Regionalized Predictions of Aligned Updrafts and their Tuning for Planning Soaring Flights

Olivier Liechti
Analysen und Konzepte
CH-8404 Winterthur
OlivierLiechtiAuK@compuserve.com

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Abstract

Persisting weather conditions with aligned lift allow for the most spectacular long-distance soaring in mainly straight and level flight. This paper presents a regionalized approach towards predicting such meteorological conditions and their application to planning soaring flights. Since the terrain slope is underestimated in numerical models due to smoothed orography, the cumulative area-elevation distribution is used to predict slope updrafts. In addition, glider flights are analyzed to determine empirically for each region the absolute terrain slope. Simulations of glider flights with tuned predictions of aligned and isolated updrafts demonstrate the potential of regionalized forecasts for the planning of long and fast soaring flights.

Nomenclature

Δx	horizontal resolution (m)
Δz	vertical resolution (m)
z	elevation (m)
z_{min}	minimum elevation in a forecast region
z_{max}	maximum elevation in a forecast region
z_s	elevation with maximum slope
a	area (m^2)
a_{region}	region area (m^2)
$a(z)$	cumulative area-elevation distribution (m^2)
Δa	area (m^2) of a terrace in the area-elevation distribution
r	radius (m)
$r(z)$	radius as a function of elevation
α	slope angle (rad)
$\tan(\alpha)$	terrain slope
$\alpha(z)$	slope profile
α_{max}	maximum slope angle
$\Delta z/\Delta r$	terrain slope (m/m)
$\Delta z/\Delta a$	terrace slope (m/m^2)
w_{sink}	sink rate (m/s)
w_{min}	minimum sink rate (m/s)
v_{flight}	flight speed (m/s)
v_{wind}	wind speed (m/s)
v_{ground}	ground speed (m/s)

Introduction

When horizontal atmospheric flow is deflected vertically by complex terrain, up- and downdrafts occur. Slope and wave updrafts are examples of such deflections by terrain[1]. Often these updrafts are aligned. Glider pilots as well as birds enjoy flying along such aligned lift, where little or no circling in isolated lift is required to gain or maintain altitude. Persisting weather conditions with aligned lift allow for the most spectacular long-distance soaring in mainly straight and level flight.

The planning of soaring flights in both isolated thermal and aligned dynamic updrafts requires forecasts of the following meteorological elements[2,3]

- the strength of isolated updrafts
- the strength of aligned updrafts
- the altitudes of both types of updrafts
- the wind at both altitudes

at spatial and temporal resolutions to be defined. A modest temporal resolution of 30 min may be considered to be sufficient for flight planning. For gliders with a typical ground speed of 100 km/h a temporal resolution of 30 min corresponds to a spatial resolution of 50 km. A possible approach would be to extract the information needed for flight planning at modest resolution from high resolution weather models by selecting and averaging the previously listed parameters over time and space.

Weather models do not, as yet, predict the updrafts needed for flight planning. Wind, however, is an element directly predicted and its terrain induced vertical component is available at full resolution of the model grid: operational weather models have a horizontal mesh size ranging from 30 km for global models down to 2 km for meso-scale high resolution models[1]. Their vertical resolution is much finer particularly near the surface.

Orography is also represented at the horizontal mesh size of the weather model. All sub-grid scale terrain structure is smoothed. Terrain slope, e.g., is underestimated in the smoothed orography of weather models. Therefore, the vertical component of terrain following near surface flow is systematically underestimated. The flow predicted on a smoothed orography interacts with the steeper, true orography.

Satellite based radar measurements of the Earth's surface are available at a horizontal resolution of 90 m. In order to assess slope updrafts WindMaps[a] follow the approach of deflecting flow predicted along smoothed terrain by higher resolution orography. This results in high resolution maps of slope updrafts.

In this paper an approach complementary to WindMaps is taken to obtain slope updrafts from smoothed flow interacting with complex terrain. Digital elevation data is used to calculate the cumulative area-elevation distribution of comparatively large forecast regions (Appendix). The cumulative area-elevation distribution has been used successfully in the numerical prediction of convection over complex topography[4]. Its potential for the prediction of slope updrafts will now be investigated.

Forecast regions

As an alternative to a high resolution grid for the weather and for the digital elevation model, a conceptual approach may be taken by defining regions (polygons) adapted to terrain features like hydrological basins or land use. The convection models ALPTHERM[4] and REGTHERM[5] take the cumulative area-elevation distribution (AED) of such regions into account for the explicit simulation of buoyancy driven mixing over complex topography. Both models predict the strength and altitude of thermal updrafts at fine vertical (Δz = 100 m) and temporal resolutions for regions typically 50 km wide, i.e. at a modest horizontal resolution. By selecting and averaging the predicted updrafts over time and altitude, the information required for flight planning with isolated, thermal updrafts is obtained. The task of locating the predicted thermal updrafts within the fairly large regions is deliberately left to glider pilots in this approach.

Area-elevation distribution and terrain slope

The cumulative area-elevation distribution of a region also contains information about the average slope of complex terrain as a function of elevation. A vertical resolution Δz of 100 m is about fine enough to catch terrain properties relevant for slope lift. A basic approach to slope updrafts in the context of forecast regions with a known cumulative area-elevation distribution is to multiply the terrain slope with the predicted wind speed at different elevations in order to obtain a vertical profile. Note that such a profile of slope updrafts is isotropic: directional information neither from the slope nor the wind is retained.

