MAXIMUM ENERGY LEGS: GLIDE SPEEDS IN AN INHOMOGENOUS AIRMASS

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SUMMARY

Glides around a turnpooint, on course or to a finish, are an important feature of sailplane racing, and the traditional approach of adopting a singlespeed to fly, or MacCready setting, for legs with different winds can be significantly sub-optimal. The best glide angle over the ground is achieved at lower speeds with a tail wind and at higher speeds with a head wind. Thus, a pilot who flies at the speed for the best glide angle over the ground for each leg can start the final glide at a lower altitude than a pilot flying the same speed for each leg. Similarly, a pilot flying at the appropriate speed for the wind on each leg will arrive at the goal sooner than the pilot flying at a constant speed. The methodology for determining the altitude required to achieve a goal and the optimal speeds to fly for multiple legs with differing wind speeds, as well as representative results, are presented.

INTRODUCTION

The altitude required and speed to fly to achieve a distance over the ground in the presence of wind have been discussed many times, e.g. Reichmann (1) and the calculation is readily performed on many graphical devices ("prayer wheels") or electronic flight computers. Frequently, particularly in competition, changes in the flight direction are imposed close enough to the goal that the comutation of the altitude required and the speed to fly for the final glide must take into account differing wind speeds; for example a tail wind before the final turn and a head wind after it. This is typically addressed by selecting a speed to fly and then calculating the altitude loss for each of the remaining course segments, or "legs," to ascertain whether or not the goal can be achieved at that speed.

Intuitively, however, one might expect that flying slowly, at a lower sink rate, with a tail wind and faster into a head wind would yield the greatest distance covered from a given altitude, or alternatively a minimum altitude required to cover a given distance. Furthermore, starting from an altitude above the minimum required to achieve the goal, flying slower with a tail wind and faster into a head wind should reduce the energy loss on each leg, or result in "maximum energy legs." The calculation below, of the minimum altitude required to achieve a distance in the presence of differing winds confirms this intuition. The difference in altitude required between flying at a constant air speed and flying so as to maximize the energy on each leg is not great for typical conditions; less than 100 meters. However, the speed achieved to the goal can be significantly greater using variable air speeds, maximum energy legs (MEL) — rather than a constant air speed (CS); as much as 10 kph for not unreasonable conditions.

After developing the methodology, results for three representative modern sailplanes (PW-5, LS-8 and ASH-25) are presented.

DISCUSSION

For simplicity, consider winds parallel to the flight path; that is ignore any cross wind component compared to the air speed as is the custom. Cross winds are not a problem, they just clutter up the discussion of the very simple idea that that is being presened here. Then

$$d_i = (v_i + w_i)\ t_i \qquad 1$$

$$h_i = s_i\ t_i = s_i \frac{d_i}{v_i + w_i} \qquad 2$$

where d_i is the length of leg i, v_i is the air speed on leg i, w_i is the wind speed on leg i, t_i is the time spent gliding on leg i, h_i is the altitude loss on leg i, and s_i is the sink rate at v_i or the "Polar."

The best glide on each leg is just given by minimizing the altitude lost on the leg:

$$\frac{\partial h_i}{\partial v_i} = 0, \text{ or } \frac{\partial s_i}{\partial v_i} = \frac{s_i}{v_i + w_i}\ , \qquad 3$$

which is the basis for the familiar graphical solution that the best L/D ratio is achieved at the tangent to the polar shifted by the wind speed.

Similarly, for varying wind speeds but constant air speed (the CS solution) on all of the legs ($v \equiv v_i$), the best glide is just given by

$$\frac{\partial \sum h_i}{\partial v} = 0\ , \qquad 4$$

while for maximum energy legs (the MEL solution) equation 3 obtains for each leg separately.

To illustrate the differences between the CS and the MEL glides, consider just two legs each of 40 km length, with the wind of the same magnitude on both legs but of opposite direction; that is, a tail wind on the first leg and a head wind on the second. Figure 1 shows the minimum height required to achieve a final glide.

