HANDICAPS AND POLARS

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ABSRACT

Methods to obtain glider polars from handicap figures were developed. Common optimization calculations like cross-country speed versus lift rate can thus be obtained with just the handicap figure of the glider known. Application to all gliders, hang gliders, paragliders, and soaring birds is possible and contributes to the assessment of their specific potential flight distance.

INTRODUCTION

Man — Met — Machine are key factors for soaring flights [1]. Meteorological forecasting for soaring flight has been improved in the last decade, particularly in forecasting lift rates [2]. The latest machines offer best glide ratios in the sixties. The present work is meant to assist Man in making the best use of Met and Machine.

Met is reduced to lift rates and wind in this context. Machines are reduced to handicap figures and Man is familiar with the handicaps to compare the performance of different gliders. The physical and mathematical background can be found in [3]. Man will have to consider lift rates, wind and handicap figures when thinking about the potential flight distance. In the days of electronic devices which will perform most of the calculations required in soaring flight [3] it is hoped that this work will contribute to simple and friendly devices.

EMPIRICAL AND UNIVERSAL POLAR

The sink rate w of a glider is related to its horizontal speed v. Two analytical expressions are in use for w(v):

$$w = a*v^2 = b*v = c \qquad \text{(1, empirical)}$$

$$2 * (w/w_o) = (v/v_o)^3 = (v/v_o)^{-1} \qquad \text{(2, universal)}$$

The quadratic polynomial (1) is used in [4]. It contains three parameters a,b, and c to be adjusted. Let us refer to (1) as the *empirical* polar.

The higher order expression (2) is used in [3] and contains only two parameters v_o and w_o. v_o is the speed for maximum glide ratio, w_o is the corresponding sink rate. It is a more fundamental expression in the sense that only two parameters are needed and that they are related to profile drag and induced drag [3]. Let us refer to (2) as the *universal* polar. Both expressions can be adjusted to glider data (Figure 1). The empirical expression with three degrees of freedom fits slightly better. The adjustment of the universal polar, however, is remarkable in view of just two degrees of freedom. Both polars (1) and (2) are useful approximations

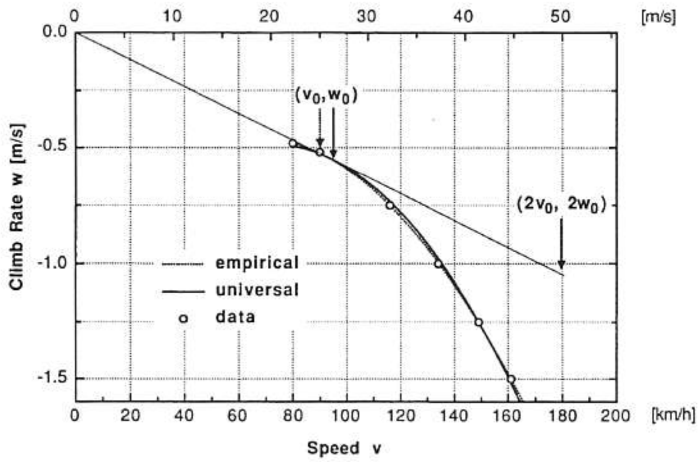

Figure 1 Empirical and universal polars adjusted to glider data

for gliding speeds v and v_o. For the *empiri cal* expression the speed v_o for maximum glide ratio is related to the coefficients a and c:

$$v_o = \text{sqrt}(c/a) \qquad \text{(3, empirical)}$$

GLIDE RATIO

The glide ratio is v/w with a maximum value of:

$$(v/w)_{max} = 1/(b = 2*\text{sqrt}(a*b)\,) \qquad \text{(4, empirical)}$$

$$(v/w)_{max} = (v_o/w_o) \qquad \text{(5, universal)}$$

The glide ratio v/w decreases in the relevant speed range from v_o to $2v_o$. For the *universal* polar thee decrease of the relative glide ratio $(v/w)/(v_o/w_o) = (v/v_o)/(w/w_o)$ is a

