SPEED TO FLY WITH MANAGEMENT OF THE RISK OF LANDING OUT

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This paper describes a theory of glider cross-country soaring. The greatest contribution on this subject is the MacCready theory. But in modern soaring technique, many glider pilots choose to set a slower speed than derived from MacCready theory. That is to use a low Ring Setting. Flying at lower speed and higher L/D allows searching over a wider range to select better thermals, and to improve mean speed. Experience has shown that the selection of thermals to use or ignore is as important as the selection of inter thermal speed. In this paper, best speed to glide, weakest thermals to use and optimum cloud chasing rules are computed based on the meteorology of a given soaring day and a pilot selected level of risk of landing-out. The mathematics underlying the stochastic decisions are shown. A strategy to get the best score at competitions is presented.

NOMENCLATURE:

C	Rate of climb in given thermal
C_1	Strength of a minimum useful thermal
C_{max}	Estimated maximum strength of thermals for this day
$\overline{C}$	Mean strength of useful thermals
D_1	Marking distance of the pilot in a competition
D_m	Task distance
h	Present altitude
h_m	Minimum altitude to make a safe landing
h_t	Altitude of leaving thermal
$\triangle h$	Height of short climb
L	Average distance to hit next thermal
L_0	Average distance to hit the next thermal producing at least zero lift
L_g	Distance that the glider can glide h to hm
n_r	Risk parameter
p_{ol}	Probability of landing out at present glide
$\overline{p_{ol}}$	Probability of landing out before completing the task
$p_{ol}(D)$	Probability of landing out at distance D
P_c	Total score of competition
$\overline{P_c}$	Expected total score of competition
P_d	Distance points
P_v	Speed points
r	Average diameter of thermals
R_g	L/D when gliding between thermals
R_{best}	Best L/D of the glider
$\overline{V}$	Inter thermal speed
V_1	Mean speed of the pilot in a competition
V_{LD}	Speed for best L/D
V_m	Best speed in a competition day
$\overline{V}$	Mean speed for the task
α	Proportion of thermals with clouds
θ	Angle of course deviation
$\rho(c)$	Numerical density of thermals

*Strength, spacing and diameters of thermals are assumed to have normal distribution.

1 INTRODUCTION

How can we speed up the cross country flight? The first theoretical answer was given by Paul MacCready [1]. This great theory is basic to all task flying glider techniques.

The second contribution was by Helmut Reichmann [2]. He considered the finite distance of glide, and found that the thermal strength is more dominant than inter thermal speed for determining the mean speed of cross-country. He mentioned stochastic thinking - the idea of applying statistical theory in competition flying. He recommended using slower inter thermal speeds than the MacCready ring.

Mitsuru Marui enlarged on these studies [3]. He studied many techniques of gliding: e.g. the dolphin, the speed setting, the cloud chase and so on. He carefully evaluated their effectiveness and distinguished between them with a value rating (whether available or unavailable). His helpful studies significantly contributed to the improved performance of Japanese pilots.

In spite of these evolutions of soaring theory, some pilots are still not using these new ideas. They sometimes persist
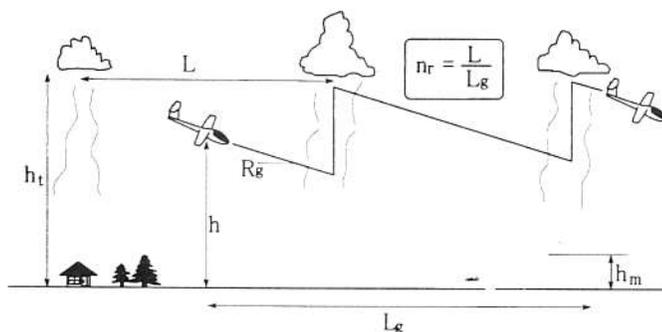
Figure 1: Model of Cross-Country

in flying on course without chasing clouds, or they may chase all clouds and wander in the sky, or they may fly too fast between thermals and land out, or they may use all thermals regardless of their strength and slow down the cruising speed, or they may concentrate their attention on the speed director while neglecting looking out, etc. The purposes of this study are to help those unfortunate pilots to realize their real ability, to help them in their decision making and to contribute to the advance of the art of soaring by answering these questions: "Should I use or should I not use this thermal? How fast should I fly between thermals? and which cloud should I go for?"