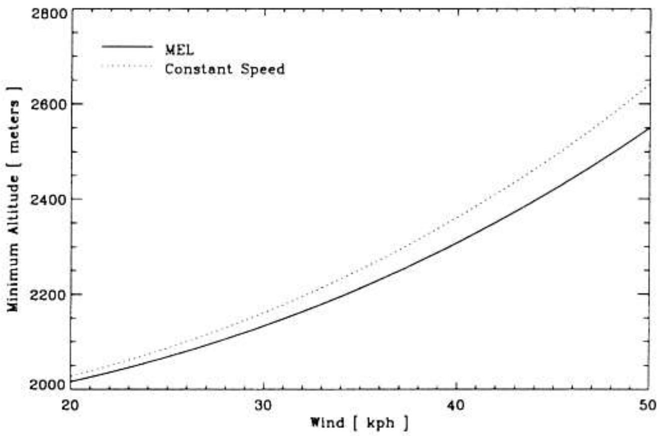

Figure 1: Minimum height to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude, for an LS-8.

The height difference is not very great.

The 100 meter difference in the minimum height required for rather extreme conditions (50 kph wind) might lead one to think that it would not be worth the trouble to worry about flying at different speeds on the two legs. However, Figure 2 shows the time required to arrive at the goal for the two solutions, and now one sees that the MEL solution can result in a minute or two difference in arrival time,

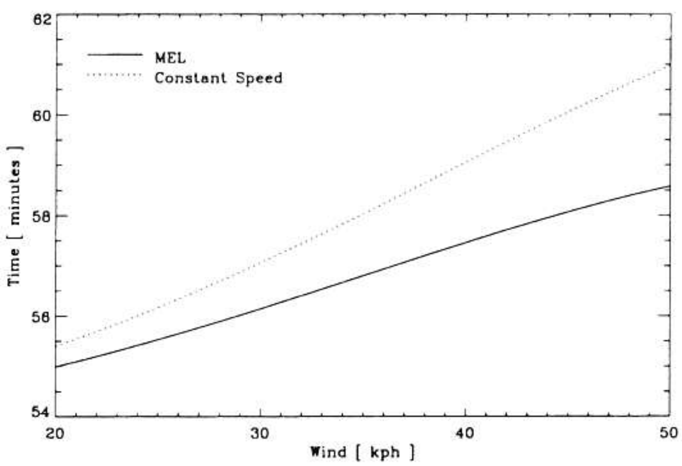

Figure 2: The time to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude, for an LS-8.

which is worth the trouble. In addition, for two pilots starting side by side, one must add, to the CS time, the time required to climb to the slightly higher altitude required. This improvement in performance is achieved by flying at significantly different speeds on the two legs as shown in Figure 3.

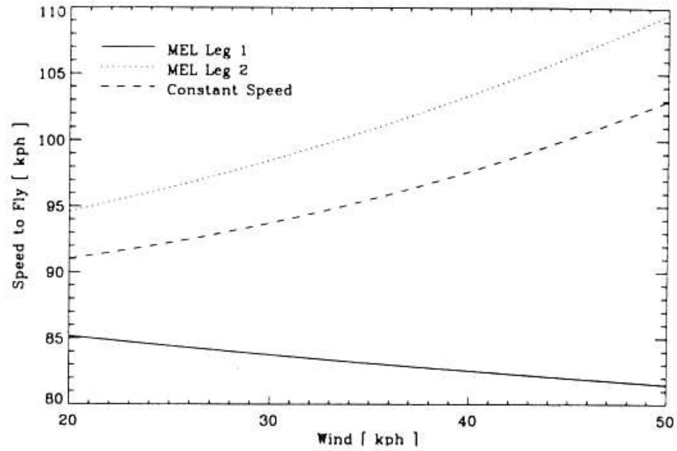

Figure 3: Speeds to fly to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude, for an LS-8.

