TURBULENCE SCALE PARAMETER AND SPECTRUM IDENTIFICATION

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Presented at the XXVI OSTIV Congress, Bayreate, 1999

SUMMARY

Four sets of atmospheric turbulence flight records have been analyzed using NAPAM natural parameter based assessment and analysis methods. Pre-processing operations included calculation of a suitable runnung mean function. Values of the standard deviation σ, of the integral scale L and of Taylor's scale λ were calculated from the definition formulae and checked by the spectral density functions. Shape parameters for the generalized Karman spectrum, too, were calculated and checked. Experimental calculations indicate the existence of a discrete frequency amplitude spectrum function and research is proceeding to verify (or to disprove) this assumption.

Notation

n	wave number	[1/m]
q	number of data points	
r(n)	amplitude spectrum	$[m^2/s]$
w	vertical component of the atmospheric turbulence	[m/s]
A	peak coefficient	
C	constant	
G(n)	(direct) spectral density function	$[m^3/s^2]$
H	wavelength	[m]
L	integrat csale	[m]
R(ς)	autocovariance function	$[m^2/s^2]$
S(n)	(autocovariation) spectral density function	$[m^3/s^2]$
α	exponent	
β	frequency limit ratio	
λ	Taylor's scale	[m]
σ	standard deviation	[m/s]
ς	space displacement	[m]
Δ	difference, error	
Ω	spatial frequency	[rad/m]
Subscript		
m	measured (finite frequency band)	
max	maximum, limit	
r	from the record	
v	from the amplitude spectrum	
w	vertical component of the atmospheric turbulence	
G	from the direct spectral density function	
S	from the autocovariance spectral density function	
O	theoretical value; $0 \le n \le \infty$	

INTRODUCTION

Turbulence analysis is one of the most important topics in aeronautical research. Theoretical and experimental investigations are equally important and must complement and assist each other. Additionally, a correct and complete analysis of the measured air/fluid flow records is required for atmospheric turbulence modeling as well as for increasing the performance of sailplanes or another type of aircraft.

A key issue in this repect is the exact determination of the natural parameters σ, L λ and complemented by the calculation of the spectral density function shape parameters α, A and β. Not being fully satisfied with the possibilities offered by the traditional statistical based stochastic record assessment and analysis methods (see e.g.: Bendat and Piersol (1,2)). A research program has been started at the TU Budapest a few years ago. From time to time the author has given account of improvements in particulars of the record assessment and analysis procedures (Gedeon (3-5)).

Recent investigations have added a few new and unexpected perceptions (or rather guesses?) to previous work (Gedeon (6,7)). So it seems the time has come to discuss these new problems with both branches of turbulence experts; with boundary layer flow specialists as well as with researchers working in the domain of atmospheric turbulence.

Basic laws and parameters of turbulence are independent of size effects and of Reynolds numbers. For this reason the difference of several orders in dimensions and in Reynolds numbers between boundary layers and free atmosphere is more assisting than hindering the success of common research projects.

1. Basic Principles of the Natural Parameter Method

Turbulence research - While making extensive use of statistical and probability calculation procedures turbulence research is based on the solution of the Navier-Stokes equations. This theoretical work cannot progress without systematic verification. Neither can the engineer do without reliable experimental data to base the dynamical analysis and stressing on. A correct and efficient analysis of stochastic measurement records is the instrument for this work.

Traditional stochastic record assessment begins with the calculation of the standard deviation σ. In the theory of turbulence there is also a second parameter, the so-called scale of turbulence or rather there are two of them. The first one is the **integral scale**

$$L = \lim_{\varsigma_1 \to \infty} \left| \frac{1}{\sigma^2} \int_0^{\varsigma_1} R(\varsigma) d\varsigma \right| \tag{1}$$

Another one of the candidates for this role is **Taylor's scale**

$$\lambda = \frac{\sqrt{2}\sigma}{\left[-\left(\frac{d^2 R(\varsigma)}{d\varsigma^2} \right)_{\varsigma=0} \right]^{1/2}} \tag{2}$$

Both scale parameters were worked out for heat transfer calculation and play simultaneously the part of a statistical index-number for wave length. Their relation to each other is not quite clear as yet. Taylor's scale has been considered to be the minor parameter, hence her French name "micro-echelle" which also indicates the belief it is associated with the upper frequency limit of the spectrum. As will be seen, our personal experience does not support this belief.

