Range Maximization by Saw-Tooth Mode Optimization for Motor Gliders with Retractable Engines

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Abstract

The range maximization of a motor glider with a retractable engine is treated as a periodic optimal control problem. The periodicity of the optimal range flight, known as saw-tooth mode, is due to cycles that consist of alternating climbing phases with the engine extended and gliding phases with the engine retracted. It is shown that the maximum range of the optimal saw-tooth flight mode is considerably larger than the greatest range achievable with the best steady-state cruise. The time histories of the state and control variables are determined as well as the optimal altitude interval of the maximum-range saw-tooth flight mode. Further, results on the achievable range are presented for saw-tooth cycles with altitude intervals that differ from the optimal value. An efficient optimization method is applied to solve the periodic optimal control problem, using realistic models of the motor-glider dynamics and of the procedure for extending and retracting the engine.

Nomenclature

C_D = drag coefficient
C_L = lift coefficient
D = drag
g = acceleration due to gravity
h = altitude
J = performance criterion
L = lift
m = mass
m_f = fuel mass
n_ρ = exponent describing power dependence on attitude
P = engine power
S = reference area
s = range
T = thrust
t = time
V = speed
α = angle of attack
γ = flight path angle
δ_P = power setting (fraction of max power)
η_P = propulsive efficiency factor
ρ = atmospheric density
σ_P = specific fuel consumption
Φ_{eng} = engine inclination angle
Dot over variable: derivative with respect to time, e.g. $\dot{V}$

Introduction

The maximum-range cruise of aircraft is usually treated as a steady-state flight at constant speed and/or altitude with a basically straight trajectory. The control and state variables are chosen such that the fuel consumption per range is minimized (e.g. Ref. 1). It has been shown, however, that this type of cruise is not generally optimal, and results have been presented for optimal aircraft flight that is of periodic nature.[1-10] This finding also holds for maximum-endurance flight.[11,12] The trajectory of such maximized range or endurance flights basically consists of periodically repeated cycles, of which each comprises of a climb followed by a descent, with the thrust alternately operated at high and low or zero throttle settings.

A reason for the superiority of periodic optimal range flight over steady-state cruise is due to the difference in the best operating condition of the engine in terms of fuel efficiency as opposed to that for the aerodynamic configuration concerning a favorable drag-to-lift ratio. There are comparable relationships with motor gliders employing a retractable engine. The difference here relates to the vehicle configurations with the engine extended and retracted.[13] When the engine is extended, it can produce thrust, while the corresponding configuration is of high drag. This is due to the extended support frame and the attached engine and propeller, yielding additional drag of considerable magnitude. By contrast, the configuration with the engine retracted has comparatively low drag. Thus, the two engine positions – extended and retracted – show a great difference in terms of the lift-to-drag ratios.

The described performance differences can be used for a periodic flight mode consisting of alternating phases with the engine extended or retracted. This periodic flight of motor gliders with retractable engines is known as saw-tooth mode. Using the saw-tooth mode, it is possible to increase considerably the achievable range compared to the best steady-state flight where the engine is extended permanently (e.g., Refs. 13 - 16). Though the saw-tooth mode is known and utilized in practical motor glider flights, detailed investigations on an elaborate optimization of this flight mode are not available.

The objective of this paper is to present a detailed optimization treatment on the maximum-range possible with the saw-tooth flight mode for a motor glider that has a retractable engine. For this purpose, realistic models are applied that describe the dynamics of a vehicle representative for modern motor gliders as well as the procedure of extending and retracting the engine. Using an efficient optimization procedure, it was possible to generate solutions of the maximum-range problem.

