quad (10ab)$$

Standard stochastic methods are used for calculation of the probability functions as shown on Fig. 9.

Analytical form of the probability function

Standard probability functions are used for the smoothing of the experimental data. In some cases, the exponential distribution function or the normal probability density formula

$$p(x) = (\sigma_x \sqrt{2\pi})^{-1} \exp \left[-\frac{x^2}{2\sigma_x^2} \right] \quad (11)$$

will serve well (Fig. 9). The two-parameter Weibull function,

$$\frac{1}{1-P(x)} = \exp \left\{ \left(\frac{x}{\beta} \right)^\alpha \right\} \quad (12)$$

being in some respect a generalization of the Gauss function, can be adopted, too. In all three of them, a lower boundary x_0 or an upper one x_m can be introduced for the distribution.

Correlation

Autocovariance function

The statistical analysis of the records in the dimension of the independent variable uses the one-sided autocovariance function

$$R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau) dt \quad (13a)$$

$$R_x(kh) = \frac{1}{m-k+1} \sum_{i=0}^{m-k} x_i x_{i+k} \quad (13b)$$

For errorless records the relation reads

$$\frac{1}{\sigma_x^2} R_x(kh) + \frac{1}{2} \delta_h^2(kh) = 1 \quad (14)$$

Therefore, introducing two non-standard notations, the local one

$$S_k = \frac{1}{m-k+1} \sum_{i=0}^{m-k} x_i x_{i+k} + \frac{1}{2(m-k+1)} \sum_{i=0}^{m-k} (x_{i+k} - x_i)^2 \quad (15)$$

and the average one

$$S_n = \frac{1}{n} \sum_{k=1}^n S_k \quad (16)$$

gives the possibility to estimate the initial random measuring error (Fig. 10)

$$\Delta_r(\zeta) = \Delta_r(kh) = S_k - S_n \quad (17)$$

Integral scale and Taylor's scale

We adopted the scale parameters from turbulence theory. The integral scale reads (Fig. 11)

$$L = \lim_{\zeta_1 \rightarrow \infty} \left| \frac{1}{R_x(0)} \int_0^{\zeta_1} R_x(\zeta) d\zeta \right| \quad [m] \quad (18)$$

In time coordinates it comes out

$$L_t = \lim_{\tau_1 \rightarrow \infty} \left| \frac{1}{R_x(0)} \int_0^{\tau_1} R_x(\tau) d\tau \right| \quad [s] \quad (19)$$

For a vehicle running over/through a stationary external load input, the transformation formula reads

$$L_t = \frac{L}{V} \quad [s] \quad (20)$$

The formula for Taylor's scale reads

$$\lambda = \frac{\sqrt{2}}{\left[-\left(\frac{d^2 R_x(\zeta)}{d\zeta^2} \right)_{\zeta=0} \right]^{1/2}} \quad [m] \quad (21)$$

The calculation scheme is shown in Fig. 12. In time coordinate it is

$$\lambda_t = \frac{\sqrt{2}}{\left[-\left(\frac{d^2 R_x(\tau)}{d\tau^2} \right)_{\tau=0} \right]^{1/2}} \quad [s] \quad (22)$$

The transformation formula is the same

$$\lambda_t = \frac{\lambda}{V} \quad [s] \quad (23)$$

Frequency analysis

Spectral density function

Spectral representation is mandatory for the analysis of turbulence as well as for the dynamic load input-output calculations. As it is well-known, the nonperiodic character of the so-called stochastic functions necessitated the extension of the Fourier calculus methods in the form of the Fourier transformation done on the autocovariance function. It results seemingly in the form and dimensions of a continuous differential-spectrum function $G_x(\omega)$ or $G_x(f)$. The autocovariance function does not include the phase angle ε . For this reason the spectral density function only is an ensemble spectrum.

The NAPAM programs prefer the so-called direct method using directly the $x(t)$ data record offered by the introduction of digital processing. It gives substantial improvements especially in case of multiple input-output calculations.

Direct spectrum

Direct spectrum methods are used but without fast-Fourier-transform (FFT) modification. The net result is a reliable determination of the function $G_x(f)$ or $G_x(n)$ over the measured base length T (see e.g. Fig. 13) including the phase angle $\varepsilon(f)$ or $\varepsilon(n)$.