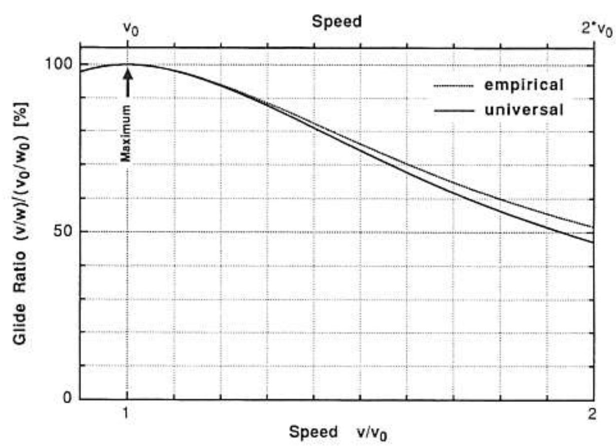

Figure 2 Decrease of glide ratio with speed

unique function of relative speed v/v_o (Figure 2). The glide ratio of an *empirical* polar decreases in a similar way.

HANDICAPS AND POLARS

Performance of different glider types varies widely. Handicap figures are in use in order to score flights achieved with different types of gliders and pilots are familiar with them. The German handicap figures, e.g., range from 84 to 124 for glider types between K6 and ASH-25.

Polars of different types of gliders (K6, ASW-15, LS-8 18m, ASH-25) were analyzed by adjusting *empirical* (a,b,c) and *universal* polars (v_o,w_o) to the glider data. Adjusted polars gave figures for the maximum glide ratio $(v/w)_{max}$ and the corresponding glide speed v_o. These were plotted against the handicap figures (Figure 3). The maximum glide ratio $(v/w)_{max} = (v_o/w_o)$ turns out to be a linear function of the handicap figure and the corresponding gliding speed v_o can be expressed as a third order polynomial of the handicap figure. v_o tends to e higher when *universal* polars are adapted to glider data, maximum glide ratios are identical for both types of polars. Wing loading accounts for most of the slope of v_o in Figure 3.

Data of hanggliders and paragliders was also analyzed by adjusting both types of polars. Adjustments was more difficult than for the glider data. Maximum glide ratios of 12 and 5 place them at handicaps of 50 and 38, respectively. Corresponding gliding speeds are reasonable. Extension to these soarers is possible. Steppe Buzzards [5] do not fit into the picture because of their wing loading.

For the *universal* polar the job is done at this point, since $w_o = v_o/(v/w)_{max}$. For a complete description of the empirical polar, however, we need more information than (v_o,w_o). *Empirical* polars are completely defined, if a second point (v,w) is known. Say we select w_2 at twice the speed of maximum glide ratio: $v_2=2^*v_o$. Figure 4 shows a linear relation between handicap figures and w_2 of all glider and the hangglider data. Paragliders and Steppe Buzzards do not fit into the picture.

The coefficients a,b, and c can be obtained from v_o,w_o, and w_2:

$$c = (w_2 - 2^*w_o) \quad (6a)$$

$$a = c/\mathrm{sqr}(v_o) \quad (6b)$$

$$b = w_o/v_o - 2^*c/v_o \quad (6c)$$

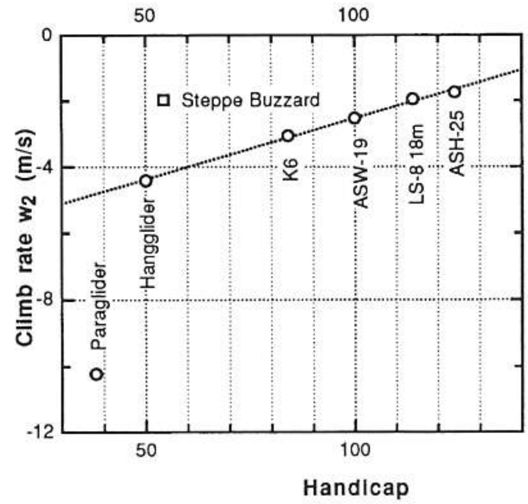

Figure 4 Vertical speed w_2 of gliders at twice the speed of best glide ratio (as obtained from adjusting empirical polars to glider data) plotted versus handicap figures.