For initial altitudes above the minimum required to arrive, which might be a bit nerve-wracking for these distances and winds, the speeds to fly are determined by minimizing the total time

$$T = \sum \frac{d_i}{(v_i + w_i)} \qquad 5$$

this is, solving for

$$\frac{\partial T}{\partial v_1} = 0 \qquad 6$$

subject to the constraint that the glide reach the goal, given by

$$H = \sum h_i \qquad 7$$

which relates the v_i to v_j.

Figure 4 shows the speed to the goal for a representative wind and a range of altitudes above the minimum for the CS solution , and Figure 5 the corresponding speeds to fly.

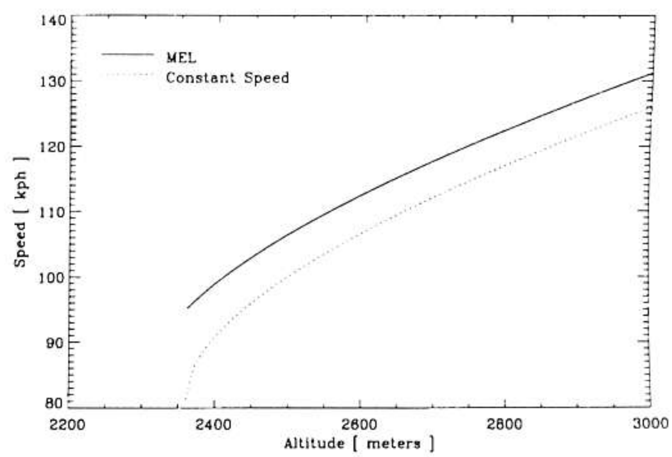

Figure 4: Speed achieved to a goal with two 40 km legs, and a 40 kph tail wind on the first leg and 40 kph head wind on the sec ond, for various altitudes above the minimum required to arrive at the goal for the Constant Speed solution, for an LS-8.

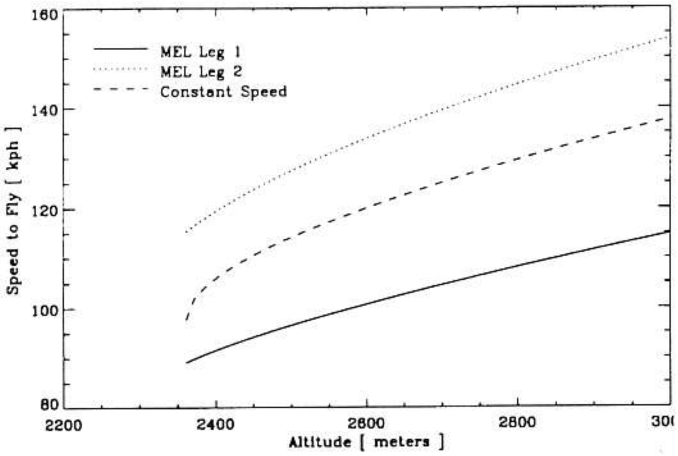

Figure 5: Speeds to fly to reach a goal with two 40 km legs, and a 40 kph tail wind on the first leg and 40 kph head wind on the second for various altitudes above the minimum required to arrive at the goal for the Constant Speed solution, for an LS-8.

Another way of visualizing the performance advantage of the MEL over the CS is shown in Figure 6, where the two solutions start at the same height above the minimum required for the CS to arrive at the goal.

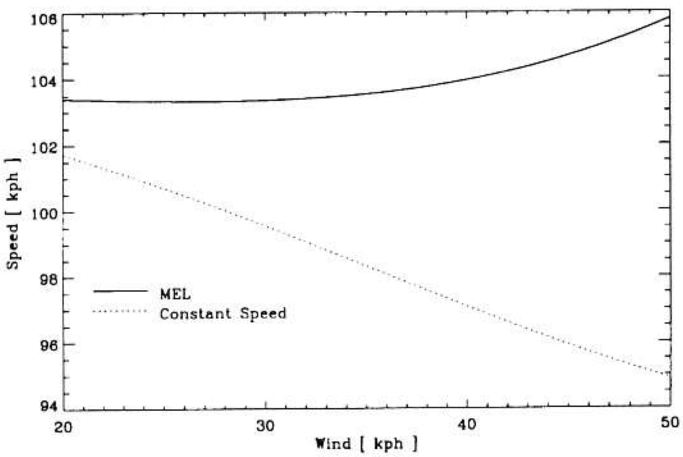

Figure 6: Speeds to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude, for an LS-8 starting at 100 meters above the minimum altitude required for the Constant Speed solution.