The generalized form of the Kármán turbulence spectrum is used for smoothing the measured spectrum amplitude points. It reads

$$G_x(n) = 4L\sigma_x^2 \frac{1 + A(\text{CL}n)^2}{[1 + (\text{CL}n)^2]^\alpha} \quad (24)$$

or

$$G_x(f) = 4L_t\sigma_x^2 \frac{1 + A(\text{CL}_t f)^2}{[1 + (\text{CL}_t f)^2]^\alpha} \quad (25)$$

In case of turbulence, the exponent reads $\alpha=11/6$, resulting in the original Kármán formula. Road/terrain unevenness spectra seem to give about $\alpha=2$ for start and landing load calculation.

Exact point for point space-time conversion is also possible

$$f_i = n_i V \quad (26)$$

$$G_{xi}(f_i) = \frac{G_{xi}(n_i)}{V} \quad (27)$$

The role and the place of Taylor's scale seems uncertain at present in the evaluation. Space and time bounds do not allow discussion of every detail of the calculation.

Multiple input-output

Knowledge of the relative input phase angles $\varepsilon_i - \varepsilon_j$ at $t = ?$ is necessary for the multiple input-output calculations. In the traditional form, the input relative phase information is in the input spectrum matrix

$$\mathbf{G}_{xx}(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & \dots & G_{1r}(f) \\ G_{21}(f) & G_{22}(f) & \dots & G_{2r}(f) \\ \vdots & \vdots & \ddots & \vdots \\ G_{r1}(f) & G_{r2}(f) & \dots & G_{rr}(f) \end{bmatrix} \quad (28)$$

and the formula reads:

$$G_{yy}(f) = \mathbf{H}_{yx}^*(f) \mathbf{G}_{xx}(f) \mathbf{H}_{yx}^T(f) \quad (29)$$

The direct spectrum calculation offers the possibility of phase angle evaluation, by giving a (complex) spectrum vector representation of the input forces. It results in substantial savings in memory space as well as in CPU time.

Discrete frequency amplitude spectrum

Field tests in Hungary confirmed the accuracy of input-output calculations using the direct spectra of the terrain unevenness³. Nevertheless, doubts arise about the theoretical correctness of the method. As shown in Fig. 13, the raw PSD points have a significant dispersion. The fact that it does not diminish with the development of instruments and data processing brought up the subject of a discrete frequency non-continuous spectrum structure.

In order to get hold of the problem, respective trial calculations were made on four atmospheric turbulence records from the DLR research program LOTREX/HIBE'89⁸. The results were reported in 1996 at Budapest⁹ and in 1999 at the XVI OSTIV Congress¹⁰. A theoretically not indisputable modification of the Fourier process confirmed the existence of the discrete frequencies (Fig. 14) but the numerical accuracy was inferior to that of the PSD calculus. Thereupon, another method was to be found.

It is easy to prove that no manner of Fourier integration type calculus can be correct if the ratios of the respective frequencies are not integer. To get around this difficulty, we developed a method based on the phase portrait

$$\frac{dx}{dt} = f(x(t)) \quad (30)$$

The simple geometric approximation

$$\left(\frac{dx}{dt} \right)_{t=t_1} \approx \frac{x_{i+1} - x_i}{h} \quad (t_1 = \frac{t_{i+1} + t_i}{2}) \quad (31)$$

could be an acceptable compromise for continuous and smooth functions but it is of no use because of the primary random measurement errors. Reducing the intervals between the samplings and introduction of a local running mean calculation solved this problem (see Fig. 15). Now, we are working on a correct method to calculate the respective frequencies and amplitudes including the initial phase angles.

Rating of geometric similarity

Similarity of forms or movements is a useful indicative of physical relations. The human sense perceives and grades it instinctively, but the Euclidean geometry accepts and defines the forms with perfect, so to say the 100%, similarity only. Therefore, to circumvent this limitation, the numerical rating of the similarity was explored. It promises new possibilities in research and development.

The work is in the initial stages. Initial concepts and basic formulae have been reported^{11, 12}.

Concluding remarks

NAPAM is a record analysis method of chaotic character starting from the stationary and ergodic process model and from the Kovászny theorem. It gives considerable improvements in accuracy as well as savings in CPU time and in memory space. At present the frequency analysis and the rating of similarity stand in the center of the development work.