Modelling of motor glider dynamics

For determining the maximum-range saw-tooth trajectory, a mathematical model based on point-mass dynamics can be used. Applying the standard conventions as in Fig. 1, this may be formulated as:

$$\begin{aligned}\dot{V} &= \frac{T-D}{m} - g\sin\gamma \\ \dot{\gamma} &= \frac{L}{mV} - \frac{g}{V}\cos\gamma \\ \dot{h} &= V\sin\gamma \\ \dot{s} &= V\cos\gamma\end{aligned} \tag{1}$$

The aerodynamic model is given by

$$\begin{aligned}D &= C_D(\rho/2)V^2 S \\ L &= C_L(\rho/2)V^2 S\end{aligned} \tag{2}$$

Realistic data are used for describing the drag-lift characteristics of a high-performance motor glider, with reference made to Ref. 17. An aerodynamic model of the drag-lift characteristics similar to that of the related vehicle has been constructed, yielding the drag polars of the vehicle with the engine retracted and extended. In Fig. 2, the drag polar of the configuration with the engine retracted is shown. Thus, the drag coefficient to be used in Eq. (2) for the configuration with the engine retracted is

$$C_D = C_{Dre}(C_L) \tag{3}$$

For the configuration with the engine extended, the drag polar is presented in Fig. 3

$$C_D = C_{Dex}(C_L) \tag{4}$$

The propulsion system consisting of the engine and the propeller is modelled as

$$T = \eta_P \frac{P_{\max}\delta_P}{V} \tag{5}$$

where the effects of the altitude range that is traversed in a saw-tooth mode cycle, can be described by [1]

$$P_{\max} = P_{\max,h=0}\left(\frac{\rho}{\rho_0}\right)^{n_\rho} \tag{6}$$

The model for the fuel consumption reads

$$\dot{m}_f = \dot{m}_{f0} + \sigma_P P \tag{7}$$

The model accounts for non-zero fuel consumption at idle, denoted by $\dot{m}_{f0}$.

The atmospheric model, which is used for describing the air density and the thrust dependence on altitude, agrees with the ICAO Standard Atmosphere.[18]

Modeling of engine extension and retraction procedure

The optimal saw-tooth flight for maximizing the range consists of repetitive cycles. A cycle, which is schematically depicted in Fig. 4, forms the basic constituent of the maximum range saw-tooth trajectory. It can be decomposed into four phases that are characteristic elements of each cycle:

1) Climbing flight with extended engine
2) Engine retraction phase
3) Gliding flight with retracted engine
4) Engine extension phase

The duration of the complete saw-tooth cycle, denoted by t_{cyc}, is subject of optimization. The times of retracting and extending the engine, Δt_{ex} and Δt_{re}, are supposed to be given. The durations of climbing and gliding flight phases, t_1 and $t_{cyc} - (t_1 + 2\Delta t_{con})$, are unknown and are subject of the saw-tooth mode optimization.

The phases of retraction and extension of the engine yield large changes in the drag-lift characteristics of the vehicle. Concerning the engine extension phase, the following modeling is applied for the drag change;

$$t_{cyc} - \Delta t_{ex} \le t \le t_{cyc}: \quad C_D = C_{Dre} + (C_{Dex} - C_{Dre})\sin\phi_{eng}(t) \tag{8}$$

where ϕ_{eng} is the inclination angle of the engine (Fig. 5). The model of Eq. (8) is based on the assumption that the effective drags of the engine, the propeller and the support frame can be related to their vertical projection.

Analogously, the change in the drag characteristic during the engine retraction phase is modeled as

$$t_1 \le t \le t_1 + \Delta t_{re}: \quad C_D = C_{Dre} + (C_{Dex} - C_{Dre}) \sin \phi_{eng}(t) \tag{9}$$

The engine inclination is operated in the following angular range

$$0 \le \phi_{eng} \le 90 \deg \tag{10}$$

It is assumed that the inclination angle changes at a constant rate, yielding

$$\dot{\phi}_{eng,ex} = -\dot{\phi}_{eng,re} = \text{const} \tag{11}$$

Thus, for the engine extending phase

$$t_{cyc} - \Delta t_{ex} \le t \le t_{cyc}: \quad \phi_{eng}(t) = \dot{\phi}_{eng,ex}[t - (t_{cyc} - \Delta t_{ex})] \tag{12}$$

and for the engine retracting phase

$$t_1 \le t \le t_1 + \Delta t_{re}: \quad \phi_{eng}(t) = \pi/2 + \dot{\phi}_{eng,re}(t - t_1) \tag{13}$$

Formulation of periodic optimal control problem

The optimal control problem is to determine the control and state variable time histories that yield the best trajectory in terms of the maximum range per fuel consumed. Because of the periodicity of the saw-tooth mode, it is sufficient to consider one cycle for optimizing the complete flight. Accordingly, the performance criterion can be formulated as

$$J = \left. \frac{s(t)}{m_f(t)} \right|_{t=t_{cyc}} \tag{14}$$

where t_{cyc} is the time of a saw-tooth cycle.

