PARAMETRIC ANALYSIS OF THE SAILPLANE LONGITUDINAL DYNAMIC STABILITY

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SUMMARY

The linearized analytic model of general longitudinal movement of the sailplane is discussed. Linearized deviation equations of motion, derived in a stability co-ordinate system, are applied to the parametric analysis of the sailplane longitudinal dynamic stability for arbitrary symetric flight conditions. The analytic model is generated in the form of an interactive programming product PODYST. This is used to show the effect of some mass and geometric parameters on the short-period and phugoid motions following the disturbance from a selected reference flight condition.

SYMBOLS AND UNITS

a (1/rad) slope of sailplane lift curve a_{sd} (1/rad) slope of wing/fuselage lift curve a_h (1/rad) slope of horizontal tailplane lift curve b (m) wing span \bar{c} (m) mean aerodynamic chord of wing c_{D0} (1) sailplane zero-lift drag coefficient c_L (1) sailplane lift coefficient c_{m} (1) sailplane lift coefficient c_{a} (1) sailplane lift coefficient c_{a} (1) resultant aerodynamic force coefficient in z-axis direction e (1) coefficient of aerodynamic efficiency (Oswald's coefficient) I_{a} (m) reference altitude of flight I_{y} (kgm ²) mass moment of inertia to y-axis L_{h} (m) horizontal tailplane arm M (kg) sailplane mass S (m2) planform horizontal tailplane area V_k (m2) reference trimmed flight velocity \bar{x}_A (1) non-dimensional sailplane aerody	А	(1)	wing aspect ratio
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μ (1)non-dimensional sailplane mass ρ (kg/m³)air density	α	(°)	sailplane angle of attack
ρ (kg/m ³) air density	λ	(1)	characteristic equation root
	μ	(1)	non-dimensional sailplane mass
Θ (°) sailplane pitch angle	ρ	(kg/m ³)	air density
o () ourfaine break angle	Θ	(°)	sailplane pitch angle

INTRODUCTION

Stability and control belongs to the dominant problems of the flight characteristics of each aerodyne, including sailplane. Stability and controllability directly affect the safety of its operation. Therefore great attention to the analysis of both phenomena from very early phases of the sailplane design must be paid. This paper deals with the longitudinal dynamic stability of the sailplane. The longitudinal dynamic stability is determined by general motions around aircraft transversal axis, occurring after disturbance of the stabilised reference flight conditions. However, the good static longitudinal stability is important, but not the only condition for showing the appropriate dynamic longitudinal stability characteristics. After disturbance of initial flight conditions the return pitching moment must arise to regain these initial flight conditions. Great number of design parameters and flight conditions affect the longitudinal dynamic stability. The aim of presented analysis is to generally investigate the effect of the selected design parameters and flight conditions on the sailplane longitudinal dynamic stability characteristics.

ANALYTIC MODEL

For investigation of the sailplane longitudinal dynamic stability the modified analytic model was created. The modification is based on the application of general equations of motion, reduced to linearised deviation equations derived in the stability co-ordinates system. The influence of air compressibility on the aerodynamic stability derivatives had not been considered. The derivatives c_{mv} and c_{Ly} were neglected.

The basic characteristics, necessary to evaluate the longitudinal dynamic stability of the sailplane, are set up from the roots of characteristic system equation, derived on condition, that the stability determinant for non-trivial solution is zero.

$$\Delta = \begin{vmatrix} (2\mu\lambda - 2c_{L_R}ig\Theta_R - c_{x\bar{y}}) & -c_{x_a} & c_{L_R} \\ 2c_{L_R} & [(2\mu + c_{L_a})\lambda - c_{Z_a}] & -[(2\mu - c_{L_q})\lambda - c_{L_R}ig\Theta_R] \\ 0 & -(c_{m_a}\lambda + c_{m_a}) & (\overline{i_y}\lambda^2 - c_{m_q}\lambda) \end{vmatrix} = 0$$

By developing the stability determinant we obtain the necessary characteristic equation of 4-th degree

$$P_{4}\lambda^{4} + P_{3}\lambda^{3} + P_{2}\lambda^{2} + P_{1}\lambda + P_{0} = 0 ,$$