Acknowledgements

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References

- ¹ Simonyi, K.: Kulturgeschichte der Physik (Akadémiai Kiadó – Verlag Harri Deutsch, 1995)
- ² Kovászny, L.S.: Revue des théories de la turbulence in Favre et al.: La Turbulence en Mécanique des Fluides (Gauthier-Vilars, Paris, 1976; pp. 91-94)
- ³ Laib, L., Gedeon, J.: A terepen mozgó járművek mozgásának elemzése (Járművek, Mezőgazdasági Gépek, Vol. 36, No. 8, 1989; pp. 285-289; Budapest, in Hungarian)
- ⁴ Gedeon, J.: The Role of the Scale Parameter in Service Load Assessment and Simulation (Proc. of the 13th ICAS Congress; ICAS-82-2.8.3; Seattle, 1982, pp. 1339-1349)
- ⁵ Wigner, E.P.: Events, Laws of Nature and Invariance principles (Nobel Prize Lecture 1963) in: Nobel Prize Lectures; Elsevier Publishing Company
- ⁶ Bendat, J.S., Piersol, A.G.: Random Data: Analysis and Measurement Procedures (Wiley-Interscience, New York, 1971)
- ⁷ Dóra, S.: Development of Visual and Numerical Methods Based on Second Order Statistics for the Analysis of Digital Measurement Records, Per.Pol.Transp.Eng., vol. 35, 2007 (p. 111-123); http://www.pp.bme.hu/tr/2007_1/pdf/tr2007_1_11.pdf
- ⁸ Jochum, A.M., Entstrasser, N., Fimpel, H.P., Morl, P., Rosster, F., Willeke, H.: Evaporation and Energy Fluxes in the Atmospheric Boundary Layer During LOTREX, AMS Symposium on the First ISLCP Field Experiment (FIFE) 70th AMS Annual Meeting (February 7-9, 1990; Anaheim California)
- ⁹ Gedeon, J.: Analysis of Low-Level Atmospheric Turbulence. A Case Study (Proc. 6th Mini Conference on Vehicle System Dynamics, Identification and Anomalies; Budapest, 1996; pp. 41-50)

- ¹⁰ Gedeon, J.: Turbulence Scale Parameter and Spectrum Identification (Technical Soaring Vol. 25, No. 1; 2001 Jan. pp. 121-125).
- ¹¹ Gedeon, J.: A mértani hasonlóság problémája (Járművek, Vol. 49, No. 6, 2002, pp. 35-38; Budapest, in Hungarian)
- ¹² Dóra, S., Gedeon, J.: Conversion from Stochastic to Chaotic Approach in Research and Design (Technical Soaring Vol.26, No.4, 2002 Oct. Pp. 124-131).

