CALCULATIONS ON SOARING SINK

Energy and Power Calculations when Soaring Sink Pockets

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As discussed in the article *Dynamic Soaring and Sailplane Energetics*, there is just as much energy in the motion of sinking air as in rising air. It is, however, quite a bit more difficult for a glider to get power from downward moving air. Getting energy from sink requires negative g's or inverted flight.

The energy a glider gets from sinking air generally appears as extra speed (kinetic energy). There is also frequently a loss of height as the kinetic energy increases. Still, the increase in kinetic energy can far exceed the loss of potential energy due to loss of height. Below are some calculations demonstrating how sailplanes can get energy from sinking air.

Before looking at the details of how the glider gets energy from sink, let's look at where the energy comes from. It comes from the atmosphere. There is kinetic energy in a mass of downward moving air. If the air pushes on something in the air's direction of motion (down in this case) the air loses energy at a rate where: HP = Force (lbs) x Velocity (f/s) / 550.

This tells us that 10 f/s moving air can lose energy at 1.8 HP for each 100 lbs of force. If the air is moving downwards at 20 f/s and pushing with a force of 800 lbs (minus 1 g in an 800 lb glider) then energy is lost by the air at a rate of 29.1 HP. Windmill designers use these kinds of calculations for sideways moving air.

How much of this energy can a glider utilize to increase its speed or height? That will depend on the L:D of the glider. A glider with an infinite L:D could get all the energy lost by the air, a real glider gets less. We can look at some vector velocity and force diagrams to see how much power a glider can actually get. We refer to the Dynamic Soaring Vector Diagram, which shows how a glider can get energy from a downward gust.

In the diagram we're looking at a glider's situation just as it has entered a strong sink pocket. Because of its inertia and momentum the glider's velocity has not yet changed significantly, remaining the same as it was in the still air. The glider velocity is shown in the diagram as a slightly downward sloping vector. A downward gust is shown as a downward pointing vector. The relative wind experienced by the glider at that moment is the glider's velocity subtracted from the gust vector. The angle between the relative wind and glider velocity is labeled (a).

In addition to showing velocities the diagram shows the aerodynamic forces on the glider. To extract energy in sinking air the glider must push upward on the down gust. So assuming it is right side up, the glider must be in a negative g situation (we use minus 1 g in this example).

As in normal practice the lift force on the wing is defined as perpendicular to the relative wind and the drag force as parallel to it. The vector sum of the lift and drag is called the total resultant aerodynamic force. The angle between the lift and resultant force is labeled (b). The tangent of angle (b) is the glider's drag to lift ratio (at -1 g).

The smaller angle (b), the better for soaring. To determine the angle (b) we need to know the glider's drag and lift forces in different situations of speed and g force. For this, we can create a chart using equations found in chapter 16 of *New Soaring Pilot* by Welch and Irving. The chart gives glider drags at different speeds and g loadings. Angle (b) is the arctangent of the L/D ratio. These drag values are very useful when calculating powers and energies.

We separate the total resultant aerodynamic force on the glider into two components or parts, one component parallel to the glider's velocity and one perpendicular to it. The component of force perpendicular to the glider's velocity changes the glider's direction of motion but not its speed or energy. The force vector component parallel to the glider's velocity results in changes of speed and energy. This dynamic soaring thrust vector is shown in bold on the diagram. If angle (b) is too large relative to angle (a) this "thrust" vector reverses and becomes a source of drag losses.

Note that gravitational forces need not be considered in this diagram. Thus the diagram can be tipped, and works in any orientation. We include the angle (c) between the horizontal and the glider's velocity vector for reference, but it is not used in the power calculations.

Gravity is represented by a conservative field, which means that any kinetic energy picked up due to gravity is precisely offset by a corresponding loss in potential energy. The aerodynamic forces are the ones that change the glider's energy. We will look at some cases that consider gravitational potential energy after going through the vector diagram.