The mass of the vehicle is treated as constant during a saw-tooth cycle. This assumption is based on the fact that the mass reduction caused by the fuel consumption during a cycle is small when compared with the overall mass of the motor glider.

With reference to the periodicity of the optimal saw-tooth flight, the following periodic boundary conditions hold

$$\begin{aligned} V(0) &= V(t_{cyc}) \\ \gamma(0) &= \gamma(t_{cyc}) \\ h(0) &= h(t_{cyc}) \end{aligned} \tag{15}$$

The control variables are the lift coefficient C_L and the power setting δ_P, that are subject to the following inequality constraints

$$\begin{aligned} C_{L\min} \le C_L \le C_{L\max} \\ 0 \le \delta_P \le 1 \end{aligned} \tag{16}$$

Further constraints relate to state variables. Limits in the altitude and the speed are accounted for, yielding

$$\begin{aligned} h_{\min} \le h \\ V \le V_{\max} \end{aligned} \tag{17}$$

The optimal control problem can now be formulated to determine the controls, $C_L(t)$ and $\delta_P(t)$, the initial states, $V(0)$, $\gamma(0)$ and $h(0)$, the time describing the beginning of the propeller retracting phase, t_1, as well as the cycle time, t_{cyc}, that maximize the performance criterion according to Eq. (14). The described problem is subject to the dynamic system, Eq. (1), the boundary conditions, Eq. (15), and the inequality constraints, Eqs. (16) and (17).

For solving the periodic optimal control problem, an efficient numerical optimization method and computational technique was used.[19,20]

Results

An optimization result showing a characteristic feature of the maximum-range saw-tooth mode in terms of the altitude profile is presented in Fig. 6. There are four phases of the saw-tooth mode cycle, as described in a previous section. The climbing and gliding phases form the main parts. Shorter portions are the phases during which the engine is retracted and extended.

A primary result of the saw-tooth mode optimization is the maximum range that can be achieved. The maximum range in terms of distance travelled per fuel consumed amounts to:

$$\frac{s_{sawtooth,\max}}{m_f} = 56.92 \, \frac{\text{km}}{\text{kg}}$$

This is considerably larger than the greatest range attainable in a steady-state cruise with the engine permanently extended and operating. Denoting the greatest range in steady-state cruise as $s_{steady,\max}$, the improvement possible with the optimal saw-tooth flight mode can be expressed as

$$\frac{s_{sawtooth,\max}}{s_{steady,\max}} = 3.05$$

A further result of the saw-tooth mode optimization is that there is an optimal altitude interval associated with the maximum range cycle. According to the altitude time history shown in Fig. 6, the optimal altitude interval amounts to

$$\Delta h_{sawtooth,\,opt} = 948.4 \text{ m}$$

Another optimization result is the optimal cycle time, yielding

$$t_{cyc} = 1761.1\,\text{s}$$

A detailed presentation of the engine retraction phase is shown in Fig. 7. The optimized time for the engine retraction lasts from $t_1 = 335.1\,\text{s}$ until 350.1s. Figure 7 shows that the retraction phase is part of the optimized transition maneuver from the climb at maximum power to the glide with zero thrust. This transition maneuver during which the maximum altitude of the saw-tooth cycle is reached involves a push over flight maneuver. Thus, the speed of the climbing part is increased to the higher value of the gliding flight in an optimal manner.

Similarly, there is a transition maneuver during which the engine is extended, as shown in Fig. 8. The engine extension phase begins at 1746.1 s and lasts until 1761.1 s. Figure 8 shows how this phase relates to the complete transition maneuver. The transition maneuver during which the minimum altitude is reached involves a pull up flight maneuver yielding a change from the glide to the climb, again in an optimal manner.