If a cumulative area-elevation distribution a(z), where $0 < a(z) < a_{region}$ and $z_{min} < z < z_{max}$, is to be used to assess the average terrain slope $\Delta z/\Delta x$ for each elevation layer Δz, some difficulties appear. The relation between Δx and Δa depends on the shape of the terraces Δa representing the complex terrain. A circular shape of the terraces, where $a(z) = \pi * r(z)^2$, implies terrain shaped like a single cone. Terrain slope $\tan(\alpha) = \Delta z/\Delta r$ is then related to the terrace slope $\Delta z/\Delta a$ by

$$\tan(\alpha) \;=\; \Delta z/\Delta r \;=\; 2 * (a(z)/\pi)^{1/2}\,(\Delta z/\Delta a)$$

Terraces of elliptic instead of circular shape lead to steeper terrain slopes. Therefore, a slope profile $\alpha(z)$ with absolute values is not readily available from the area-elevation distribution. Slope profiles $\alpha(z)$ of relative values, however, can be obtained by assuming single cone shaped terrain. The elevation z_s with the steepest slope $\alpha_{max}(z_s)$ then can be identified in an area-elevation distribution.

Flight data reveal the slope of the flow

The flight polar of a glider[1,7] describes the sink rate w_{sink} corresponding to any flight speed v_{flight}. The minimum sink rate w_{min} of modern gliders is on the order of 0.5 m/s at flight speeds v_{flight} on the order of 25 m/s. If such a glider can soar at constant altitude along a ridge being blown at perpendicularly by a wind of strength v_{wind} = 10 m/s (about 20 KT or 36 km/h), then the slope of the flow (vertical deflection of the 20 KT wind) is

$$w_{sink} / v_{wind} > \; 0.5 \text{ m/s} / 10 \text{ m/s} = 0.05$$

The absolute terrain slope $\tan(\alpha) = \Delta z/\Delta r$ must meet or exceed this threshold for an elevation layer to be useful for ridge soaring in a 20 KT flow being deflected vertically to follow the terrain slope.

As the glider flight speed (v_{flight} > 25 m/s) normally exceeds the wind speed (v_{wind} > 10 m/s), the glider soars along the ridge at a *crab angle*. The *crab angle* appears between the flight speed vector and the ground speed vector. The ground speed v_{ground} of a glider following the updraft aligned along a ridge perpendicular to the wind is obtained from

$$(v_{ground})^2 = (v_{flight})^2 - (v_{wind})^2$$

Flight data reveal the ground speed of gliders soaring at a constant flight level along ridges in slope updraft. Recorded glider ground speed combined with predicted wind speed yields the glider flight speed. The flight polar, then, provides the glider sink rate, which must be equal to the slope updraft for the glider to fly at a constant altitude. The ratio of the deduced slope updraft to the predicted wind speed is the flow slope.

Calibration of terrain slope with flow slope

The difficulty of obtaining absolute terrain slopes from the area-elevation distribution may be overcome by analyzing glider flights in aligned ridge lift. Presumably relative terrain slopes deduced from the area-elevation distribution are proportional to absolute flow slopes extracted from recorded flight and predicted weather data. The proportion factor is to

[a] www.aerodrome-gruyere.ch/windmap/

be determined empirically for each region from soaring flights in ground-fixed aligned updrafts.

In Europe long distance soaring flights in ground-fixed aligned updrafts are predominantly made in mountain wave systems generated on the downwind side of mountain ranges like the Scottish Mountains, the Pyrénées, the Cévennes, and the Alps. The USA mid-Atlantic states, however, are known for fast, long, and low altitude soaring flights along the Appalachian ridges in north-westerly flow.

Soaring flights along the Appalachians in weather conditions producing both aligned and isolated updrafts were simulated on the basis of regional forecasts of both types of updrafts in order to calibrate the relative terrain slope obtained from the regional area-elevation distribution with flow slope deduced from glider trajectories.

On-line forecasts and flight planning for soaring

Regionalized forecasts for gliding are produced operationally for Central Europe and experimentally for parts of the USA (mid-Atlantic states and Colorado) by the German Weather Service (DWD). The convection model REGTHERM[6] is coupled to the global and continental weather models of DWD. The global model GME has a trigonal grid with a mesh size of 25 km (Figure 1), the COSMO-EU model for Europe a rectangular grid with a mesh size of 7 km. In Europe, REGTHERM feeds upon hourly vertical profiles (temperature, moisture, wind, solar radiation) for one grid point per region taken from the COSMO-EU model (nested into the GME model), whereas, in the USA coarser GME profiles are used. GME runs are updated once a day, COSMO-EU runs twice a day (00 UTC and 12 UTC) and so are the REGTHERM runs. In the morning the final REGTHERM runs are updated hourly based on the 00 UTC run of COSMO-EU and completed by hourly (03 to 06 h UTC) surface observations of temperature and dewpoint. The assimilation of surface observations modifies the predicted profiles near the surface and improves the REGTHERM predicted onset of cumulus clouds. No surface observations are assimilated by REGTHERM coupled to GME for the experimental forecasts in the USA.

The REGTHERM model calculates thermal climb rates and the depth of convection for complex terrain represented by its area-elevation distribution. Additionally, the hourly predicted wind profiles from the weather model (GME or COSMO-EU) are interpolated to the vertical resolution of 100 m and multiplied by the terrain slopes of the area-elevation distribution in order to estimate the magnitude of aligned slope updrafts. The elevation with the maximum slope updraft is selected as the preferred flight level of a glider in aligned lift. The strengths of both thermal and slope updrafts are listed together with the corresponding flight levels and COSMO/GME winds at a 30 min temporal resolution for all regions (horizontal resolution about 50 km).