So first let's work a numerical example with the vector diagram using a glider velocity of 100 f/s and a down gust of 20 f/s. This gives an angle (a) between the glider velocity and relative wind of about 11.3 degrees. If we push forward on the stick to get minus 1 g we will have a generally downward lift force on the wing of 800 lbs. The lift force is perpendicular to the relative wind and thus points forward relative to the glider's velocity. This is similar to how a sailboat gets forward force from a crosswind.

For an inverted L/D = 25 we add a drag force vector that is 1/25 of the lift and is parallel to the relative wind. The resultant total aerodynamic force is tipped back from the lift vector by an angle (b) of 2.3 degrees (the arc tan of 1/25).

To find the component of the resultant aerodynamic force which is pushing the glider in its direction of motion; we subtract angle (b) from angle (a), 11.3 - 2.3 = 9 deg.

Multiplying the total resultant force (very close to 800 lbs) by the sine of 9 degrees gives us the force in the direction of motion. The result in this example is 125 lbs. To get the power extracted we multiply the force times the speed and divide by 550.

Calculated Glider Drags at Different Speeds and G Loads

Glider Specifications:

Best L:D	40	Drag @ Best speed & 1 g	
@ Speed of	100	Induced Drag	10
Weight	800	Friction Drag	10
Stall Speed	60	Negative G drag Factor	1.4
Inverted Stall Speed	90	Chart g start	-3
Chart Speed Increment	25	Chart g Increment	0.5

Drag Chart:

Speeds on top row

G Load	50	75	100	125	150	175	200	225
-3	-	-	-	-	-	84	88	96
-2.5	-	-	-	-	70	71	78	88
-2	-	-	-	-	56	61	70	82
-1.5	-	-	-	42	46	53	64	77
-1	-	-	28	31	38	47	60	74
-0.5	-	14	18	24	33	44	57	72
0	4	8	14	22	32	43	56	71
0.5	13	10	13	17	24	31	41	51
1	+	23	20	22	27	34	43	53
1.5	+	46	33	30	33	38	46	55
2	+	+	50	41	40	44	50	59
2.5	+	+	73	56	50	51	56	63
3	+	+	+	73	63	60	63	68
3.5	+	+	+	94	77	71	71	75
4	+	+	+	118	94	83	80	82
4.5	+	+	+	+	113	97	91	91
5	+	+	+	+	134	112	103	100
5.5	+	+	+	+	157	129	116	110
6	+	+	+	+	183	148	130	122
6.5	+	+	+	+	+	169	146	134
7	+	+	+	+	+	191	163	147
7.5	+	+	+	+	+	214	181	162
8	+	+	+	+	+	240	200	177

Notes: Units can be varied, though Lbs and feet/second could represent a modern 1 seater.
Friction drag assumed proportional to speed squared.
Induced drag assumed proportional to lift squared and inverse to speed squared.

125 lbs x 100 f/s / 550 = 22.7 HP. This is the power delivered to the glider's kinetic energy while it is in the down gust. The immediate drag losses are already accounted for by angle (b). The additional latter losses due to zooming up and converting the additional kinetic energy into height (after leaving the sink gust) should now be considered.

An 800 lb sailplane with a 40:1 glide angle requires 3.6 HP to fly at 100 f/s in a normal glide. When making a 2 g pull up the drag and the power consumed increases. Looking at our drag chart for a speed of 100 we see that between one and two g's the drag increases from 20 lbs to 50 lbs. So at 2 g's the power consumption is about 9 HP.

Now let's look at the whole cycle of a glider soaring in and out of sink pockets. We need to fly a lift/g cycle that provides enough average upward lift to support the glider — by combining one second of minus one g push over in sink and two seconds of two g pull up in still air we get an average positive acceleration of one g. This will sustain the glider against the normal one g acceleration of gravity.