In this article, using mathematics and stochastic models of the spatial and strength distribution of thermals, concrete values of these flight techniques are discussed based on management of the risk of landing out.

2 MODELLING OF CROSS-COUNTRY FLIGHT WITH STOCHASTIC PROCESS

The model of glider cross-country flight developed here starts with a glider flying with glide ratio R_g and leaving a thermal at altitude h (see Fig.1). The distance L_g that the glider can fly is $R_g(h-h_m)$, here h_m is the minimum altitude to make a safe circuit and land.

Next, a random spatial distribution of thermals is assumed based on an arbitrary numerical density (Fig.2). Then, the sky is partitioned into squares of boundary length r, an assumed value representing the average size of thermals (Fig.3). A fraction of the squares now contain thermals. Assuming the density of thermals where the strength of thermals is LARGER than C is $\rho(c)$ (number/area), the probability that a square contains a thermal is $r^2\rho(c)$.

As the pilot flies along the course for a distance not longer than L_g, the number of squares the glider passes through is no greater than $\frac{L_g}{r}$. The probability p_{ol} that the glider will NOT hit any thermal that the strength is LARGER THAN C is

$$p_{ol} = (1 - r^2\rho_{(C)})^{\frac{L_g}{r}} \tag{1}$$

This expression can be reduced to

$$p_{ol} = e^{-\frac{L_g}{L}} \tag{2}$$

Here, L is defined as

$$L \equiv \frac{-r}{\log(1 - r^2\rho_{(C)})} \tag{3}$$

L is the average distance to glide until hitting the next useful thermal stronger than C. It corresponds to the average thermal spacing. L increases with C. Here, it should be remarked that $\rho(c)$ is affected by not only weather conditions but also on the skill of the pilot and the performance of the glider. On an actual cross country, the probability of hitting thermals can be increased by chasing clouds and using horizontal dolphin. Skilled pilots can find strong thermals and locate the centre, but a novice pilot will be apt to lose a thermal or turn a *4m/sec* thermal into *2m/sec* by poor centering.
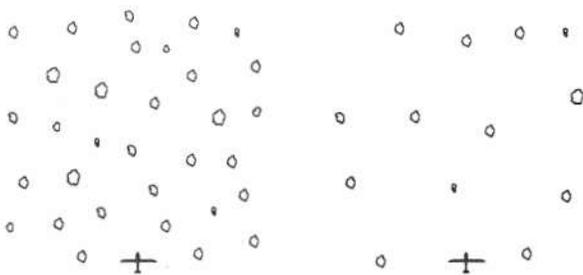
High density   Low density

Figure 2: Numerical Density of Thermal

To keep the risk of landing out low, the glide length of the glider L_g should be larger than the thermal spacing L. Here, the Risk Parameter n_r is introduced as

$$n_r = \frac{L}{L_g} \tag{4}$$

The probability of being forced to land out without finding any thermal, p_{ol} takes a value of $e^{-1/nr}$. The Risk Parameter nr is a function of the probability of landing out. For successful soaring, it should be less than 1. For example, the probability of landing out when leaving a thermal is 37% at $n_r = 1$, and 0.67% at $n_r = 0.2$. So, this parameter is proportional to the aggressiveness of the pilot. If the pilot is aggressive, he discards weak thermals to get high speed, the risk of landing out increases and the Risk Parameter takes a large value. On the other hand, if the pilot is conservative and uses almost all thermals, including the weak ones to avoid landing out, then the Risk Parameter takes a small value.

Cross-country soaring speed is improved by selecting only strong thermals. A compromise using weak thermals reduces the danger of landing out. Here, we need the value of the minimum thermal strength to use while keeping the risk of landing out to an arbitrary value of $e^{-1/nr}$ selected by the pilot. Substituting $L_g = R_g(h-h_m)$ and eq.(3) to (4), the relation of the present altitude and the minimum thermal strength C_1 is given from
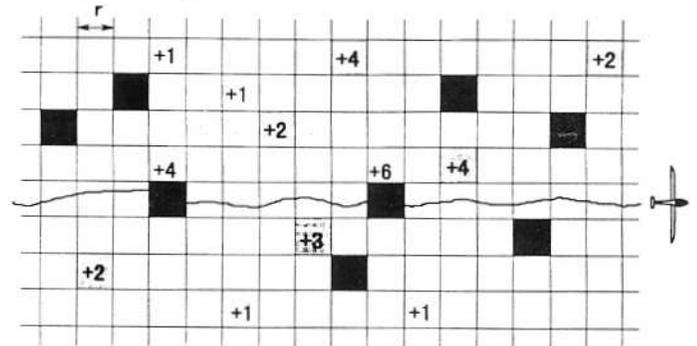
Figure 3: Model of Thermal distribution