Region forecasts are accessible on-line[a] through Java TopTask[6], a weather browser for the interactive visualization of TOPTHERM[b] forecasts (Figure 2). Additionally, the TopTask flight planning algorithm[2] is implemented in the interactive weather browser.

TopTask with isolated and aligned lift

Initially[2] only isolated thermal updrafts were used by TopTask to calculate the task speed from the predicted updrafts and winds. Comparisons of predicted task speeds with task speeds scored in gliding championships demonstrated the skill and limits of flight planning with isolated thermals predicted for regions by different numerical models[8]. Hindman and colleagues initiated regional thermal forecasts with RAMS for the USA (Colorado and mid-Atlantic states) and compared glider ground speeds along the recorded flight trace with TopTask simulations[9]. This allowed for the calibration of converting RAMS predicted convective boundary layer depth to thermal climb rate. For the mid-Atlantic states, however, the glider ground speeds simulated by TopTask with RAMS predictions of isolated thermal lift were insufficient when flights were made in – unpredicted – aligned ridge lift.

Recently, updrafts aligned along the flight track have been included in the optimization of the average glider flight speed[3]. TopTask now uses both isolated updrafts and updrafts aligned along the flight track for flight planning. The optimized average flight velocity is oriented at such a crab angle to the predicted wind vector that the resulting ground velocity is oriented along a desired course. For crosswind courses this implies that the aligned updrafts are ground fixed. Isolated lift, however, is assumed to move with the flow.

Flight simulation with predicted weather

Recorded flight data are published on the Internet[c] in the form of IGC-files[d]. Downloaded IGC-files are imported into Java TopTask. The recorded glider trajectories are displayed as flight traces on the map and the barogram with predicted climb rates in the background (Figure 3).

Moreover, the recorded flight traces are used to simulate the recorded trajectories with the TopTask algorithm by converting predicted updrafts (isolated and aligned), flight altitudes and winds into ground speed along the recorded flight trace. This conversion depends on the flight polar[3,7] of the glider and assumes the pilot to optimize the average horizontal speed. The simulated trajectory is superposed on the recorded trajectory (Figure 4). Predictions of aligned updrafts are optionally included in the simulations. This opens the

[a] http://www.flugwetter.de

[b] DWD disseminates the REGTHERM forecasts under the name TOPTHERM

[c] http://www.onlinecontest.org/olc-2.0/gliding/,
http://www.netcoupe.net/
http://sis-at.streckenflug.at/2010/

[d] standardized text format for GPS based flight recorder data

possibility to evaluate predictions of aligned updrafts for flight planning.

Simulations of flights in ridge and thermal lift

The experimental GME-TOPTHERM forecasts for the USA regions started on 24 May 2009 and were continued throughout 2010. Spectacular soaring flights were made in the Appalachians in this period, e.g.:

- 30 May 2009 1200 km @ 114 km/h
- 28 April 2010 1600 km @ 149 km/h
- 9 May 2010 1800 km @ 155 km/h
- 17 June 2010 1100 km @ 120 km/h

The GME profiles for the four days were archived in order to calibrate the regional factors relating relative terrain slope to the flow slope deduced from the recorded flights.

The first simulations of the published 30 May 2009 flights indicated that the thermal climb rates were appropriate. However, the aligned lift predictions seemed to be slightly too conservative. The day was not ideal for calibrating the aligned lift calculation because thermal lift was too strong: this reduces the sensitivity of the simulated ground speeds to the predicted aligned lift according to the best-speed diagram for isolated and aligned lift[3].

Then came the windy days 28 April and 9 May 2010 with recorded ground speeds of high performance sailplanes on the order of 150 km/h. Such ground speeds on crosswind courses indicate that aligned ridge lift is the dominant type of lift[3]. The initial simulations of 15 long-distance flights (average length 1348 km, standard deviation 290 km) were indeed too slow by 20% (standard deviation +/-10%) on average (Figure 6). Additionally, ridge lift flights of older, low performance gliders could not be simulated fully. The simulations occasionally had these gliders land out as their crab angle reached 90 degrees in strong winds. Erroneously simulated outlandings indicate that either the flow slope encountered by the glider is higher than the predicted one or the effective wind speed is weaker than predicted. The proportion factor between the relative terrain slope and flow slope was recalibrated individually for each region in order to remove erroneous outlandings from the simulations by making the slopes steeper. Simultaneously, the simulated ground speeds of the high performance gliders increased. The effective flow slopes of the six regions allowing for the highest ground speeds in ridge lift with flow from the north-west turned out to be $\tan(\alpha) = 0.1215$ which corresponds to a slope angle of $\alpha = 7$ degrees.

As a result of the flight-based calibrations of the effective regional terrain slopes performed in May 2010, the mean bias between recorded and TopTask simulated ground speeds was reduced from 20% to 4% (standard deviation +/-7%) on average (Figure 6) for the 15 flights considered. The standard error of both mean biases is significantly smaller than their difference. The applied calibration of relative terrain slope - deduced from a regional area-elevation distribution - with predicted weather and recorded flight data is robust in the sense that about 20,000 km of soaring flight in ridge lift with quite different sailplanes (best L/D from 25 to 56) and winds (from 20 to 45 KT) are adequately simulated with a fixed calibration of the terrain slope for each of the 25 regions involved. The average region size of 50 km indicates that each region was calibrated by 16 verifications on average. Further, long-distance soaring flights on the 17 June 2010 were properly simulated by the recalibrated GME-TOPTHERM forecasts for the USA mid-Atlantic states.