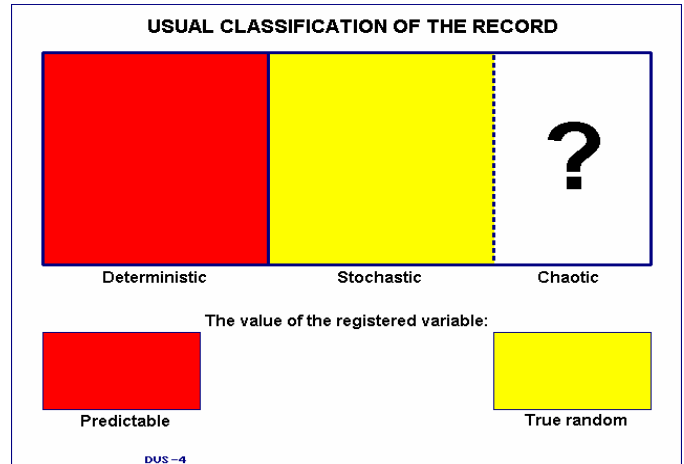


Figure 1 The usual classification scheme for records

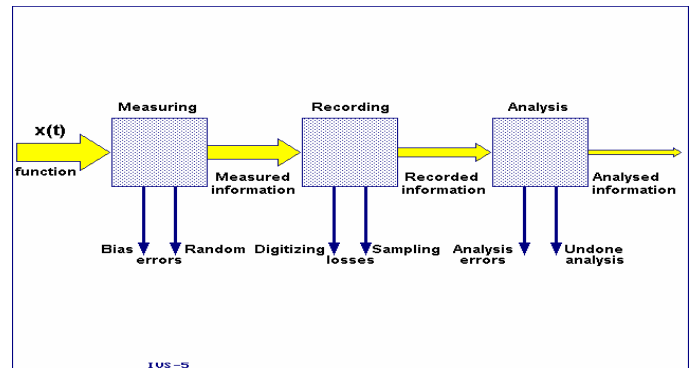


Figure 2 Loss of information in the analysis of records

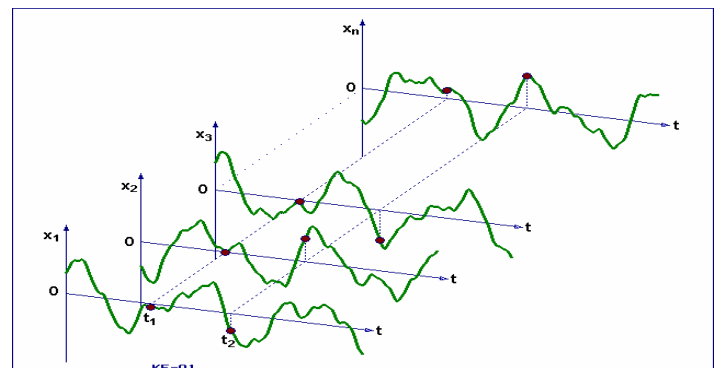


Figure 3 Scheme of the stochastic process in style of Bendat and Piersol³

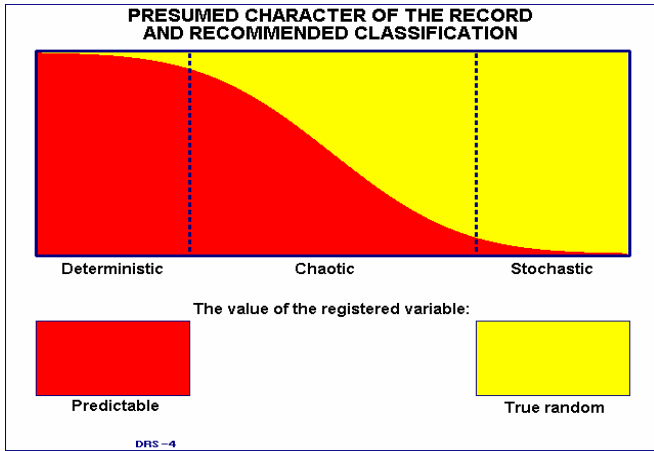
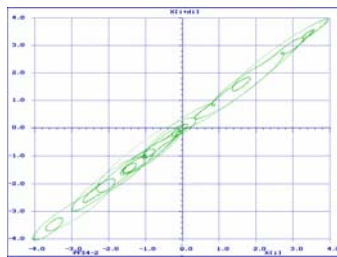
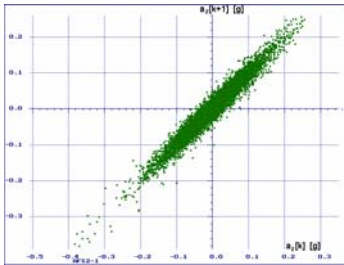


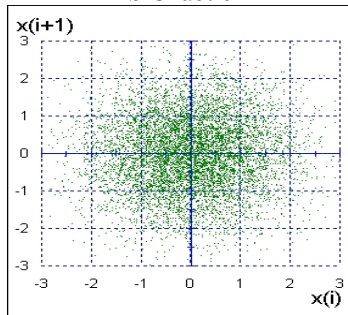
Figure 4 The NAPAM classification scheme