Professor Kovasznay (8) starting from the Wiener-Hintsin relationships proved that the zero value of the spectral density function reads:

$$G(\Omega)_{\Omega=0} = G(0) = \frac{2}{\pi} L\sigma^2 \qquad (3a)$$

In his interpretation this implies the integral scale L to be more than a special turbulence characteristic being a natural parameter for all stationary stochastic processes, equivalent with and complementary to the standard deviation σ. Our concept started from this thesis with the difference that we previously worked with the spectral density as function of the wave number n. Our zero formula reads therefore:

$$G(n)_{n=0} = G(0) = 4L\sigma^2 \qquad (3b)$$

Being based at the start on the Kovasznay theorem, our stochastic analysis method was given the name Natural Parameter Method, or for short the NAPAM method.

Practical calculation of the integral scale L using Eq. (l) proved to be difficult or even impossible in some cases because the numerical integration gave diverging oscillations. The problem could be traced back to the problem of a correct mean value calculation before the correlation process (Gedeon (6)). Pre-processing of the flight records includes therefore, calculation of a suitable running mean function as shown schematically on Fig. 1.

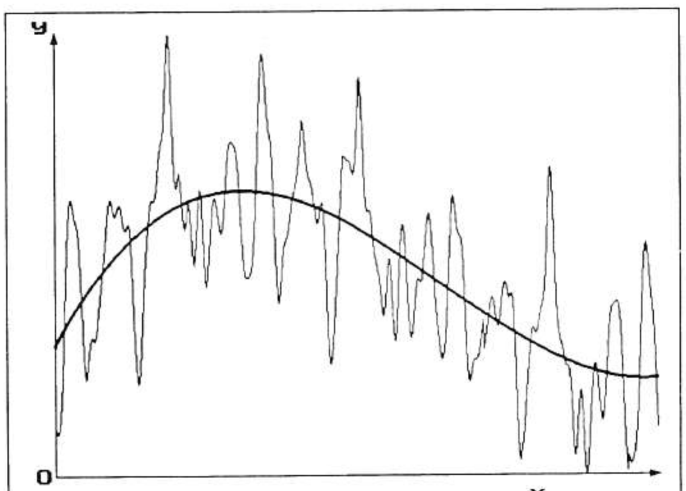

Fig. 1: Calculation of the running mean function

Presumably induced by the aforementioned extrapolation problems it is customary to run the numerical integration in (l) only up to the first zero crossing of the autocovariance function and accept this value as correct. After introduction of the running mean calculations this expectation proved to be wrong. Running the extrapolation process as pictured on Fig. 2 previously give end values 2.5 to 3 times less than the first peak on the integration curve.

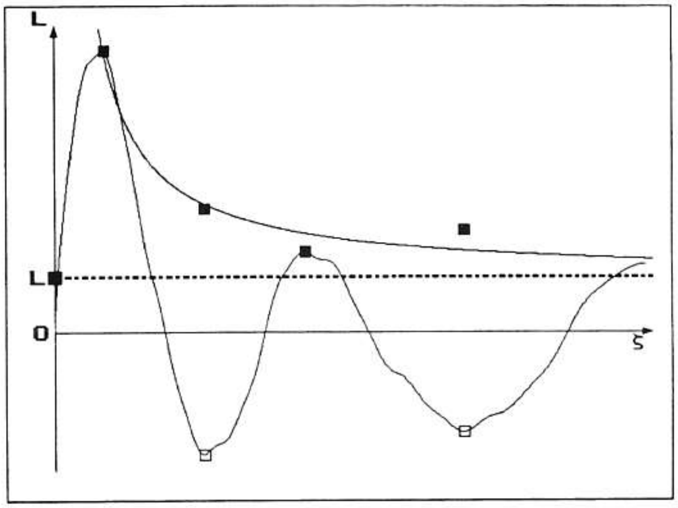

Fig. 2: Determination of the integral scale L

Calculation of Taylor's scale λ starts with a smoothing of the beginning of the autocovariance function curve (Fig. 3). Duplicate differentiation of the smoothing curve equation then provides the value of the second derivative needed for substitution in (2). The geometrical form corresponding to this formula is a vertical parabola. Intersection of this parabola with the horizontal axle gives the value of λ.