Concluding remarks

The assumption that vertically deflected atmospheric flow remains parallel to the sloping terrain seems to be useful for the prediction of regional flow slopes. Such sloping flow leads to ground-fixed updrafts which allow birds and sailplanes to soar along ridges aligned perpendicular to the wind. Glider flight data in combination with regional forecasts have been shown to be useful for the calibration of the effective terrain slope. As a result, the planning of soaring flights along the Appalachian mountains in ridge and thermal lift is useful – even with TOPTHERM coupled to GME, a global weather model with limited spatial resolution.

The flow slopes obtained from the regional ground speeds of gliders flying in aligned ridge lift depend on factors like the quality of the predicted wind profile below the ridge elevations and the pilot skill in the flights used. This implies that the flow slopes must be recalibrated if any of these factors change. Alternatively, pilots may work out their individual skill factor by simulating many of their flights with calibrated predictions.

The isotropy of the terrain slope obtained from the area-elevation distribution results in false alarms for predicted ridge lift in regions with long ridges when the wind is parallel to the ridges. The wind direction should be perpendicular or at least strongly oblique to the aligned ridges for the predicted ridge lift to be useful for long-distance soaring. The false alarms might be reduced by combining the wind direction with the dominant slope direction. The slope directions occurring in a forecast region 50 km wide could be analyzed with a high resolution digital elevation model. For the time being, pilots should check the TOPTHERM aligned lift predictions against their experience with favorable wind directions.

Simulations of wave flights with predicted regional slope updrafts were attempted. The flight level predicted in aligned ridge lift was increased by 2500 m above the predicted ridge lift altitude for wave flights to be simulated with vertically displaced ridge lift predictions (Figure 5). TopTask adjusts the flight polar to the density altitude when optimizing the flight speed. The wind used to calculate the crab angle and the ground speed, however, is still the wind predicted at ridge lift altitude. The differences between predicted and real ground speeds turned out to be larger than for flights made in combinations of isolated thermal and aligned slope lift. Improvements can be expected, if the winds predicted at the flight levels in wave were used in the simulations. In the long run, the rather basic concept of flow being parallel to the terrain slope might be combined with predicted profiles of the Scorer parameter in order to forecast climb rates and the flight levels with the highest aligned wave lift. Thus, planning of soaring flights in wave lift might become possible.

Acknowledgments

Edward Hindman, USA, initiated regional soaring forecasts for the USA (based initially on RAMS, finally on GME-TOPTHERM) in order to bring region-based flight planning with TopTask from Europe to the USA.

DWD supported the experimental GME-TOPTHERM runs and provides on-line pilot self-briefing with Java TopTask at www.flugwetter.de. In 2010 *DWD* made the GME-TOPTHERM predictions available through Java TopTask free-of-charge to USA glider pilots.

Ralf Thehos (DWD) set up the operational GME-TOPTHERM and COSMO-EU/TOPTHERM runs.

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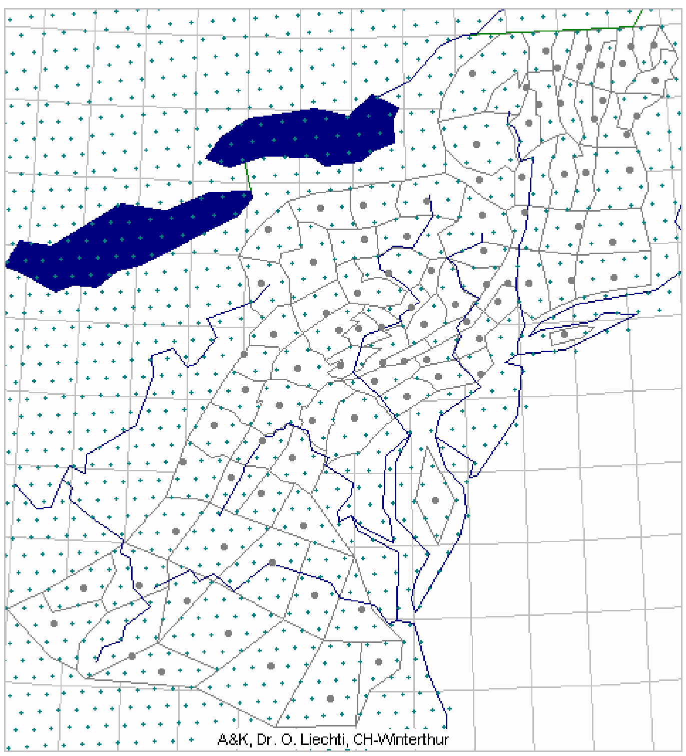

Figure 1 Forecast regions for the USA mid-Atlantic states and grid of the GME weather model. Circles indicate the grid points used for the regional forecasts.