The optimal time history of the speed is presented in Fig. 9. There are two different speed levels which are relating to the climbing and gliding flight phases. With reference to the altitude time history (Fig. 6), the speed is less during climb. This is a characteristic feature of the optimal saw-tooth mode. The different speed levels are pertaining to the aerodynamic characteristics of the propeller extended and retracted configurations as well as to the thrust and fuel consumption characteristics during climbing. This means that for the climbing phase, when the drag-to-lift ratio is comparatively large and excess thrust is required, the speed is comparatively low. The lift coefficient during gliding flight is the value that corresponds to the maximum lift-to-drag ratio. Thus, the maximum contribution to the total range can be achieved during gliding flight.

The optimized control inputs are graphically presented in Figs. 10 and 11, showing the time histories of the lift coefficient and the powers setting. Furthermore, the time history of the engine inclination angle is depicted.

The time history of the optimal lift coefficient presented in Fig. 10 shows that the lift coefficient is higher during the climbing than during the gliding phase. Furthermore, the lift coefficient is at a constant level in both phases. This suggests that the indicated air speeds show a similar behaviour.

The time history of the optimal engine power setting is presented in Fig. 11, yielding

$$0 \le t < t_1 : P = P_{\max}$$
$$t_1 \le t \le t_{cyc} : P = 0$$

The inclination angle also is depicted in Fig. 11, showing how it is associated with turning-off and turning-on the engine.

A summary of results is presented in Fig. 12 that shows the achievable range for altitude intervals differing from the optimal value. The altitude interval is referenced to the altitude at the end of the climb where the engine power is reduced to zero. The optimal altitude interval yields the minimum fuel consumption per range using the saw-tooth flight mode. For intervals that deviate from the optimum value, the fuel consumption penalties are minor, especially when increasing the altitude interval.

Conclusions

The range performance maximization is investigated for the saw-tooth flight mode of a motor glider with a retractable engine. An efficient optimization method is used to determine the optimal saw-tooth trajectory. Such a trajectory consists of periodically repeated flight cycles showing alternating climb and glide phases. As a basic result, it is shown that the optimal saw-tooth flight mode yields a maximum range that is significantly larger than the greatest range achievable in a steady-state cruise. The optimal saw-tooth flight cycle has several characteristic features. The climbing flight phase is conducted at a comparatively low speed with maximum power of the extended engine. The gliding phase where the engine is retracted shows a higher speed, with the lift coefficient corresponding to the maximum lift-to-drag ratio. The results yield also the optimal altitude interval of the maximum range saw-tooth flight mode. Furthermore, results are presented that show the sensitivity in the achievable range with regard to altitude intervals differing from the optimal value.

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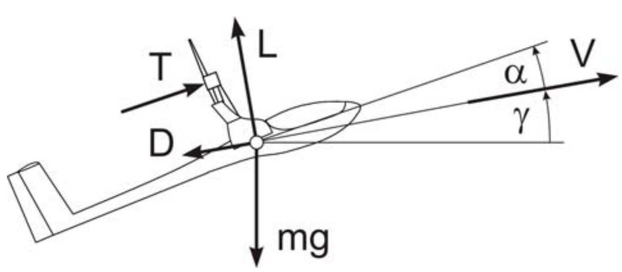

Figure 1 Forces acting on motor glider with extended engine.

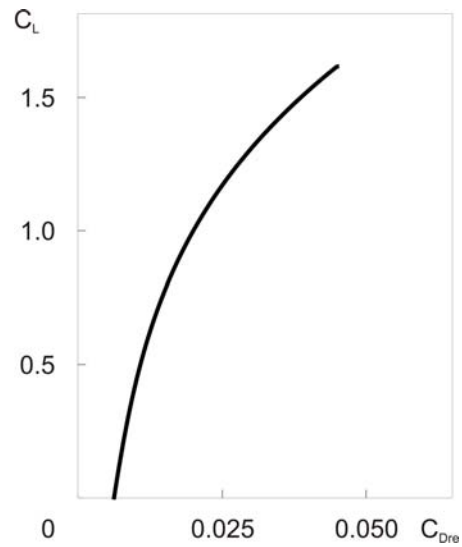

Figure 2 Drag polar of motor glider with retracted engine.