Finally, it is of interest to see how these results depend upon the performance of the sailplane, and Figure 7 shows the polars of the PW-5, LS-8 and ASH-25 taken from

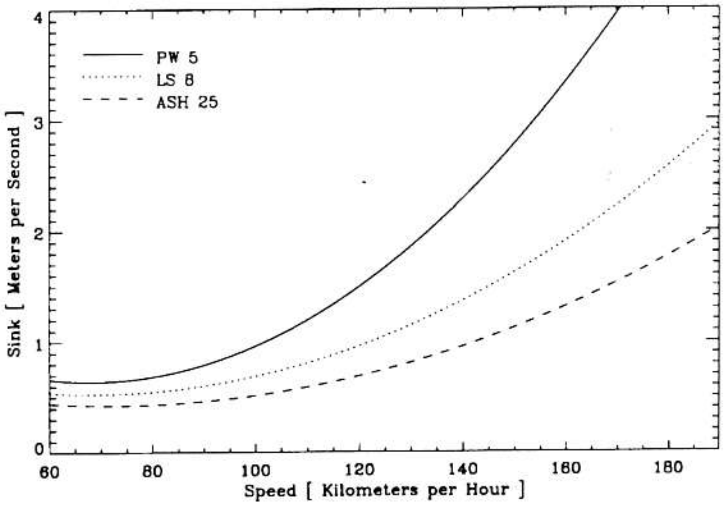

Figure 7: The quadratic representations of the polars used for the PW-5, LS-8, and ASH-25.

Johnson's flight test evaluations (2,3,4). The quadratic representation matches the best L/D ratio and the speed at which it is achieved, as well as the speed at which a sink rate of 2 mps occurs. Figures 8 and 9 show the results corresponding to Figure 6.

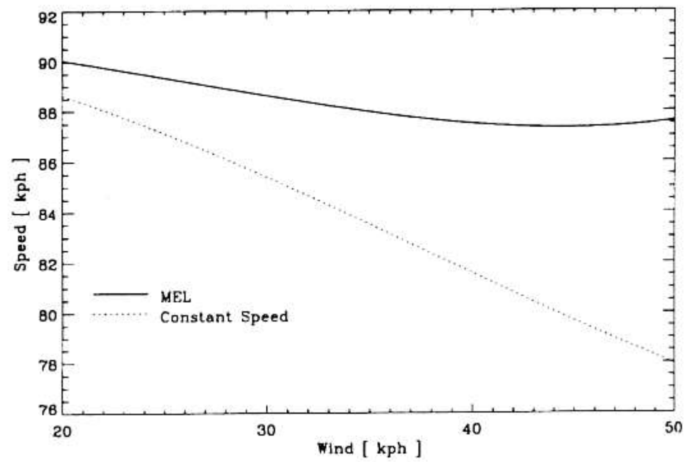

Figure 8: Speeds to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude for a PW-5 starting at 100 meters above the minimum altitude required for the Constant Speed solution.

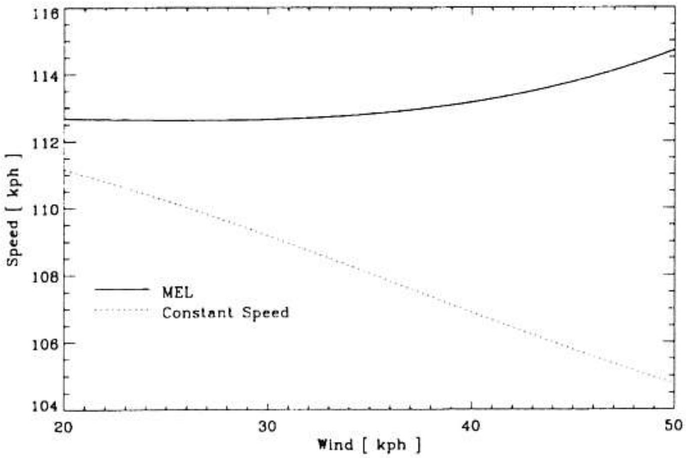

Figure 9: Speeds to reach a goal with two 40 km legs, and a tail wind on the first leg and head wind on the second of the same magnitude for an ASH-25 starting at 100 meters above the minimum altitude required for the Constant Speed solution.