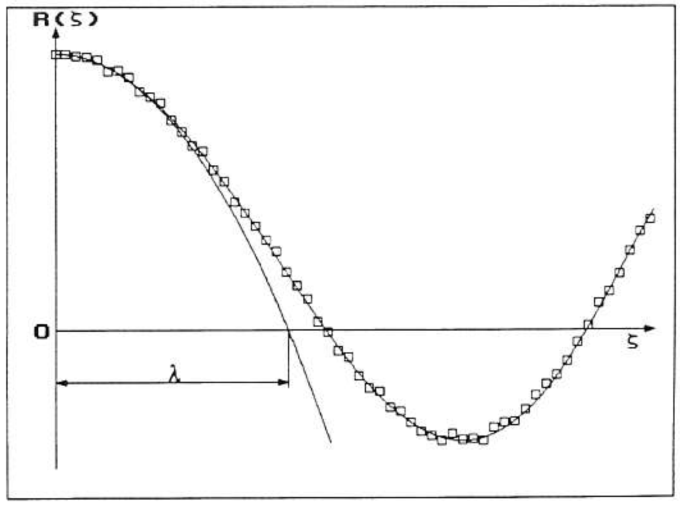

Fig. 3: Calculation of Taylor's scale λ

Statistical and correlation analysis is to be complemented with the calculation of spectra. As reported already earlier (Gedeon (4)) direct PSD spectra keeping the respective phase angle values as well are preferred, (but this time both), correlation spectra as well as direct ones, were calculated. This duality was advisable for checking the values of the natural parameters and of the spectrum shape parameters.

For smoothing of turbulence spectra we prefer to use the original Karman spectrum (Karman (9)):

$$S_w(\Omega) = \sigma_w^2 \frac{L}{\pi} \frac{1+\frac{8}{3}(1.339\Omega L)^2}{\left[1+(1.339\Omega L)^2\right]^{11/6}} \qquad (4)$$

in a generalized form:

$$G_w(n) = 4L\sigma_w^2 \frac{1 + A(CLn)^2}{\left[1 + (CLn)^2 / (1 - \beta Ln)\right]^{\alpha}} \tag{5}$$

In (5) the exponent α, the peak coefficient A and the frequency limit ratio

$$\beta = \frac{1}{Ln_{\max}} \tag{6}$$

are to be determined by smoothing of the raw spectrum. The constant C is function of α, A and β, its numerical value being determined by the postulate

$$\int_0^{\infty} G_w(n)dn = \sigma_0^2 \tag{7a}$$

respective by

$$\int_{-\infty}^{\infty} S_w(n)dn = \sigma_0^2 \tag{7b}$$

Shape parameters in Eq. (5) are the exponent a, the peak coefficient A and the frequency limit ratio, β, respectively. A correct smoothing of the raw PSD spectra using Eq. (5) is expected to give values for α and A close to the theoretical ones in (4).

The spectral density functions calculated nominally by Fourier transform of the autocovariance function are supposed - at least implicitly - to be continuous. Excessive random fluctuations found permanently in the sequence of the calculated spectrum points are hardly compatible with this. Investigation in this line has provided the following results so far.

As a matter of fact, the course of a continuous and stationary stochastic function can be approximated within acceptable error margins by several different harmonic series. Practical adoption of PSD spectra including input-output calculations seems to give satisfactory results. Nevertheless, research on the possibility of a discrete amplitude spectrum and (if its existence should be proved) on its fine structure promises to give substantial benefits.

It is easy to prove that Fourier series expansion is giving correct results if (and only if) the period is exactly known. Otherwise the integration giving the Fourier coefficients is not extended over whole periods resulting in nonzero values even for non-existing components. But this investigation also showed a possibility to develop a procedure for finding the "true" Fourier components. First preliminary results were reported recently by Gedeon (7).