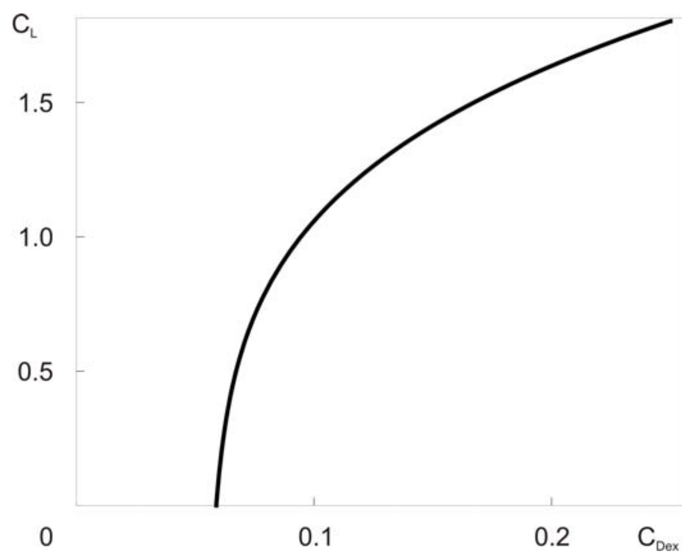

Figure 3 Drag polar of motor glider with extended engine.

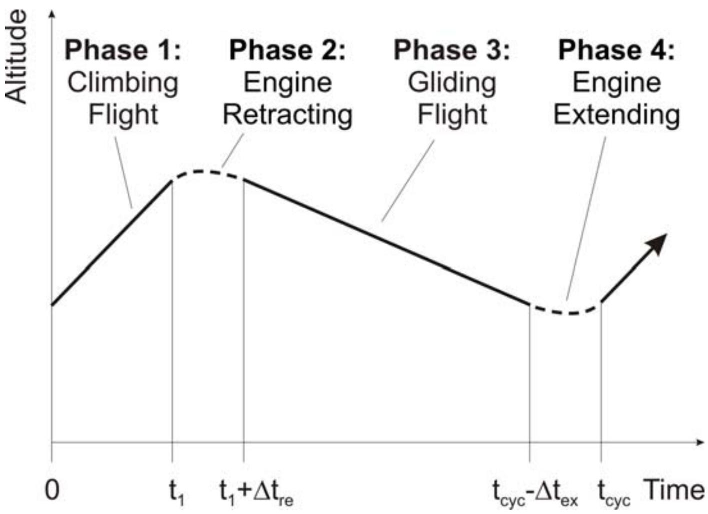

Figure 4 Structure of optimal saw-tooth flight mode for range maximization.

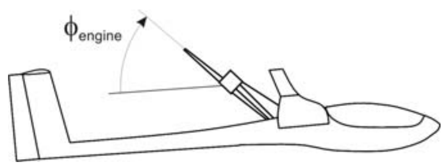

Figure 5 Inclination angle of engine.

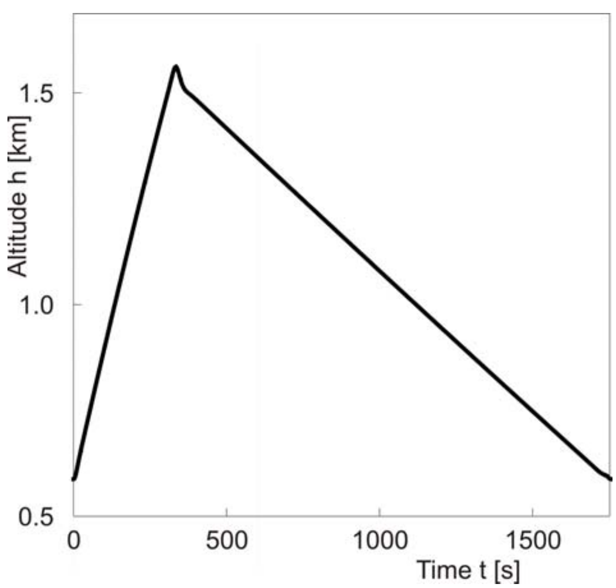

Figure 6 Optimal altitude time history.