following equation

$$e^{-\frac{r}{n_r R_g(h-h_m)}} = 1 - r^2\rho_{(C_1)} \tag{5}$$

This equation can be simplified using the approximation of the exponent function $e^x \simeq 1+x$, for small values of x as

$$\frac{1}{n_r r R_g(h - h_m)} = \rho_{(C_1)} \tag{6}$$

This equation will be used to obtain the strength of the minimum useful thermal by solving for C_1.

3 MINIMUM STRENGTH OF THERMAL

To solve the minimum strength of the thermal C_1, the thermal distribution $\rho(c)$ must be estimated. Here, a very simple model is used. If we wanted to deal with it exactly, we should refer to the study of meteorology. A prediction of the thermal distribution $\rho(c)$ in nature is beyond the scope of this study. (See [6],[7],[8],[9] and [10].)

The assumptions used here are 1) Thermal strength has normal distribution, 2) Truncated on the high end by an arbitrary value of C_{max} and 3) Thermal density $\rho(c)$ (inverse of thermal spacing) decreases uniformly with an increase of thermal strength (C).
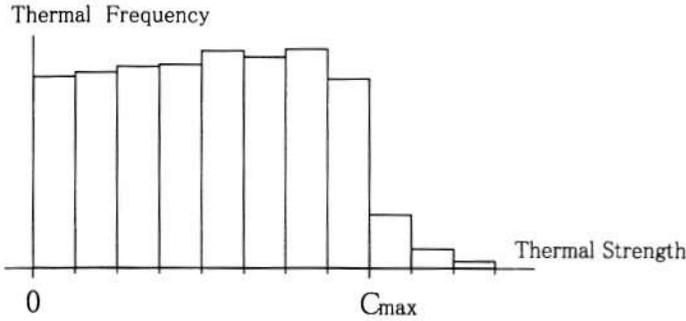
Figure 4: Histogram of Thermal Strength

Figure 5: Strength of Minimum Useful Thermal

An example of function $\rho(c)$ is given as

$$\rho_{(C)} = \rho_0\left(1 - \frac{C}{C_{max}}\right) \qquad (7)$$

Substituting equation (7) into equation (6), the minimum strength of the thermal C_1 can be obtained as a function of the altitude h

$$C_1 = C_{max}\left\{1 - \frac{L_0}{n_r R_g(h - h_m)}\right\} \qquad (8)$$

Here, L_0 is defined as

$$L_0 \equiv \frac{1}{r\rho_0} \qquad (9)$$

L_0 is the average spacing of thermals producing at least zero lift. This may be a few kms at a standard good cross country condition.

The result of setting $L_0 = 3km$, $h_m = 300m$, $Rg = 30$, $C_{max} = 4m/sec$, and $n_r = 0.3, 0.2$ and then solving for the minimum strength of the thermal C_1 is shown in Fig.5. (The reason for the values 0.3,0.2 of n_r shown is mentioned in a latter section.)

The pilot can make a decision according to this Figure. For example, if the glider is flying at 900m AGL and met a 1.5m/sec thermal, the pilot can discard the thermal, if the pilot was aggressive ($n_r = 0.3$), but the pilot should use the thermal, if the pilot was conservative ($n_r = 0.2$).

4 INTER THERMAL SPEED SETTING

In conventional theory, the inter-thermal speed setting was calculated from the MacCready ring. But in the modern view, slower speeds are recommended because they allow enough altitude to select strong thermals and to keep the risk of landing out low [3]. Many pilots agree that cross country mean speed is more affected by thermal strength than by inter thermal speed [2]. Serious performance penalties are inflicted when the pilot is forced to use weak thermals at low altitude due to having flown at high speed with low L/D's.