a Periodic



b Chaotic



c Random

Figure 5 Characteristic neighborhood figures

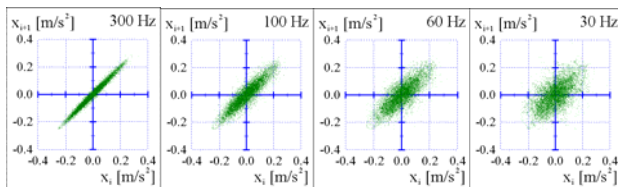
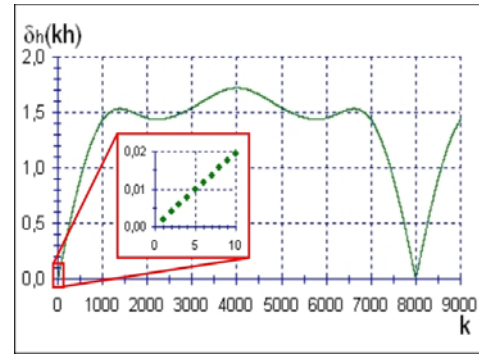
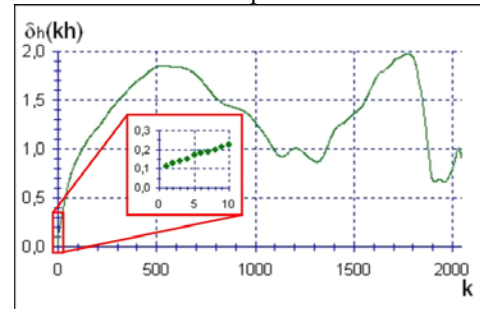


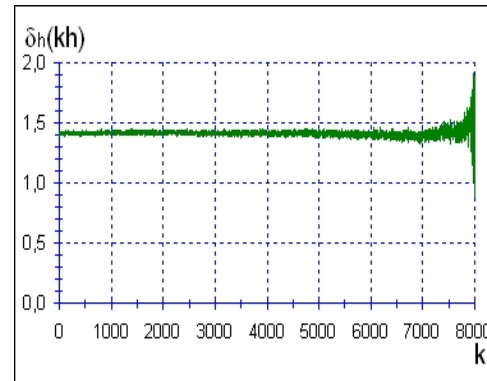
Figure 6 Neighborhood figures of a record relating to various sampling frequencies