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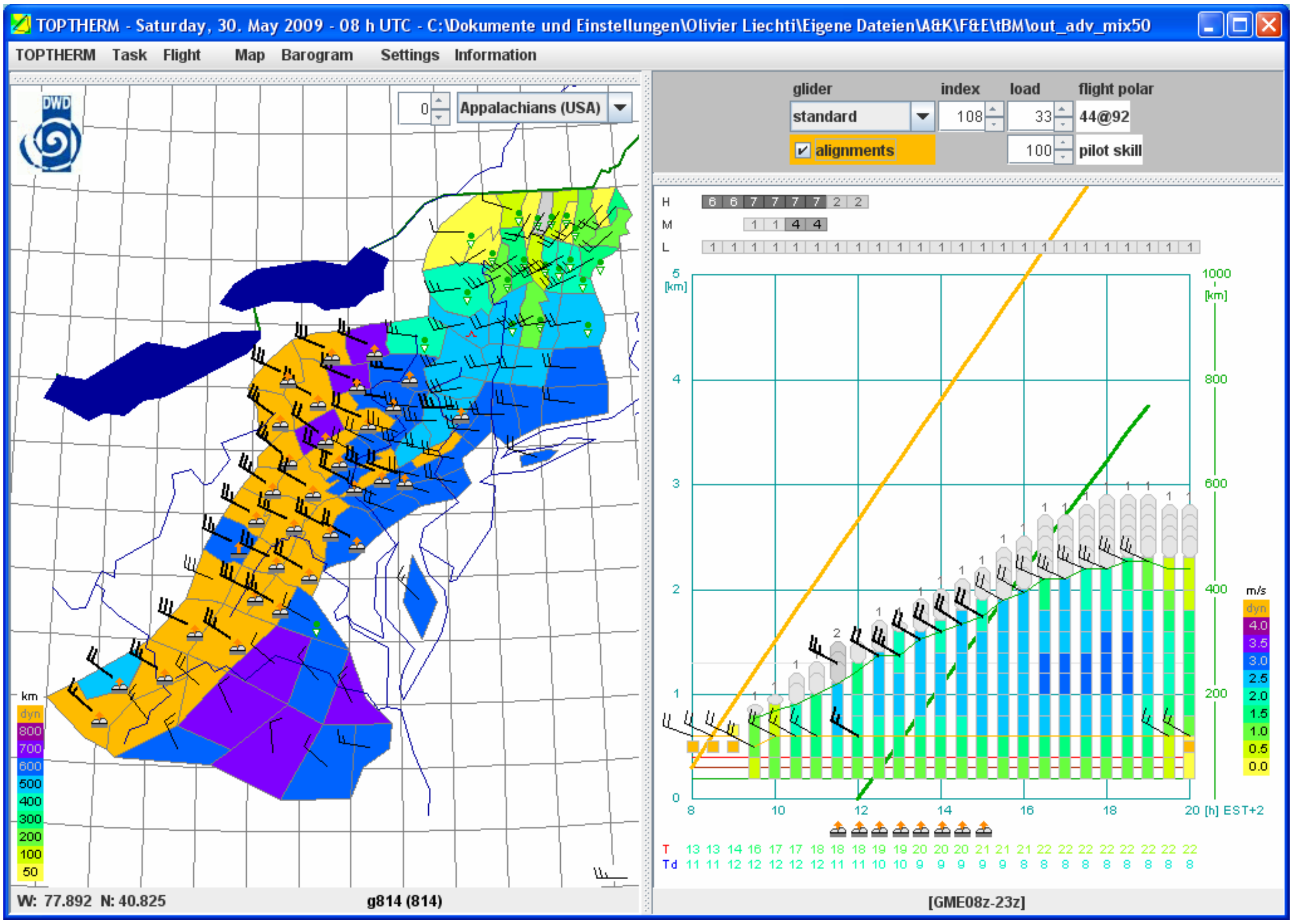

Figure 2 Browsing a TOPTHERM forecast on a map (left panel) and a barogram (right panel, left axis). The cursor position on the map interactively selects the region to be shown in the barogram. The cursor position on the barogram selects the time and altitude shown on the map. The two diagonal lines on the right panel represent the accumulation of the potential flight distance (right axis) in aligned ridge lift (upper line) and in thermal lift (lower line). The color on the map reflects the accumulated potential flight distance in each region.