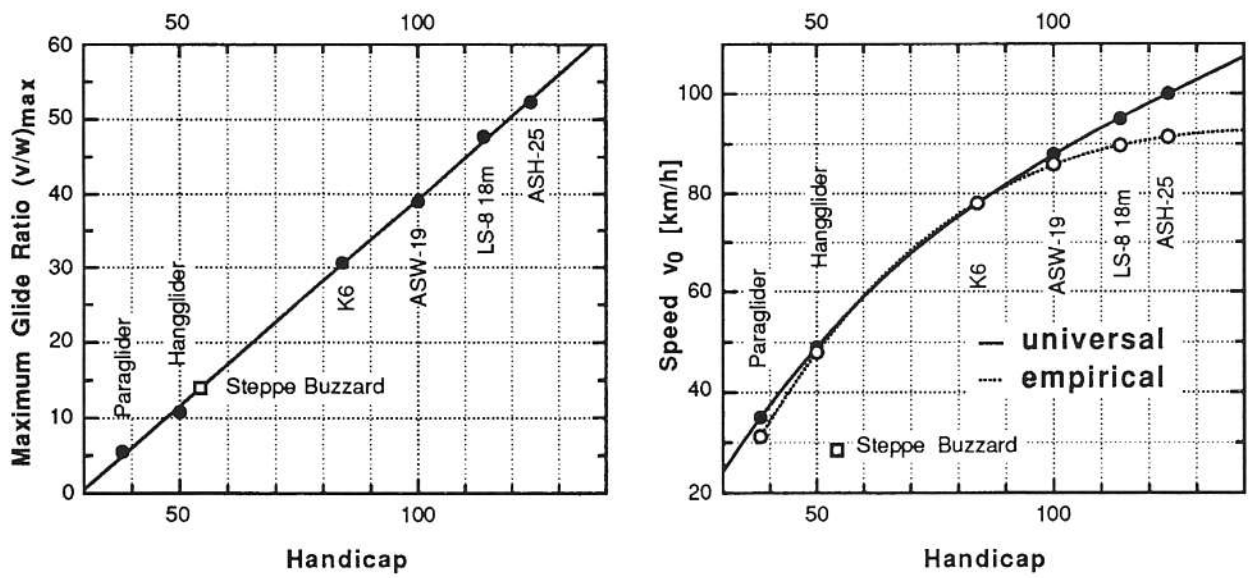

Figure 3 *Maximum glide ratio and corresponding gliding speed (as obtained from adjusting* universal *and* empirical *polars to glider data) plotted versus handicap figures.*

Empirical polars can also be deduced from handicap figures.

CROSS-COUNTRY SPEED

Under classical analysis (climbing and gliding at optimum speed, see [3, 4]) cross-country speed v_{av} may be obtained from the polar by suitable tangents. For an *empirical* polar cross-country speed v_{av} depends on a,b,c, and the rate of climb w_c (see [4]) as follows:

$$y = \mathrm{sqrt}((c - w_c)/a) \qquad (7a)$$
$$v_{av} = w_c/(2^*(w_c - c) - b^*y) \qquad (7b)$$

The universal polar is more reserved about revealing its cross-country speed. The geometrical problem to be solved is recalled in Figure 5:

$$v_{av}/w_c = v_G/(w_G - w_c) \qquad (8)$$

is the slope of the tangent to the polar at (v_G,w_G). The slope of the tangent is obtained by differentiating (2) at $v=v_G$ and substituting the left side of (8). w_G on the right side of (8) can be substituted by (2). The resulting eqution for v_G is:

$$(v_G/v_o)^3 - (v_G/v_o)^{-1} + w_c/w_o = 0 \qquad (9)$$

An analytical solution is not obvious. A numerical solution, however, is obtained after multiplying with $x = v_G/v_o$ and applying Newton's method (more tangents...) for finding the root of:

$$x^4 + (w_c/w_o)x - 1 = 0 \qquad (10)$$

by starting an iteration at $x_o = 2$ ($v_G = 2v_o$). Very few steps of:

$$x_{n+1} = x_n - (x_n^4 + (w_c/w_o)x_n - 1) / (4x_n^3 + (w_c/w_o)) \qquad (11)$$

will reveal the root $x_G \geq 1$ for any value of $w_c/w_o<0$. Programmable devices can handle this. The cross-country speed v_{av} follows from (8):

$$v_{av} = v_o^*w_c^*x_G/ (1.5w_c - w_o/x_G) \qquad (12)$$

(12) and (7) were used for Figure 6. Both *empirical* and *universal* polars reveal the same characteristics in cross-country speeds for different handicaps. Differences are on the order of 2 km/h or two handicap points at mximum. Handicap figures can be used for the calculation of cross-country speed with both types of polars.