a Exact periodic



b Chaotic



c Random

Figure 7 Characteristic neighborhood functions

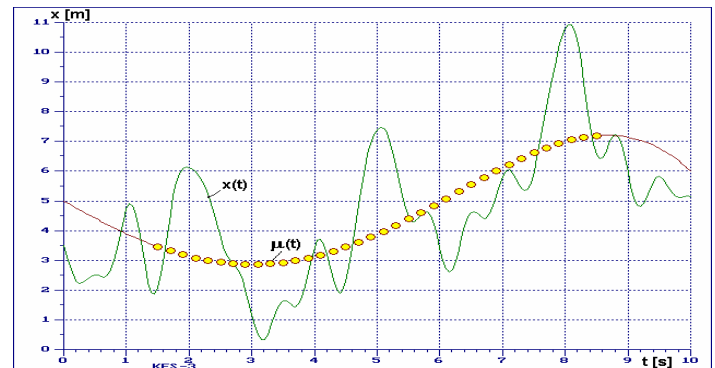


Figure 8 Calculation of the running mean

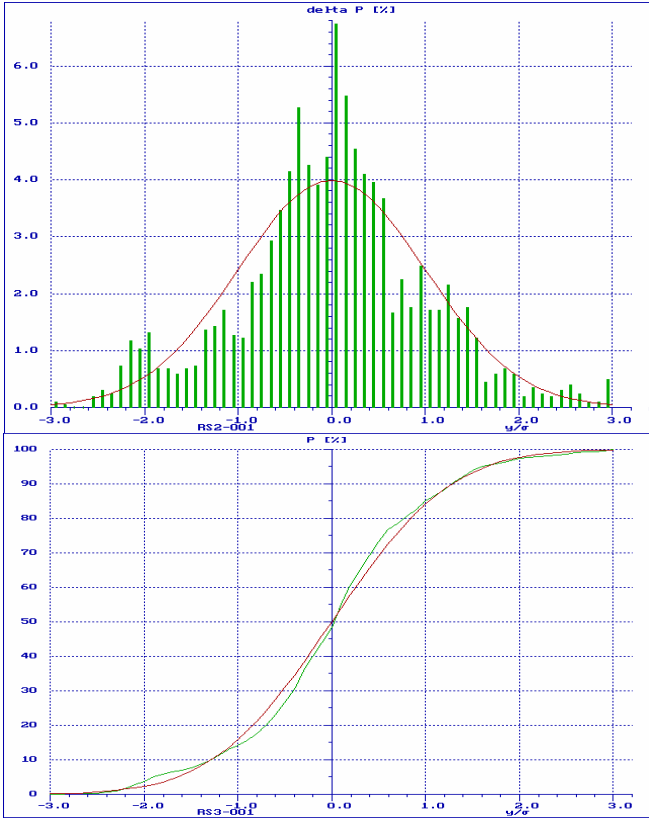


Figure 9 Calculation of the probability distribution and density

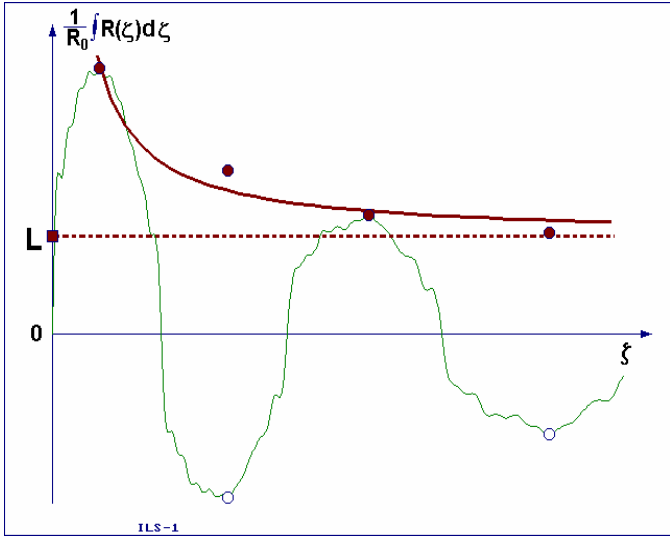


Figure 11 Calculation of the integral scale

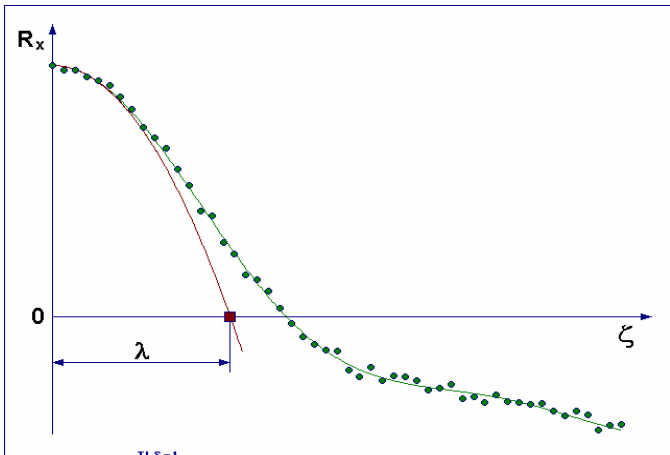