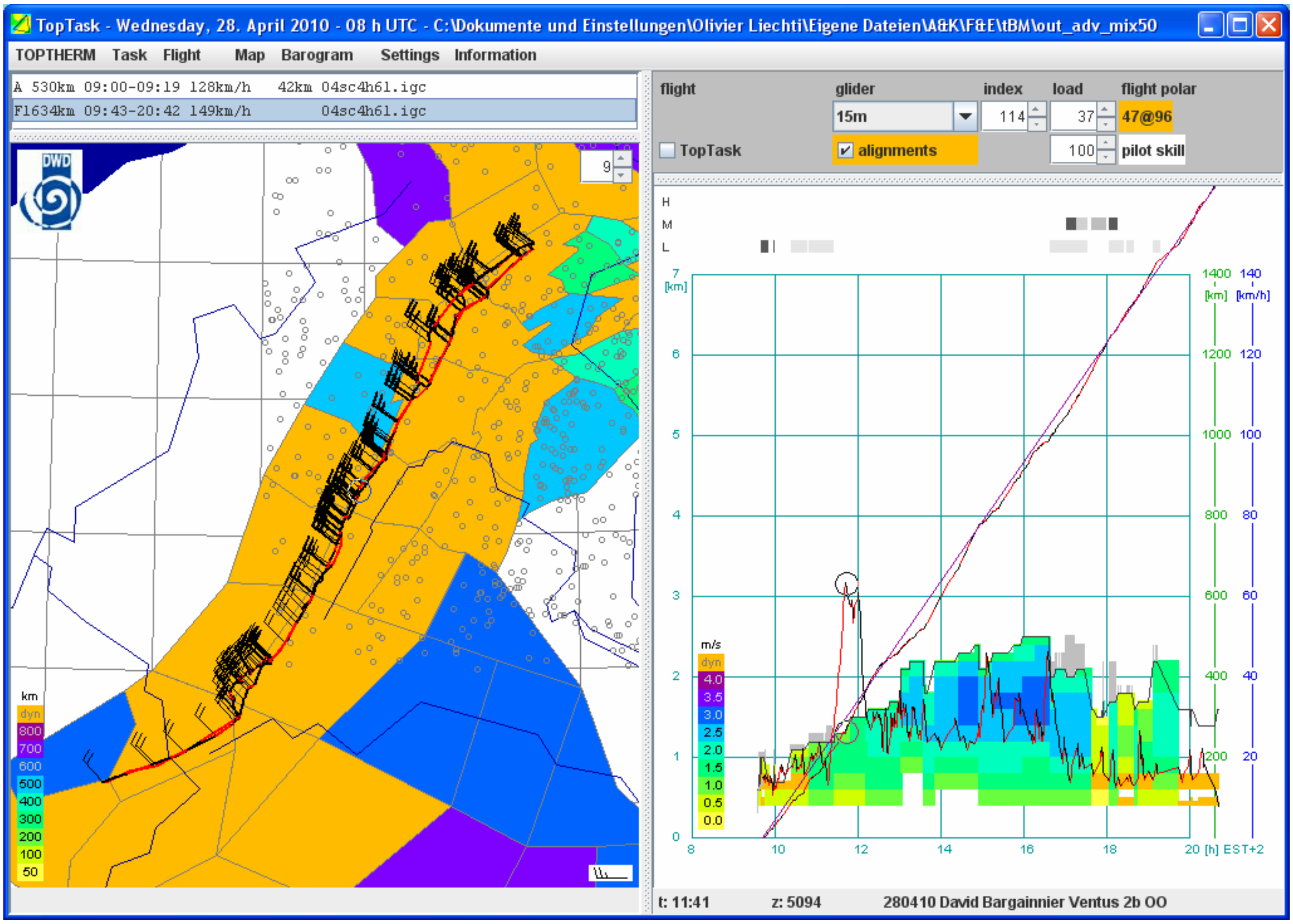

Figure 3 Recorded flight along the Appalachian Mountains (USA) displayed on map (left panel) and barogram (right panel) by Java TopTask. The cursor position on the barogram (circle) selects the time and the position shown on the map (circle). In the right panel the length of the flight trace (total 1634 km, right axis) is displayed as a function of time (horizontal axis). The slope of the diagonal curve reflects the ground speed (average ground speed 149 km/h) along the trace. The flight begins at ridge lift altitude, goes temporarily to wave, comes back to mixed ridge and thermal lift, and finishes at ridge lift altitude

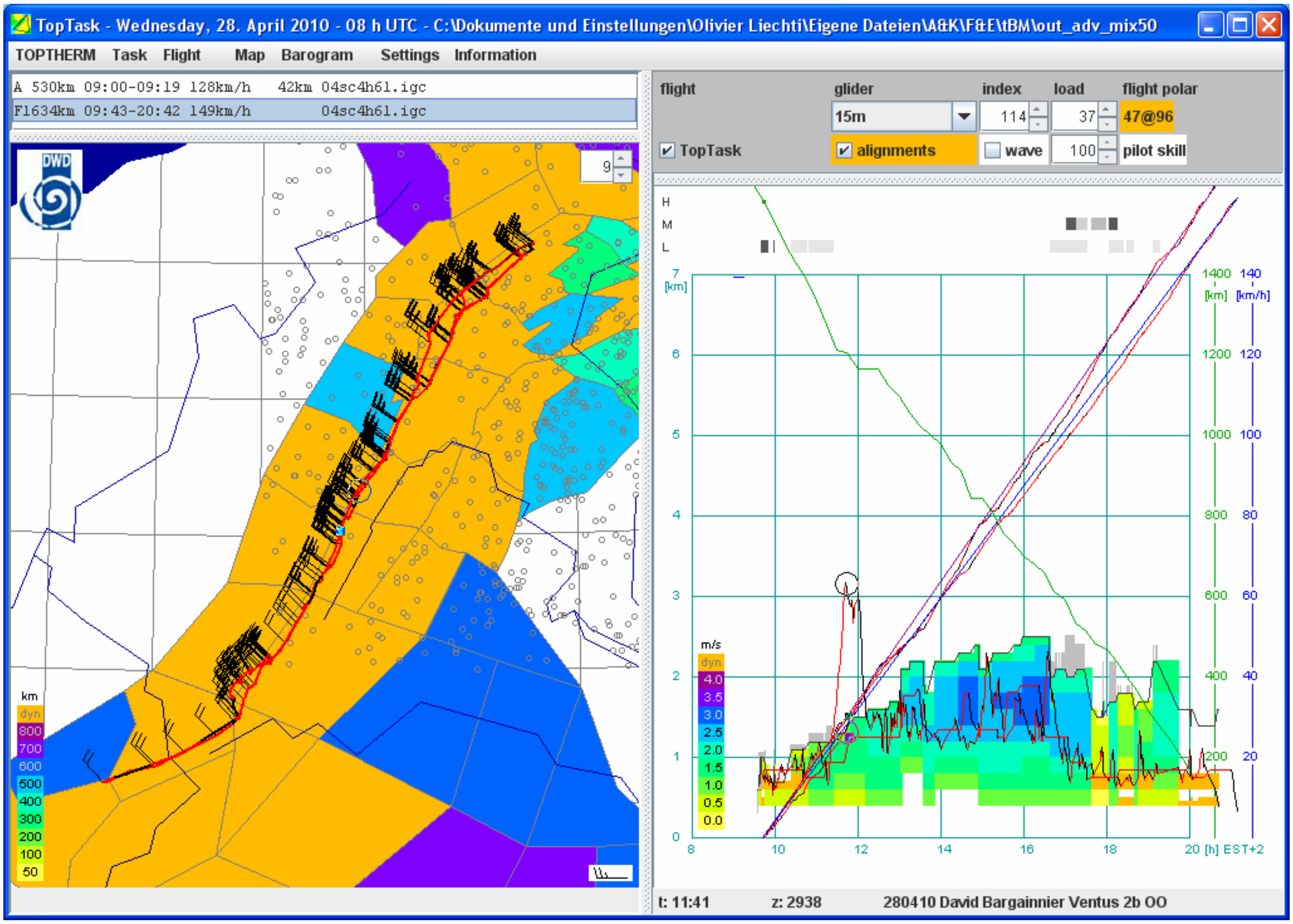

Figure 4 Recorded and simulated flight displayed on map (left panel) and barogram (right panel) by Java TopTask. The simulation of the recorded glider trajectory with isolated thermal and aligned ridge lift predictions and a 15m class flight polar is 6% slower than the ground speed of the real flight (149 km/h). The predicted flight level follows the recorded flight level at the exception of a temporary climb into wave (circle).