AIR DENSITY AND WING LOADING

Both *empirical* and *universal* polars can be transformed to other air densities p and other wing loadings W/S, W being the weight of the glider and S the wing area. Higher wing loading $W>W_o$ and lower air density $\rho<\rho0$ increase the

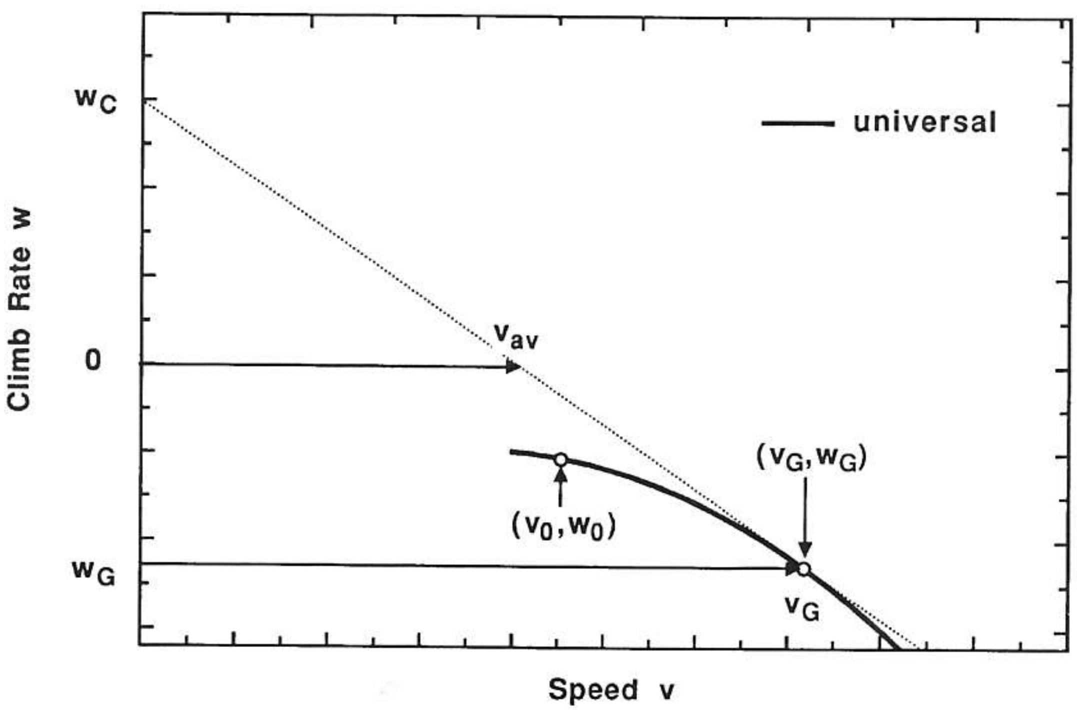

Figure 5 *Obtaining optimum cross-country speed v_{av} for climb rate w_c involves finding (v_G, w_G) on the polar*

transformation factor:

$$f = \mathrm{sqrt}(\,(W/W_o) * (\rho_o/\rho)\,)$$

for the parameters of both polars [4,3]:

$$a' = a/f \qquad (13, \text{empirical})$$
$$b' = b$$
$$c' = f*a$$

$$v_o' = f*v_o \qquad (14, \text{universal})$$
$$w_o' = f*w_o$$

For flights in the Alps with common flight altitudes of 3800 mMSL an increase of cross-country speed on the order of 10% can be expected in comparison with flights near sea level. Higher wing loadings work in the same sense.

CROSS-COUNTRY SPEED IN COMPETITIONS

Today's GPS documentation of soaring flights provides extensive flight data from which achieved cross-country speeds can be extracted. In 1991 these things were different. The Swiss national championship in Schänis took place during a period of unusually good soaring weather. In nine flying days tasks between 220 km and 660 km were set in the Open Class and a total distance of 4000 km was covered. The nine tasks consisted of 34 legs and a Cambridge S-NAV on board a glider registered average climb rates, fraction of climbing time, average speeds etc., for each leg. Pilots still had to mark the turnpoints to tell their instruments about the next leg.