Figure 12 Calculation of Taylor's scale

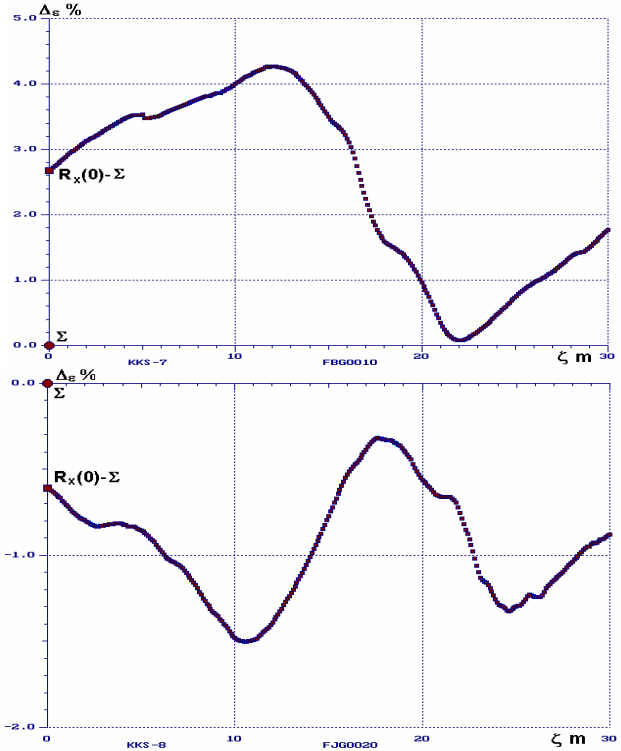


Figure 10 Two estimation of the measuring error

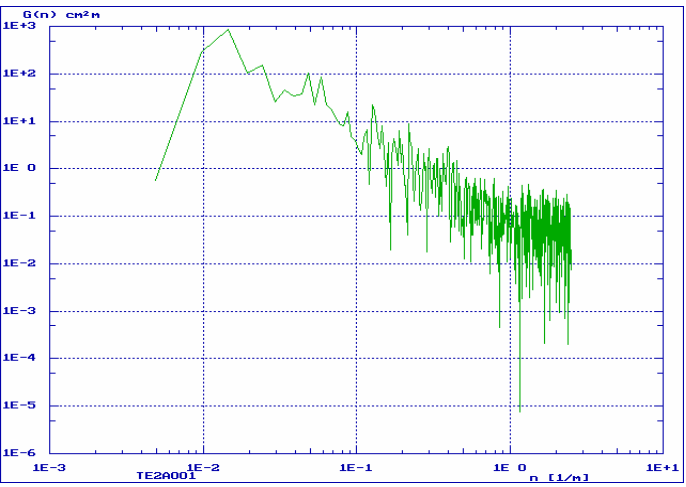


Figure 13 Power spectral density function

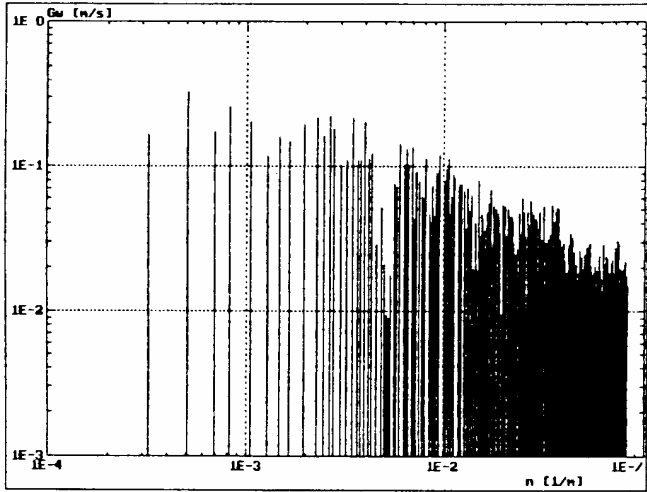


Figure 14 Experimental discrete-frequency atmospheric turbulence spectrum

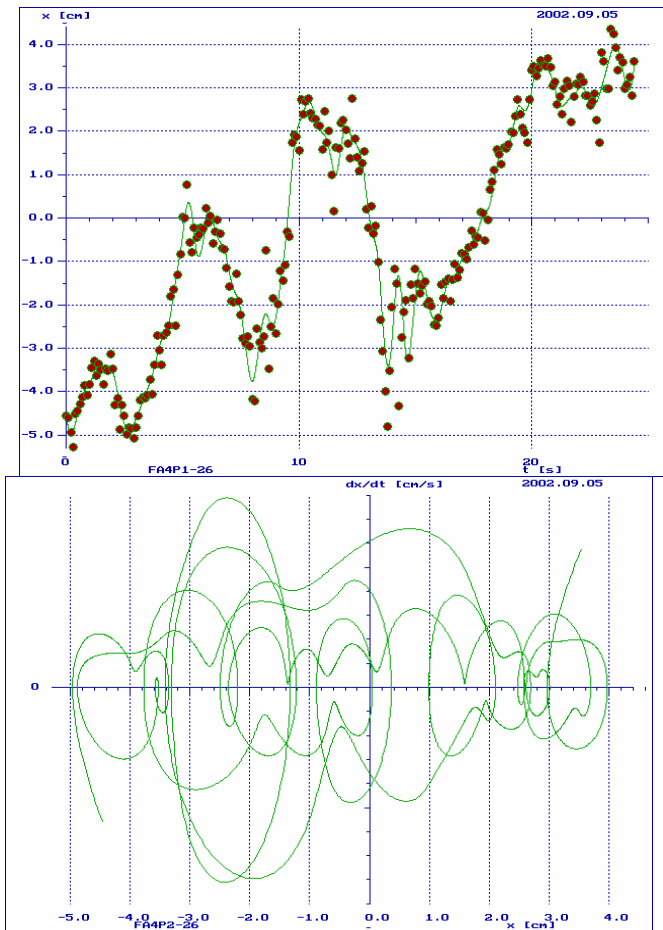


Figure 15 Calculation of the phase portrait