It is probable that atmospheric turbulence records can be expanded to discrete amplitude harmonic series too (Fig. 4). These harmonic series have a peculiar frequency sequence, linear altogether but not following the sequence of whole numbers. Research is on going and more records of measurements are needed to make the details clear.

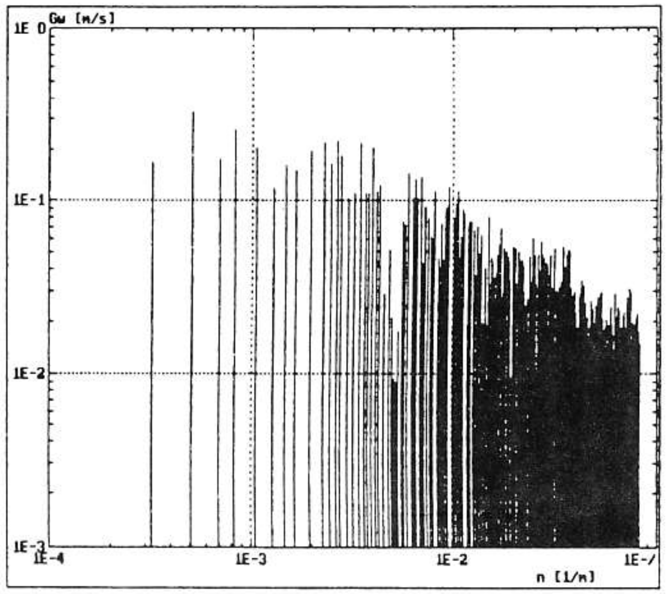

Fig. 4: The shape of the discrete amplitude spectrum

2. RESULTS OF THE ANALYSIS

Four atmospheric turbulence vertical velocity records were analyzed from the research program LOTREX/HIBE'89 (Jochum et al. (10)) measured in runs FJl, FJ2, JFl and JF2 respectively. Standard deviation as calculated from the records:

$$\sigma_{mr} = \left[\frac{1}{q}\sum_{i=1}^{q} w_i^2\right]^{1/2} \tag{8}$$

is given in the first row of Tab. 1.

Pre-processing by calculation of a running mean function gave good convergences for the calculation of the integral scale using (1). The nominal values for the four flights are given in the first row of Tab. 2. As shown by the diagram in Fig. 2, they turned out to be substantially smaller than in most publications. A considerable scatter in the values given, indicates however some problems still remaining in the exactness of this calculations.

Taylor's scale λ determined according to (2) is given in the first row of Tab. 3. Range as well as scatter of the results is up to expectations. We do not regard as yet values obtained tentatively for the upper frequency limit n_{max} of the power spectra to be definitive but nevertheless, they exclude the possibility of Taylor's scale being the inverse of them.

Direct spectral density functions $G_w(n)$ as well as correlation spectra $S_w(n)$ were smoothed using (5). Shape parameters α and A determined this way are given in Tab. 4 and Tab. 5. While the exponents given by the least squares procedures seem to be correct beyond expectations, no such claim for the peak coefficients can be made. The integral scale L and the theoretical standard deviation σ_{OG} and σ_{os} were determined also using the Karman formula and least squares procedures.

Taylor's scale is associated to the second momentum of the PSD function:

$$\lambda_G = \frac{\sigma}{\sqrt{2}\pi}\left[\int_0^\infty n^2 G(n)dn\right]^{-1/2} \tag{9}$$

A similar formula can be composed for the correlation spectrum $S_w(n)$.

As it can be shown, the measured standard deviation can be calculated from the amplitude spectrum $r_w(n)$ the following way:

$$\sigma_{mv} = \frac{1}{\sqrt{2}}\left[\sum_{i=1}^{q} r_i^2\right]^{1/2} \tag{10}$$

Scale parameters can also be checked using the amplitude spectrum. In order to make the formulae more descriptive the designation for the wave length

$$H = \frac{1}{n} \tag{11}$$

will also be used. The formula for the integral scale then reads

$$L = \frac{1}{2\pi}\frac{\sum_{i=1}^{q}\frac{r_i^2}{n_i}}{\sum_{i=1}^{q} r_i^2} = \frac{1}{2\pi}\frac{\sum_{i=1}^{q} r_i^2 H_i}{\sum_{i=1}^{q} r_i^2} \tag{12}$$

Taylor's scale can be calculated using the formula:

$$\lambda = \frac{1}{\sqrt{2}\pi}\left[\sum_{i=1}^{q}\frac{1}{n_i^2}\right]^{1/2} = \frac{1}{\sqrt{2}\pi}\left[\sum_{i=1}^{q} H_i^2\right]^{1/2} \tag{13}$$

This relation also proves Taylor's scale not to be the inverse of the limit wave number n_{max}.. Moreover, it may turn out to be the right choice for a statistical scale of wave length.