The mean speeds of cross country flight are shown on Fig.(6). If the top of the climb is h_t, the mean altitude of arrival at next useful thermal h_b is given as

$$h_b = h_t - \frac{L(\bar{C})}{R_g} \qquad (10)$$

Here, $L(\bar{C})$, is the mean length of glide until finding next thermal of strength $\bar{C}$ or more, can be calculated from equation (3) and using the approximation of the exponent function

$$L(\bar{C}) \simeq \frac{1}{r\rho_{(\bar{C})}} \qquad (11)$$

Figure 6:
Means Speeds

a:Top of thermal

b:Thermal spacing

c:Maximum lift for the day

d:Best L/D

e:Air mass vertical motion

f:Head/Tail wind
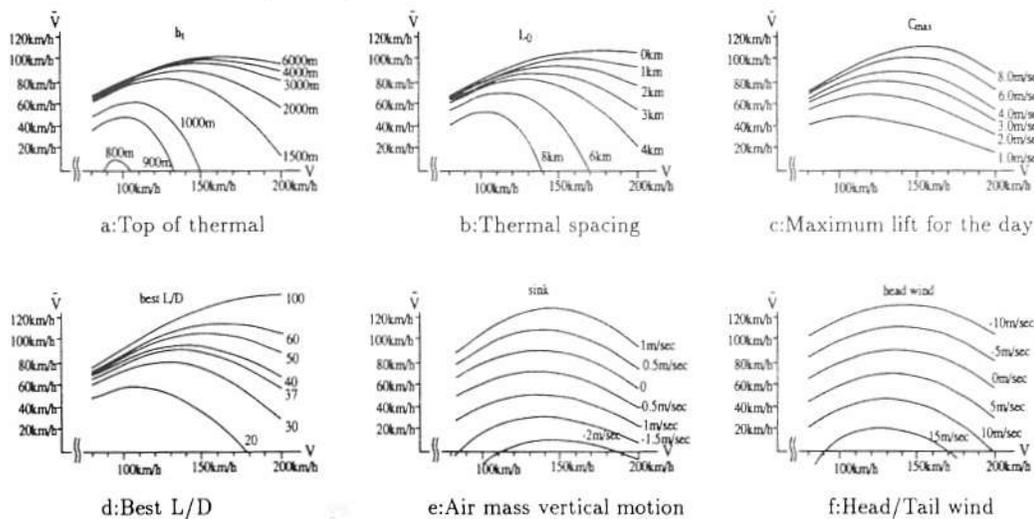
Except as noted,
Top of climb $h_t = 2000m$, Thermal spacing $L_0 = 3km$, Mximum lift $C_{max} = 4m/sec$
Risk $n_r = 0.2$, Best L/D = 37 at 95km/h, no air mass motion

Now, Using a rough assumption that the mean strength of thermals $\bar{C}$ coincide with the minimum strength C_1 at the altitude of arrival altitude h_b, $\bar{C}$ can be obtain from equation (8) by substituting h_b to h as

$$\bar{C} = C_{max}\left\{1 - \frac{L_0}{n_r R_g(h_b - h_m)}\right\} \qquad (12)$$

Using equation (10), (11) and (12), $\bar{C}$ can be found as

$$\bar{C} = C_{max}\left\{1 - \frac{(n_r + 1)L_0}{n_r R_g(h_t - h_m)}\right\} \qquad (13)$$

Next, the mean speed of cross country V is given by the inter thermal speed $\bar{V}$ and the mean strength of the thermals $\bar{C}$ as

$$\bar{V} = \frac{V}{1 + \frac{V}{R_g \bar{C}}} \qquad (14)$$

(For more exact calculation, a stochastic procedure should be used [4].) The glide ratio R_g is given as a function of speed V from the glider performance. For numerical analysis, an adequate model of glider performance is given as

$$\frac{2}{R_g} = \frac{V^2}{R_{best}V_{LD}^2} + \frac{V_{LD}^2}{R_{best}V^2} \qquad (15)$$

Here, R_{best} is the best L/D of the glider, and V_{LD} is the air speed that the glider can get the best L/D.

For the performance of a glider having best L/D R_{best} = 37 at V_{LD} = 95km/h, the calculated values of the mean speed $\bar{V}$ and the inter thermal speed V are shown on Fig.6, with various conditions of the top of the climb h_t (Fig.6.a), the thermal spacing L_0 (Fig.6.b), and the maximum strength C_{max} (Fig.6.c). According to this result, the best inter thermal speed setting can be found. For example, in the meteorologic condition of L_0 = 3km, C_{max} = 4m/sec and the thermal top h_t = 2000m, the best inter thermal speed is about 137km/h.