(-1 g x 1 sec + 2 g x 2 sec) / 3 sec total time = 1 g average

To calculate energy we look at the average power over the cycle:

(+22.7 HP x 1 ec -9 HP x 2 sec) / 3 sec = 1.6 HP

The net positive power of 1.6 HP corresponds to an average climb rte of about 1.1 f/s for an 800 lb glider (65 f/min). In this example we are *climbing* by using "sink!"

Now let's look a little more closely at the effect of gravity on the sailplane's energy while dynamic soaring. As noted above the effects of gravity are not of primary importance when dynamic soaring. Gravity does however limit how much height change we can make before reaching excessive speeds and as we noted above resisting gravity requires maintaining an average upward lift equal to the glider's weight.

The mechanical energy of a glider has two parts: it's potential energy due to height and its kinetic energy due to speed. The potential energy equals mass times height times the gravitational constant. The kinetic energy equals one half of the mass times the speed squared.

$$E_{potential} = M \times h \times g$$
$$E_{kinetic} = 1/2 \times M \times V^2$$

The Pythagorean relationship tells us that in a right trianble; side A squared plus side B squared equals the length of the hypotenuse squared. We can use this relation to separate the kinetic energy into two parts one due to horizontal speed and one due to vertical speed. If we choose examples that keep the horizontal speed constant then the calculations are easier.

Let's calculate the total energy changes for a glider in two different cases both where: horizontal speed = 100 ft/sec, initial vertical speed = -3 ft/sec, weight = 800 lbs, mass = 25 slug (one slug equals about 32 pounds mass).

First, normal glide,

In a time interval of one tenth of a second the glider descends 0.3 feet. The loss of potential energy equals M x g x h. The weight of 800 lbs is equal to M x g. Thus the potential energy change is -240 ft-lbs (-0.3 feet times 800 lbs). There is no change in vertical or horizontal kinetic energy. The total energy loss of 2400 ft-lbs/sec coresponds to -4.36 HP.

Now let's look at a second case where we encounter a down gust which allows us to fly at minus one g and accelerate rapidly downward without losing forward speed. In order to not effect horizontal speed, we require a total aerodynamic resultant force that is vertical. This means that the angle (a) is equal to the sum of angles (b) and (c). This corresponds to a down gust velocity of about 7 feet/second. If we fly minus 1 g without a down gust, we will lose forward speed.

At minus one g we accelerate downward at 64 feet per second squared (32 from gravity and 32 from the aerodynamic forces). In this example, we do it for one tenth of a second, resulting in an increase of sink rate from 3 f/s to 9.4 f/s. The average sink rate is 6.2 f/s and the height lost; in 1/10 second is 0.62 feet. The loss in potential energy is 0.62 feet times 800 lbs or -496 ft-lbs. There is no change in horizontal speed or horizontal kinetic energy.

The change in vertical kinetic energy is from 3 f/s squared to 9.4 f/s squared (both times 1/2 Mass). Which is: 9.4 squared - 3 squared = 88.4 - 9 = 79.4 feet squared/seconds squared. Multiplying by a 1/2 Mass of 12.5 slugs gives a kinetic energy increase of 992 ft-lbs. We subtract the loss in potential energy from the gain in kinetic energy. The total energy change is a gain of: 992 - 496 = 496 ft-lbs (in one tenth of a second). 4960 ft-lb/sec corresponds to 9 HP. So while in the sink at minus one g we are getting energy at a rate of 9 HP.

As we saw looking at whole cycles of dips and zooms 9 HP for only part of the time may not be enough to produce a complete cycle that gains net energy. It may take stronger sink than 7 ft/sec for that. In the 20 ft/sec down gust case figured earlier there is considerable forward dynamic thrust that increases the horizontal velocity and kinetic energy. With 20 f/s gusts there is enough power to overcome the losses of the periodic pull ups. If the pull ups can be made in upward gusts then the dynamic soaring is particularly effective.

The above calculations demonstrate how a high performance sailplane can stay up on a day with no lift, but only periodic pockets of strong sink.