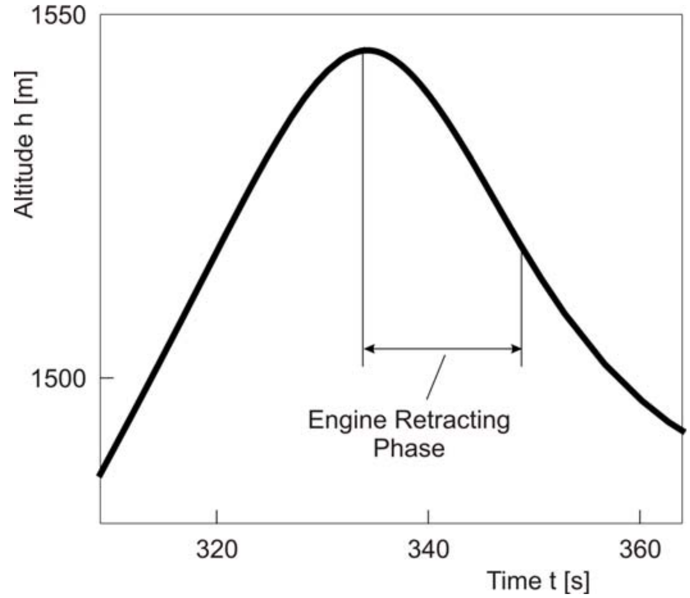

Figure 7 Optimal transition flight maneuver from climb to glide.

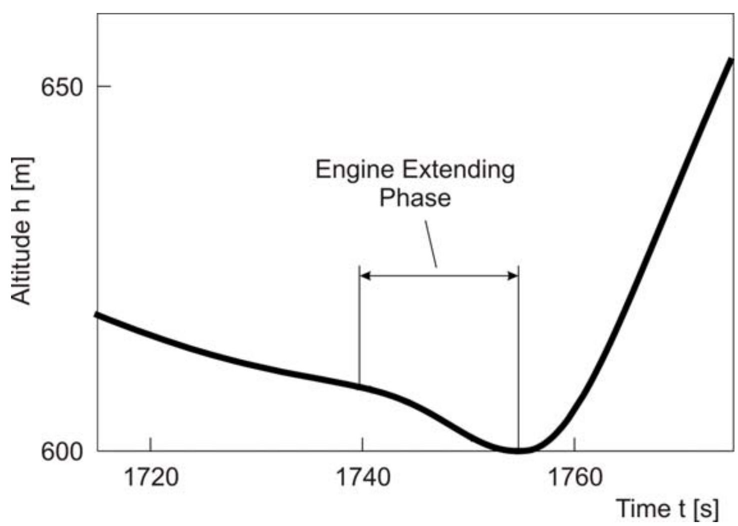

Figure 8 Optimal transition flight maneuver from glide to climb.

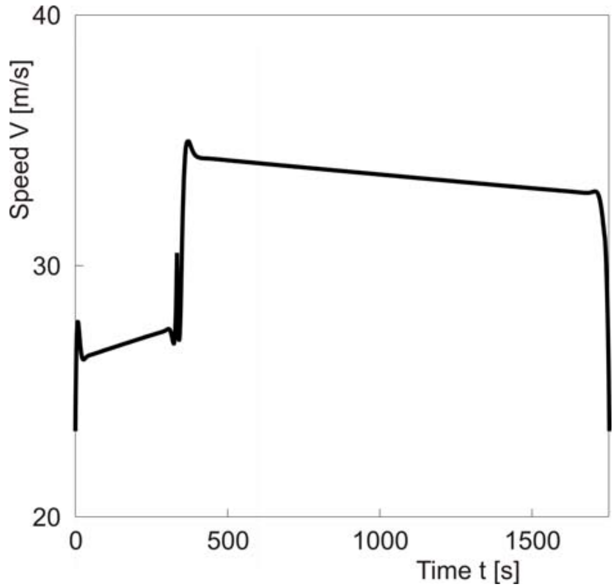

Figure 9 Optimal speed-time history.