Note that a large setting of the Risk parameter n_r has the same effect as L_0 becoming small in equation(13). The mean speed can be higher with aggressive flight, but the risk of landing out will increase.

In Fig.6.d, the air speed for best L/D is 95km/h for all gliders. The performance significantly contributes to the mean speed.

Fig.6.e represents the effect of air mass vertical motion. The optimum speeds appear to contradict the classical rule of "dolphin" or "porpoising" in rising or falling air. Apparently, this fact seems inconsistent with MacCready theory. But, the deterioration of available thermal strength with head wind or sink causes a shift of the best inter thermal speed to slower speed, and cancels the shift of the polar curve due to the head wind or sink.
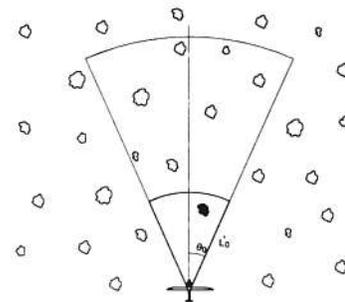
Figure 7: The Course Deviation

5 COURSE DEVIATION WITH CLOUD CHASE

Assume the glider flies under a "rule" that says to go to the nearest cloud such that the angles of course deviation are smaller than a maximum deviation angle θ_0 (Fig. 7). In contrast to flying straight ahead on course, the thermal spacing L_0 changes to a smaller value $\acute{L}_0$. From the definition of the spacing L_0, there should be one thermal in the sector with the radius $\acute{L}_0$ and the angle $2\theta_0$. Considering the finite size of the thermals and the existence of blue thermals, ρ_0, θ_0 and $\acute{L}_0$ should satisfy the following equation

$$\rho_0(\theta_0 \acute{L}_0^2 + \alpha r \acute{L}_0) = 1 \qquad (16)$$

where α is a compensation for the existence of blue thermals ($\alpha > 1$). This equation can be solved for $\acute{L}_0$.

$$\acute{L}_0 = \frac{2L_0}{R_L}\left(\sqrt{1 + R_L} - 1\right) \qquad (17)$$

Here,

$$R_L \equiv \frac{4L_0\theta_0}{\alpha r} \qquad (18)$$

The mean speed is improved by an equivalent shortening of the thermal spacing $\acute{L}_0$, and deteriorated by the course deviation. In the case of course deviation by the angle θ, the spacing will be lengthened by $1/\cos\theta$, then the mean speed will be deteriorated by $\cos\theta$.
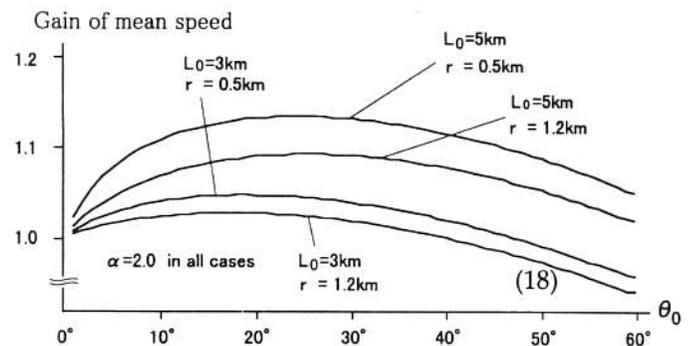
Figure 8: Ratio of Mean Speed Deviating to Mean Speed on Course

Hence, the mean speed is replaced with a equivalent mean speed $\acute{V}$ as

$$\acute{V} = \overline{\cos\theta}\bar{V} \qquad (19)$$

Assuming the clouds are distributed at random, the angles of off-track θ are distributed at random from $-\theta_0$ to θ_0. Then, the mean value of speed deterioration $\overline{\cos\theta}$ is calculated as

$$\overline{\cos\theta} = \frac{1}{2\theta_0}\int_{-\theta_0}^{\theta_0}\cos\theta d\theta = \frac{\sin\theta_0}{\theta_0} \qquad (20)$$

The best equivalent mean speed can be obtained substituting $\acute{L}_0$ into (13) and multiplying $\overline{\cos\theta}$ the mean speed calculated by (14). Fig. 8 shows the numeric result of a speed gain of cloud chase relative to no cloud chase. The best maximum deviation angle exist between $20\sim30deg.$ and becomes wider as the soaring condition become worse (see the case of $L_0 = 5km$). The performance improvement of cloud chase is about 10%. Generally speaking, too much deviation (exceeding $50deg\sim60deg$) is useless.