. where the coefficients of the characteristic equation are defined by the following expressions:

$$\begin{split} P_{0} &= -c_{L_{R}} [(2c_{L_{R}} tg \Theta_{R} + c_{x_{\bar{v}}}) tg \Theta_{R} + 2c_{L_{R}}] c_{m_{\alpha}} \\ P_{1} &= 2c_{L_{R}} (\mu c_{m_{\alpha}} tg \Theta_{R} - c_{m_{q}} c_{x_{\alpha}} - c_{L_{R}} c_{m_{\alpha}}) + \\ &+ (2c_{L_{R}} tg \Theta_{R} + c_{x_{\bar{v}}}) [(2\mu - c_{L_{q}}) c_{m_{\alpha}} - c_{m_{q}} c_{Z_{\alpha}} - c_{m_{\alpha}} c_{L_{R}} tg \Theta_{R}] \\ P_{2} &= 2\mu (c_{Z_{\alpha}} c_{m_{q}} + c_{m_{\alpha}} c_{L_{R}} tg \Theta_{R}) + (2c_{L_{R}} tg \Theta_{R} + c_{x_{\bar{v}}}) [c_{Z_{\alpha}} \overline{I_{y}} + (2\mu + c_{L_{\alpha}}) c_{m_{q}} + (2\mu - c_{L_{q}}) c_{m_{\alpha}}] + \\ &+ 2[c_{L_{R}} c_{x_{\alpha}} \overline{I_{y}} - \mu (2\mu - c_{L_{q}}) c_{m_{\alpha}}] \\ P_{3} &= -2\mu [c_{Z_{\alpha}} \overline{I_{y}} + (2\mu - c_{L_{q}}) c_{m_{\alpha}}] - (2\mu + c_{L_{\alpha}}) [2\mu c_{m_{q}} + (2c_{L_{R}} tg \Theta_{R} + c_{x_{\bar{v}}}) \overline{I_{y}}] \\ P_{4} &= 2\mu (2\mu + c_{L_{\alpha}}) \overline{I_{y}}. \end{split}$$

Both the input aerodynamic stability derivatives and other necessary data are given or calculated directly during the process. Derived analytic model may be applied for investigation of the stability in the straight stable gliding flight in vertical plane. This, so called "reference flight condition", stability of which is investigated, is defined by speed VR, or the lift coefficient C_{LR} respectively, and flight altitude H. The pitch angle of sailplane Θ_R , that is in time t=0 identical with the flight-path angle γ_R , is calculated automatically.

×.

APPLICATION AND RESULTS

The above-mentioned analytic model was used in the semi-interactive so~ware product PODYST. The analytic model is programmed in FORTRAN language for PC application. The original and modified version of two-seater training sailplane L-13 Blanik was chosen for this application of parametrical analysis of the longitudinal dynamic stability. The basic input parameters (geometrical, aerodynamic and mass-) are as follows:

S	= 17,35	(m ²)	S_h	= 2,65	(m2)	C _{DO}	= 0,025	(1)
\overline{c}	= 1,3	(m)	L_{h}	= 5,19	(m)	e	= 0,80	(1)
b	= 3,85	(m)	m	= 500	(kg)	\overline{x}_{cg}	= 0,23	(1)
a _{wb}	= 5,07	(l/rad)	I_v	= 1236	(kgm2)			
$\overline{x}_{A, wb}$	= 0,216	(1)	ah	= 3,86	(l/rad)			

The basic parameters of gliding flight are: $V_R = 90 \text{ (km/h)}$, or $C_{LR} = 0.735 \text{ (1)}$ respectively,

$$H = 0 (m),$$

$$\rho = 1,225 \, (\text{kg/m})$$

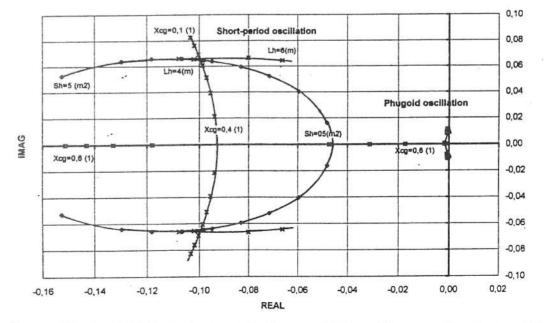
The reference lift coefficient C_{LR} and the flight altitude H (the variable parameters of the flight conditions) were gradually changed. As for the mass parameters, the mass of the sailplane "m" and the relative position of the sailplane centre of gravity x_{cg} were varied. As for the geometrical parameters the change of the horizontal tailplane area S_h and the arm between horizontal tailplane aerodynamic centre position and wing-fuselage combination aerodynamic centre position L_h were investigated. The effect of the change of the sailplane moment of inertia, depending on the horizontal tail arm, was simplified to the influence of the total fuselage length upon this moment.

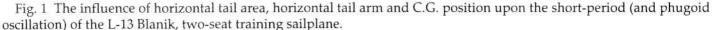
The influence of the above mentioned parameters upon the sailplane longitudinal dynamic stability is considered according to the course of the root hodograph. The hodograph illustrates positions of generally complex roots (λ =h+ik); their real parts corresponding to the damping and the imaginary parts to the frequency, depending on the varied parameter. All other parameters are constant and correspond to the basic input parameters, mentioned above. The results of parametric analysis are presented in following figures.