3. ESTIMATION OF CALCULATION ERRORS

If turbulence would be a pure statistical phenomenon then error estimation procedures as worked out for example by Bendat and Piersol (l) could be used to check the calculations. More strong laws than these apply to the chaotic solutions of nonlinear simultaneous differential equations, so it is advisable to look also for other reliability estimation methods. One method might be a check on the natural parameters and spectrum shape parameters.

The value σ_{mr}, calculated directly from the record by (8) is regarded to be the norm for standard deviation. Nominally the same value should be given by (7a) or (7b). If not, then the spectrum calculation formulae could be blamed first for the difference. The direct spectra $G_w(n)$ passed this test with flying colors (Tab. 1, rows 2 and 3). The correlation spectra $S_w(n)$ are not quite up to this high standard (rows 4 and 5) but the sign of the differences are right, and the amount of bias errors are acceptable. The ratios of the theoretical standard deviations (rows 6-8) also support this theory.

Errors of the experimental amplitude spectra $r_w(n)$ (rows 9 and 10) indicate the basic concept to be probably right but need some refinement in the calculation procedures. Work is going on in this line.

The integral scale L (Tab, 2) did not figure well in the checks. Even the scatter of the values given by (l) between the individual runs (row 1) is too much. Quite unacceptable differences for the spectrum checks (rows 2-5) may probably be traced back to uncertainties in the low frequency end of the spectra. Scale parameter calculations for the amplitude spectra using (12) and (13), respectively, have been postponed until the revision of the spectrum calculation procedures.

Taylor's scale λ (Tab. 3) passed the test substantially better. Scatter/error values for (2) (row 1) as well as for Eq. (rows 2-3) are within acceptable 5% error margins. The correlation spectrum calculation (rows 4-5) is showing an amount of bias otherwise in the 5% band.

Spectral density function shape parameters 'a' and A were checked against their theoretical values as given by (4). Best fit smoothing values close to theoretical were obtained for the exponent (Tab. 4). Regression values for the peak coefficient A show unacceptable differences to the theoretical value 8/3 as well as between themselves. This strengthens the suspicions raised by the problems with the integral scale concerning the reliability of the low frequency end of the PSD functions.

CONCLUSIONS

- It is advisable to use a suitable running mean function for pre-processing because it provides reliable and reproducable natural and spectrum shape parameters.
- Calculation of the integral scale L from the autocovariance function and by regression analysis from the spectral density functions does not give acceptable conformity. The source of the errors may be uncertainties in the low-frequency end of the PSD functions. In spite of these problems the analysis indicates values substatially lower than customary for the atmospheric turbulence.
- Values for Taylor's scale λ given by (2) and (9) respectively are in good congruence, confirming the theoretical models. There is no sign of λ being the inverse of the limit wave number n_{max}.
- Regression values obtained for the exponent 'a' in the Karman spectrum formula were in good agreement with the theoretical value of 11/5. No such agreement has been found for the peak coefficient A, the source of the errors being probably the same as for the integral scale L.
- There are strong suggestions for to the existence of a special discrete amplitude spectrum, too. The experimental spectrum calculation procedure isn't perfect yet, but further development may correct the insufficiencies.

The author would gratefully receive any contribution and/or turbulence records helping to clear up these problems.

ACKNOWLEDGEMENTS

The research reported herein has been sponsored in part by the Orszagos Tudomanyos Kutatasi Alap under the research grant T-025075. Thanks are due to Dr. Manfred M. Reinhardt, director (ret.) of the DLR Instiut für Physik der Atmosphare and his co/workers Mrs. A.M. Jochum and Mr. N. Entstrasser for flight records. The author also wishes to express his thanks to Dr. Sandor Horvath for his expert help in computer problems and in editing the paper.