Although this approach is completely different from other studies, the result that the most profitable course deviation angle lies between $20\sim30deg$ is very similar to the results of Ref. [5], lending credence to this part of the study.
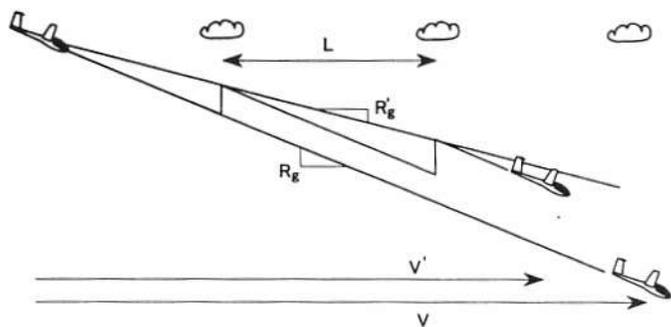
Figure 9: Utilization of Weak Thermal

6 UTILIZATION OF WEAK THERMALS

If the glider meets a weak thermal, it is a good idea to climb only a little bit [3].

Consider a case that the glider departs from a thermal, then meets a weak thermal (strength c) at a point of distance L, and climb a short time t (Fig. 9). In this case, the apparent glide ratio is improved, and the apparent glide speed is retarded. The apparent glide ratio is changed as

$$\acute{R}_g = \frac{R_g}{1 + \frac{tcR_g}{L}} \qquad (21)$$

And the apparent glide speed is changed as

$$\acute{V} = \frac{V}{1 + \frac{tV}{L}} \qquad (22)$$

Substituting $(21)\sim(22)$ to V, R_g to V, R_s of (13), (14), the mean speed is changed to

$$\acute{V} = \frac{V\left(1 + \frac{N_r L_0 R_g}{L_1}z\right)}{1 + \frac{(h_t - h_m)V}{L_1 C_{max}} + Az + \frac{N_r L_0 R_g}{L_1}\frac{V}{c}z^2} \qquad (23)$$

Here,

$$z \equiv \frac{tc}{L} \qquad (24)$$

and,

$$A \equiv \frac{V}{c} + \frac{R_g}{L_1}\{N_r L_0 - \frac{V}{C_{max}}(h_t - h_m)\} \qquad (25)$$

$$N_r \equiv 1/n_r + 1 \qquad (26)$$

$$L_1 \equiv R_g(h_t - h_m) - N_r L_0 \qquad (27)$$

To get the maximum effect of the "short climb", the value of the climb height tc that provides the best value of the mean speed $\acute{V}$ should be calculated. It can be given by a differential of $\acute{V}$ by z. The value of z that the differential equals zero is the solution. Hence,

$$\frac{d\acute{V}}{dz} = 0 \qquad (28)$$

This equation can be reduced to $\qquad$ (29)

$$z^2 + \frac{2L_1}{N_r L_0 R_g}z + \frac{L_1^2}{N_r^2 L_0^2 R_g^2} - \frac{c(h_t - h_m)^2}{C_{max}N_r^2 L_0^2} = 0$$

Since the climb is sufficiently small, an approximation $z \ll 1$ can be fair. So, the term z^2 can be neglected, and the solution is derived as

$$\frac{tc}{L} = z \simeq \frac{h_t - h_m}{2N_r L_0}\left(\frac{c}{\bar{C}} - \frac{\bar{C}}{C_{max}}\right) \qquad (30)$$

Here, the equation (13) was used.