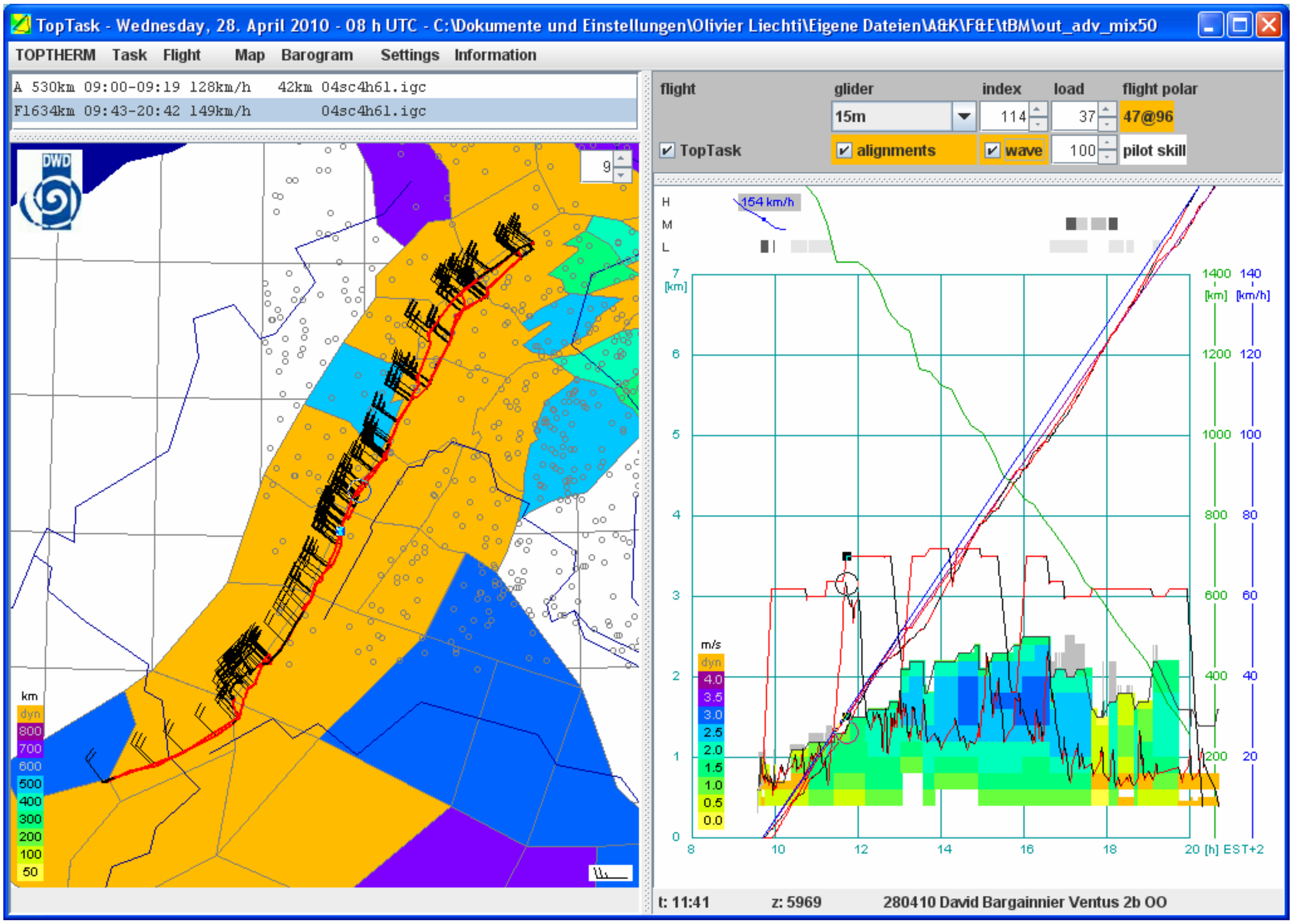

Figure 5 Recorded and simulated flight displayed by Java TopTask. The simulation of the recorded glider trajectory with predicted lift and wind, a 15m class flight polar, and the wave option selected (the flight in aligned lift is simulated at a flight level 2500 m above the predicted ridge lift altitude) is 3% faster than the ground speed of the real flight (149 km/h).