Fig. I expresses the influence of geometrical and mass parameters upon the short periodic oscillation with a relatively short period (about several seconds). From the course of roots it is obvious, that position of the centre of gravity remarkably affects the short period oscillations, mainly the oscillation frequency, whereas the size of horizontal tail area affects both frequency and damping. The diagram further shows, that the rearwards c. of g. position leads to disintegration of the complex root into two real roots, positioned on the horizontal axis of the hodograph. The horizontal tail arm affects the damping, but this effect is less significant than the one of horizontal tail area. To compare, note the influence of the same parameters upon the phugoid oscillations, typical for their long periods lasting dozens of seconds (up to 1-2 minutes), on the Figure 1.

We cannot compare these two types of oscillations on the single graphical scale. This is the reason, why we present them separately. In Fig. 2 the influence of the same parameters upon the phugoid oscillation is shown. The size of horizontal tail area has small effect on the frequency of oscillation whereas the influence upon the damping is more noticeable. The c. of g. position affects both the damping and frequency, but not significantly.

Further couple of figures shows the influence of the flight conditions and sailplane mass upon both components of the oscillation after the disturbance. It is obvious (Fig. 3) that the tested parameters have almost no influence on the frequency of shortperiod oscillations.





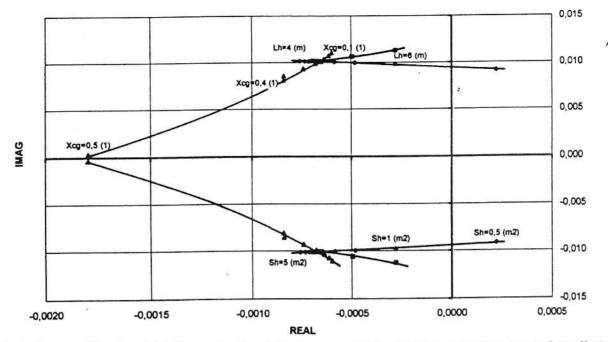


Fig.2 The influence of horizontal tail area, horizontal tail arm and C.G. position upon the phugoid oscillation of the L-13 Blanik, two-seat training sailplane.

The damping of short-period oscillations is changing rapidly and it drops with the increasing flight altitude and sailplane mass. The lift coefficient (flight speed) does not influence neither the damping nor the frequency of short-period oscillations. The influence of the same parameters upon the phugoid oscillations (Fig.4) has similar character when the influence of sailplane mass and flight altitude is considered, but the influence of lift coefficient (flight speed) upon the frequency and damping is more remarkable in comparison with the short period oscillations case. Both frequency and damping of phugoid oscillations drop with increasing airspeed.

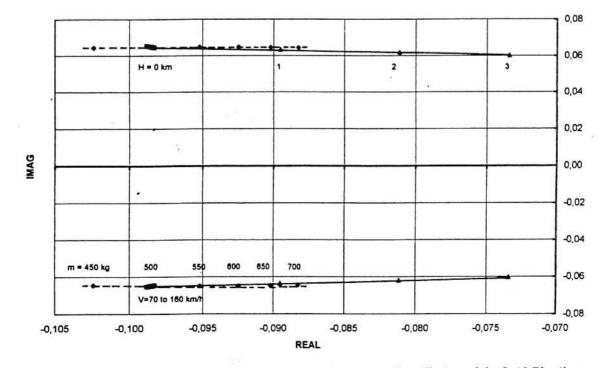


Fig. 3 The influence of mass, altitude and airspeed upon the short-period oscillation of the L-13 Blanik, two-seat training sailplane.

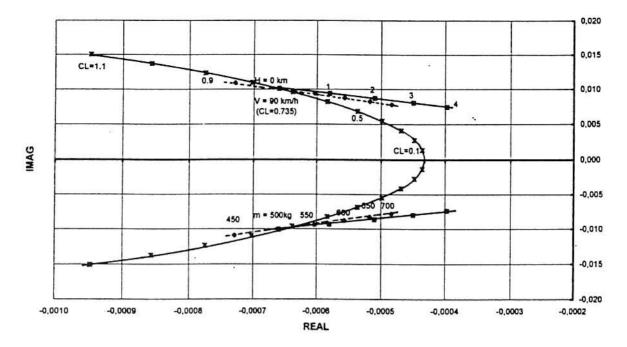


Fig.4 The influence of mass, altitude and lift coefficient (airspeed) upon the phugoid oscillation of the L-13 Blanik twoseat training sailplane.

CONCLUSIONS

The presented programme product PODYST enables to generally investigate the influence of some selected design parameters of the sailplane and the flight conditions on the longitudinal dynamic stability by the application of the root hodograph. Considering the longitudinal dynamic stability, more detailed and acceptable characteristics can be calculated for some special selected cases. Among these characteristics belong e.g. time, necessary for damping of short-period and phugoid oscillations amplitude, frequency, periods, number of oscillations, logarithmic decrement of damping, etc. The presented programme is now in its prototype phase. We are prepared to gradually precise the programme and transfer it to the user's friendly form.

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