The interval of weak thermals $L_{(c\sim\bar{C})}$ can be estimated with consideration of the average thermal spacing that is weaker than $\bar{C}$ and stronger than c as

$$L_{(c\sim\bar{C})} = \frac{1}{r\left(\rho_{(c)} - \rho_{(\bar{C})}\right)} = \frac{L_0 C_{max}}{\bar{C} - c} \qquad (31)$$

Finally, the best climb height $\Delta h \equiv tc = zL_{(c)}$ can be derived as

$$\Delta h \simeq \frac{C_{max}}{\bar{C} - c}\frac{h_t - h_m}{2N_r}\left(\frac{c}{\bar{C}} - \frac{\bar{C}}{C_{max}}\right) \qquad (32)$$

Maximum lift in the day $C_{max}(m/sec)$	Thermal spacing $L_0(km)$	Minimum thermal strength for short climb c (m/sec)
6	1	4.94
6	3	3.13
6	8	0.40
3	1	2.47
3	3	1.56
3	8	0.20

Except as noted,
Top of climb $h_t = 2000m$, Risk $n_r = 0.2$,
Best L/D = 37 at 95km/h, no air mass motion

Table 1: Minimum thermal strength for the short climb

Obviously, the value of $\triangle h$ is negative at the condition of too small of a value of $c < \frac{\bar{C}^2}{C_{max}}$. This shows that climbing in a thermal weaker than this is useless. The minimum value of the thermal worth utilizing is shown in Table 1.

Evaluating the performance advantage of using weak thermals by substituting (30) into (23) is mathematically awkward. A trial and error numerical computation produced the results shown in Table 2. This shows the most effective value of $\triangle h$ and improved mean speed $\acute{V}$ for each value of c. The best glide speeds are changed to V'_{glide}. The condition was $L_0 = 3km$, $n_r = 0.2$, $h_t = 2000m$, $C_{max} = 4m/sec$, $h_m = 300m$. In this case the mean used thermal strength C_1 is about 2.5m/sec. And a thermal weaker than 1.5m/sec is useless. So, the proper value of minimum strength of available thermal for "short climb" might be about 2m/sec. The profit of short climb is about 1%, and the best climb height $\triangle h$ is

Lift of short climb $c(m/sec)$	Mean speed $\acute{V}(km/h)$	Best speed for inter thermal glide V'_{glide}	Best height of short climb $\Delta h(m)$
1.5	90.16	136	0.0
1.7	90.23	138	10.4
1.8	90.48	140	24.6
2.0	91.34	144	55.8
2.2	92.55	147	90.7
2.5	94.78	152	151.2
3.0	98.07	154	231.6

Except as noted,
Top of climb $h_t = 2000m$, Thermal spacing $L_0 = 3km$,
Maximum lift $C_{max} = 4m/sec$, Risk $n_r = 0.2$,
Best L/D = 37 at 95km/h, no air mass motion

Table 2: Improvement of the mean speed by short climb

some tens of metres, which might be equal to one or two turns in the thermal.

The principle of utilization of weak thermals is to be able to improve thermal selection by an apparent improvement of the glide ratio. This result is consistent with the technique of top pilots. They say their barograph traces show short climbs, probably the result of a few turns in weak thermals (see [3]). They arrived at the best technique by experimentation.

7 SCORES AT THE WORLD CUP COMPETITION

The calculation of score P_c at the world cup championship is shown as

$$P_c = F(P_v + P_d) \tag{33}$$

Here, F is the day factor (usually 1), P_v is the speed point and P_d is the distance point.

The speed point P_v is calculated as

$$P_v = 2P_m R_n \left(\frac{V_1}{V_m} - \frac{2}{3} \right) \tag{34}$$

Here, P_m is the day factor (usually 1000). V_1 is the competitor's speed and V_m is the best speed of the day. R_n is the ratio of the number of competitors exceeding 2/3 of best speed and the number of competitors with a competition launch on the day.

The distance point P_d is

$$P_d = P_m \frac{D_1}{D_m} \left(1 - \frac{2}{3} R_n \right) \tag{35}$$

D_1 is the marking distance and D_m is the length of the task.

Considering the possibility of landing out, the expected value of the scores can be calculated. The probability of landing out at distance D_1, is expressed as $p_{ol}(D1)$. And the probability of landing out during a contest day is expressed as the integral of all of the probabilities of landing at all possible landing distances.

$$\overline{p_{ol}} = \int_0^{D_m} p_{ol}(D_1) dD_1 \tag{36}$$

Then, the expected score $\bar{P}_c$ can be calculated as

$$\bar{P}_c = (1 - \overline{p_{ol}})P_v + \int_0^{D_m} p_{ol}(D_1)P_d(D_1)dD_1 \tag{37}$$

For the simplicity, the weather condition is regarded as constant during the task. The probability of landing out $\overline{p_{ol}}$ and the speed V_1 are dominated by the Risk Parameter n_r.