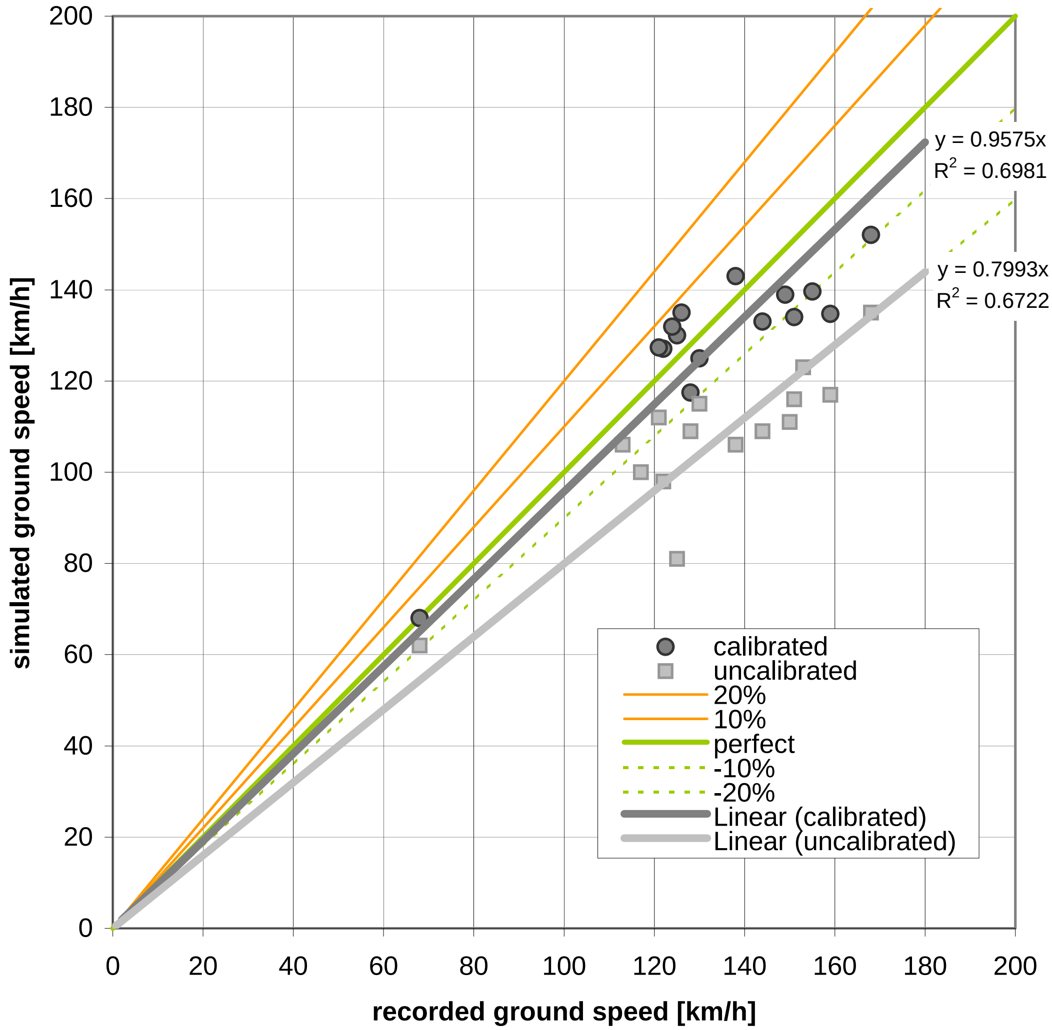

Figure 6 Calibration of regional terrain slopes with glider flight data and GME-TOPTHERM predictions. TopTask simulated ground speeds (left axis) of 15 long-distance soaring flights (average length 1348 km) in ridge and thermal lift along the Appalachian Mountains on 28 April 2010 (five flights) and 9 May 2010 (ten flights) versus recorded ground speeds (bottom axis). Perfectly simulated ground speeds would appear along the diagonal. After increasing the regional terrain slopes individually (calibration) the average error of the simulated ground speeds was reduced from –20% to –4% (slope deviation of linear regression line forced through the origin from the perfect simulation). Ground speed in aligned lift depends strongly on the aerodynamic performance of sailplanes

Appendix

The cumulative area-elevation distribution a(z) reflects the orographic structure of a region: in complex terrain elevations have a vertical distribution between the minimum and the maximum elevation of the region. As a result the interface between the ground and the atmosphere has a vertical distribution: physical processes like radiative transfer, deposition, evapo-transpiration, and mechanical turbulence occur at various elevations and terrain slopes. At altitudes below the maximum elevation the atmospheric volume fraction is reduced as the orographic volume fraction increases towards the lowest elevation. Buoyant mixing takes place in the available atmospheric volume fraction.

The cumulative area-elevation distribution a(z) of a region is obtained from a digital elevation model z(x,y) and the polygon p defining the region. All grid points of the elevation model located within the polygon p are selected (Figure 1 shows region polygons and the grid of a *weather* model) and sorted according to their elevation z. If the polygon p contains n grid points of the digital elevation model, the selected elevations z are numbered as z_i according to decreasing elevation:

$$z_{max} = z_1 > z_2 > z_3 > .... z_{n-1} > z_n = z_{min}$$

with $0 < i < n+1$.

Each elevation z_i represents a surface element of size δa which depends on the spatial resolution of the digital elevation model. The area a_{region} of the polygon p is approximately

$$a_{region} = n * \delta a$$

The cumulative area-elevation distribution a(z) is illustrated by plotting z_i against the accumulated area $a_i = i * \delta a$. The mathematical way would be to plot z on the horizontal axis and a on the vertical axis. The terrace slope of the cumulative area-elevation distribution a(z) is seen when elevation is plotted on the vertical axis and the accumulated area on the horizontal axis. Finally, the vertical elevation axis z is divided into equally spaced slots (or bins) of width Δz according to the required vertical resolution and the areas Δa_i . of the corresponding terraces are obtained. $\Delta z/\Delta a_i$ is the terrace slope and varies with elevation.

An alternative procedure would be to select the elevation model grid points located within the polygon p and to count the elevations falling into each elevation bin of width Δz. The number n_i of grid points in each bin represents the terrace surface $\Delta a_i = n_i * \delta a$. By accumulating the numbers n_i in each bin the cumulative area-elevation distribution a(z) is also obtained.