CONCLUSIONS

Flying the correct speed to fly for the wind on each leg of a multiple leg glide can lead to significant performance improvements. The maximum energy legs (MEL) speeds differ substantially from those of the traditional, constant speed, the final glide. The algebraic details for the determination of these speeds are given in the Appendix.

REFERENCES

1. Reichmann, H. "Cross-Country Soaring," 1978, 120-122.

2. Johnson, R.H. "A Flight Test Evaluation of the PW-5 World Class Sailplane," Soaring, April, 1997, 19-23.

3. Johnson, R.H. "An FTE of an LS-8a Standard Class Sailplane," Soaring, July 1998, 17-21.

4. Johnson, R.H. "A Flight Test Evaluation of the ASH-25," Soaring, May 1988, 34-40.

APPENDIX - ALGEBRAIC DETAILS

Polar:

The results presented in the body use a quadratic polar of the form:

$$s = a + b\,v + c\,v^2 \qquad \text{A1}$$

Minimum height to reach the goal:

For the MEL solution, each leg is determind by equation 3 and the speed to fly on leg i is given by the roots of

$$c\,v_i^2 + 2\,c\,w_i\,v_i + (\,b\,w_i - a\,) = 0\,. \qquad \text{A2}$$

The minimum height required is:

$$H_{MEL} = \sum s_i \frac{d_i}{(\,v_i + w_i\,)} \qquad \text{A3}$$

For the CS solution, the speed to fly is given by equation 4, that is the roots of

$$\frac{\partial s}{\partial v} \sum \frac{d_i}{v + w_i} - s \frac{d_i}{(\,v + w_i\,)^2} = 0 \qquad \text{A4}$$

or

$$(\,b + 2\,c\,v\,) \sum \frac{d_i}{v + w_i} - (\,a + b\,v + c\,v^2\,) \sum \frac{d_i}{(\,v + w_i\,)^2} = 0 \qquad \text{A5}$$

While the roots of this quartic are analytic, in practice a simple newton-Rapheson iteration is simpler to implement, and given v,

$$H_{CS} = s \sum \frac{d_i}{v + w_i}\,. \qquad \text{A6}$$

Speeds to fly from an arbitrary altitude above the minimum:

For the *MEL solution, equation 6 becomes:*

$$-\frac{d_1}{(\,v_1 + w_1\,)^2} - \sum \frac{d_i}{(\,v_i + w_i\,)^2} \frac{\partial v_i}{\partial v_1} = 0 \qquad \text{A7}$$

and $\dfrac{\partial v_i}{\partial v_1}$ is given by differentiating equation 7.

$$\frac{\partial s_1}{\partial v_1} \frac{d_1}{v_1 + w_1} - \frac{s_1\,d_1}{(\,v_1 + w_1\,)^2} + \sum \frac{\partial v_i}{\partial v_1} \frac{d_i}{(\,v_i + w_i\,)} \left(\frac{\partial s_i}{\partial v_i} - \frac{s_i}{v_i + w_i} \right) = 0 \qquad \text{A8}$$

which for two legs is just

$$\frac{\partial v_2}{\partial v_1} = - \frac{\left(\dfrac{d_1}{v_1 + w_1} \right) \left(\dfrac{s_1}{v_1 + w_1} - \dfrac{\partial s_1}{\partial v_1} \right)}{\left(\dfrac{d_2}{v_2 + w_2} \right) \left(\dfrac{s_2}{v_2 + w_2} - \dfrac{\partial s_2}{\partial v_2} \right)}\,. \qquad \text{A9}$$