What we want in this section is the best value of the Risk Parameter n_r that gives best expected score $\bar{P}_c$.

Regarding the risk of landing out is constant during the task, $\overline{p_{ol}}$ should be formed as

$$\overline{p_{ol}} = 1 - e^{-\frac{D_m}{D_0}} \qquad (38)$$

Here, D_0 is a constant. $p_{ol}(D_1)$ can be derived from (36) and (38)

$$p_{ol}(D_1) = \left. \frac{\partial}{\partial D_m} \overline{p_{ol}} \right|_{D_m = D_1} = \frac{1}{D_0} e^{-\frac{D_1}{D_0}} \qquad (39)$$

Assuming the glider makes its start from the top of a thermal $\overline{p_{ol}}$ should satisfy an equation based on the definition of the Risk parameter

$$\left. \overline{p_{ol}} \right|_{D_m = R_g(h_t - h_m)} = e^{-1/n_r} \qquad (40)$$

Then, the constant D_0 can be decided from (39), (40)

$$D_0 = \frac{-R_g(h_t - h_m)}{\log(1 - e^{-1/n_r})} \qquad (41)$$

Since e^{-1/n_r} is usually small, this formula can be reduced using the approximation $\log(1 + x) \simeq x$

$$D_0 \simeq R_g(h_t - h_m) e^{1/n_r} \qquad (42)$$

D_0 can be considered as an average attained distance without landing out.

Expanding (37) with (33)~(42), we can obtain the expected score as
$$(43)$$
$$\bar{P}_c = P_m \left[2R_n(\frac{V_1}{V_m} - \frac{2}{3})e^{-s} + \frac{1}{s}(1 - \frac{2}{3}R_n)(1 - e^{-s}) \right]$$

Here,

$$s \equiv \frac{D_m}{D_0} \qquad (44)$$
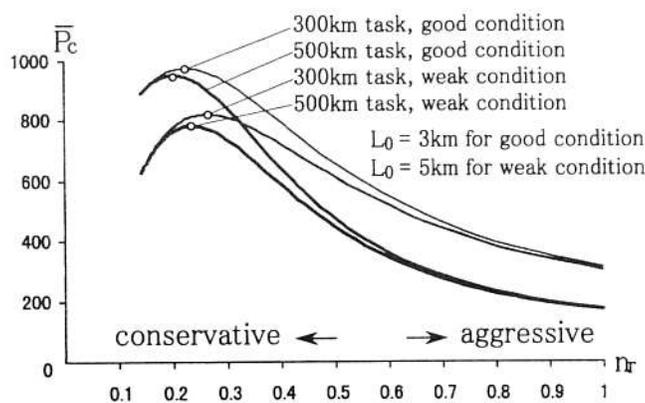
Figure 10: Expected Score and The Risk Parameter

Fig. 10 shows the result of the expected score $\bar{P}_c$ and the Risk Parameter n_r for each task distances, in the case of

$R_n = 0.7$, $V_m = 90km/h$ and substituting the best mean speed obtained by Fig. 6 to V_1 in (43). The best value of the Risk Parameter n_r becomes smaller as the task distance becomes longer, and as the soaring condition gets worse. It appears from this study that the best value of the Risk Parameter is around 0.25.

8 CONCLUSION

The big Factors of cross-country soaring
1. Minimum strength of thermal to use,
2. Speed setting of inter thermal glide,
3. Acceptable course deviation,
4. Utilization of weak thermals,
5. Setting of the risk, were solved using stochastic process.
Some results of this study are different from past theory, e.g., the best inter thermal speed is hardly affected by wind and sink. It is a good idea to "install" this theory in the "glide computer" to build a smart soaring aid.

It should be pointed out that the skill of the pilot is included in this theory since, by judicious course deviation and good centering he can affect the density and strength of thermals, the most important factor of cross-country soaring (the "compensation" for the pilot skill in the glide computer is feasible by introducing adequate parameter representing the skill).

Matters for further study include, meteorology, thermal spatial distribution in the sky and means of predicting thermal strength and distribution.

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