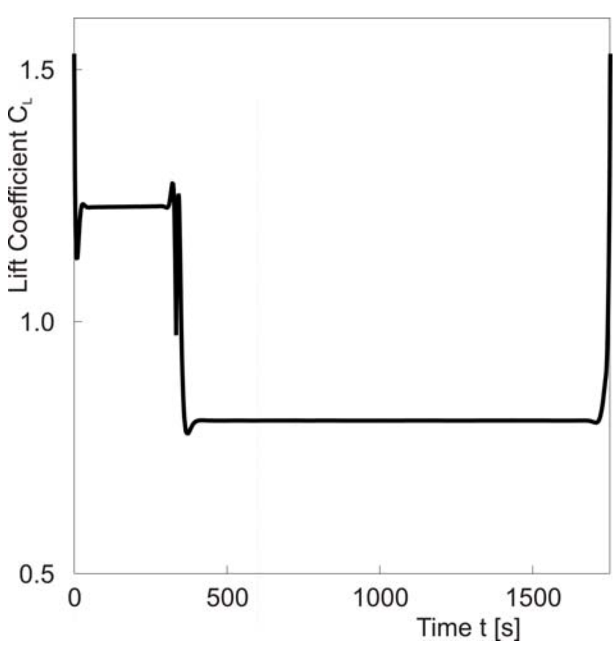

Figure 10 Optimal lift-coefficient time history.

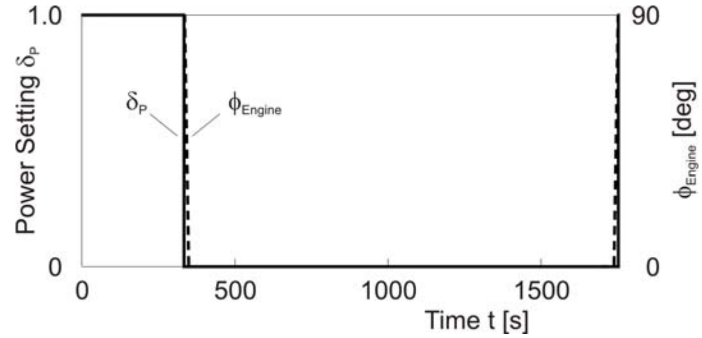

Figure 11 Optimal time histories of power setting and engine inclination angle.

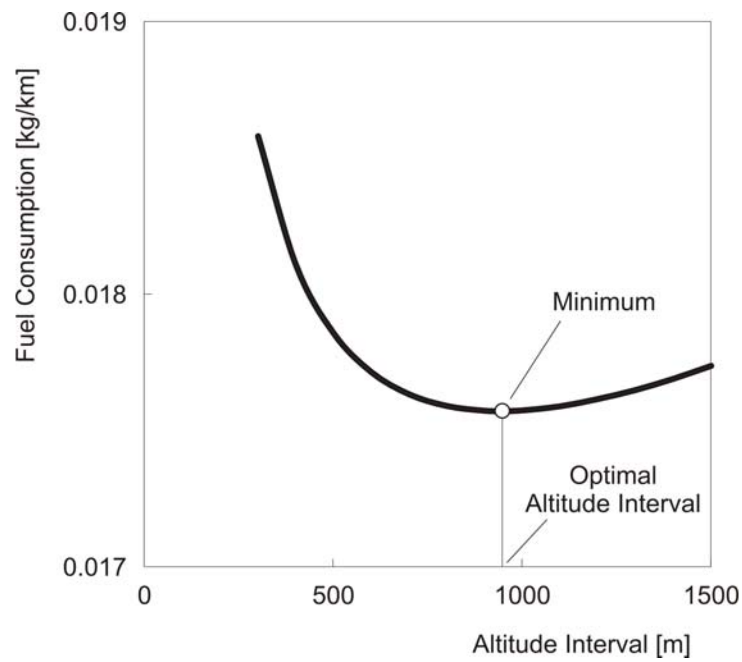

Figure 12 Effect of altitude interval of saw-tooth cycle on fuel consumption.