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Appendix 1: Natural Parameters

Tab. 1: Standard Deviation σ

			R u n:			
Origin:	Parameter:	Dim.:	FJ1	FJ2	JF1	JF2
Record	σ_{mr}	m/s	0.8292	0.8544	0.7392	0.8282
Direct sp.	σ_{mG}	m/s	0.8290	0.8545	0.7392	0.8282
$G_w(n)$	$(\sigma_{mG}-\sigma_{mr})/\sigma_{mr}$	%	-0.025	+0.006	-0.011	-0.009
Covariance	σ_{mS}	m/s	0.8852	0.9173	0.7815	0.8822
sp. $S_w(n)$	$(\sigma_{mS}-\sigma_{mr})/\sigma_{mr}$	%	+6.757	+7.353	+5.712	+6.519
$G_w(n)$	σ_{0G}	m/s	0.8309	0.8587	0.7409	0.8301
$S_w(n)$	σ_{0S}	m/s	0.8885	0.9233	0.7836	0.8850
	σ_{0G}/σ_{0S}		0.9351	0.9310	0.9455	0.9380
Amplitude	σ_{mv}	m/s	0.7882	0.7630	0.7553	0.7886
sp. $r(n)$	$(\sigma_{mv}-\sigma_{mr})/\sigma_{mr}$	%	-4.94	-10.70	+2.18	-4.79

Tab. 2: Integral Scale L

			R u n:			
Origin:	Parameter:	Dim.:	FJ1	FJ2	JF1	JF2
$R_w(\varsigma)$	L	m	64.87	119.96	81.59	96.55
Direct sp.	L_G	m	42.22	60.03	49.17	49.93
$G_w(n)$	$(L_G-L)/L$	%	-34.91	-49.96	-39.73	-48.29
Covariance	L_S	m	41.67	56.64	45.90	47.23
sp. $S_w(n)$	$(L_S-L)/L$	%	-35.76	-52.78	-43.63	-51.09
	L_G/L_S		1.0132	1.0598	1.0692	1.0517

Tab. 3: Taylor's Scale λ

			R u n:			
Origin:	Parameter:	Dim.:	FJ1	FJ2	JF1	JF2
$R_w(\varsigma)$	λ	m	29.16	33.22	30.43	29.88
Direct sp.	λ_G	m	28.01	31.83	29.44	28.95
$G_w(n)$	$(\lambda_G-\lambda)/\lambda$	%	-3.92	-4.18	-3.25	-3.09
Covariance	λ_S	m	26.35	27.84	27.55	26.43
sp. $S_w(n)$	$(\lambda_S-\lambda)/\lambda$	%	-9.62	-16.19	-9.48	-11.53
	λ_G/λ_S		1.063	1.143	1.069	1.095

Appendix 2: Spectral Density Function Shape Parameters

Tab. 4: Exponent α

Theoretical value: $\alpha = 11/6 = 1.8333$

			R u n:			
Origin:	Parameter:	Dim.:	FJ1	FJ2	JF1	JF2
Direct sp.	α_G		1.80164	1.79876	1.79804	1.80308
$G_w(n)$	$\Delta\alpha_G$	%	-1.729	-1.886	-1.925	-1.650
Covariance	α_S		1.77633	1.76827	1.76710	1.77866
sp. $S_w(n)$	$\Delta\alpha_S$	%	-3.109	-3.549	-3.613	-2.982
	α_G/α_S		1.014	1.017	1.0175	1.014

Tab. 5: Peak Coefficient A

Theoretical value: $A = 8/3 = 2.6667$

			R u n:			
Origin:	Parameter:	Dim.:	FJ1	FJ2	JF1	JF2
Direct sp.	A_G		2.124	1.423	1.544	1.571
$G_w(n)$	ΔA_G	%	-20.35	-46.63	-42.09	-41.07
Covariance	A_S		1.108	1.074	1.097	1.104
sp. $S_w(n)$	ΔA_S	%	-58.45	-59.71	-58.85	-58.61
	A_G/A_S		1.917	1.325	1.407	1.424