When plotting average speed against average climb rate for all legs, there was quite a bit of scatter in the data due to differences in altitude at the turnpoints. Averaging entire tasks reduced the scatter in the remaining nine points significantly - but we were doing better than theory said in most cases. Finally, the known differences in altitude between start and finish line were used to correct the speeds and for six tasks average speed agreed quite well with the theoretical values for the glider (Figure 6, full circles, handicap 124). Three remaining offsets could be attributed to substantial deviations from the legs resulting in lower speeds (two flights) and to partial dolphin flight conditions resulting in higher speeds (one flight). It turned out that theoretical curves (7) and (12) come close to real cross-country speeds - even when flying systematically a little slower than v_G and when deviating slightly for better lift. To advance slower than the theoretical values is about as difficult as to beat them when flying tasks. Achieved climb rates tell the truth. Theory can then take care about converting them to cross-country speeds. This experience may come in handy when thinking about flight planning with today's

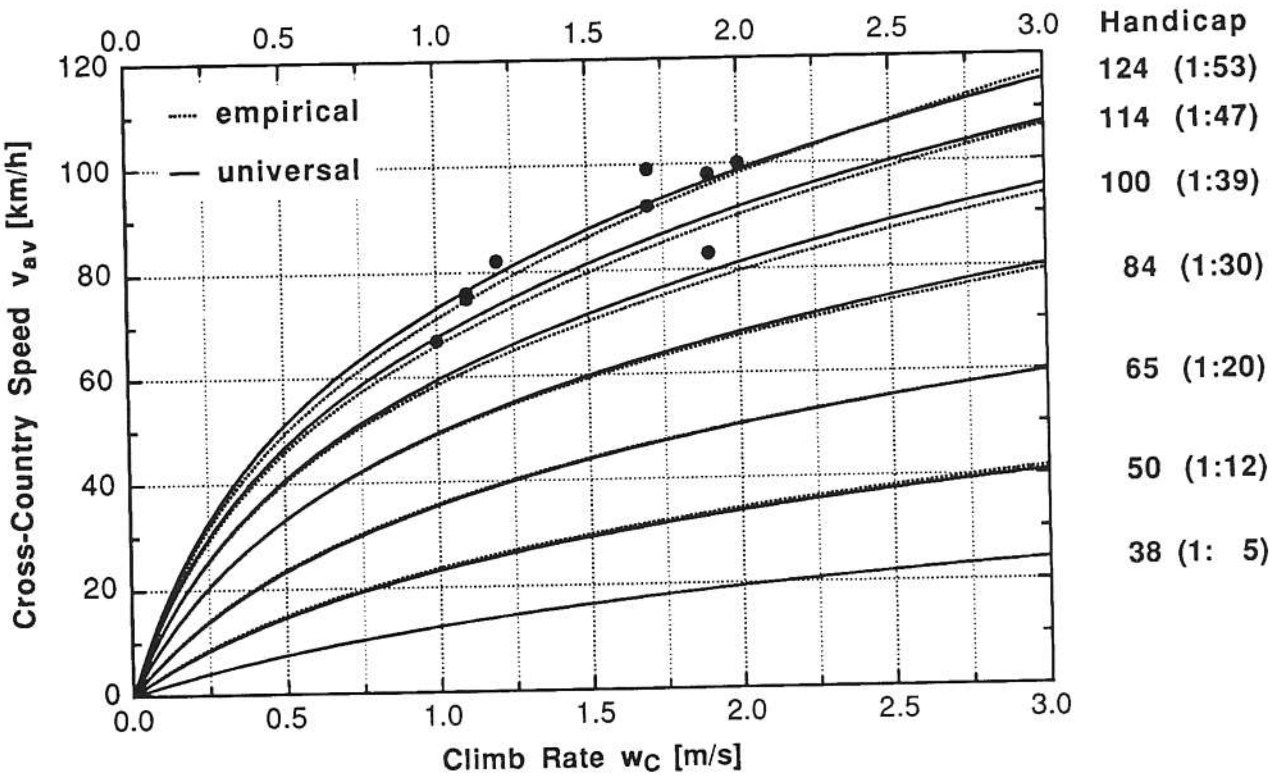

Figure 6 *Cross-country speed for different types of gliders obtained from handicap figures only. Results using* universal *and* empirical *polars fitted to glider data and speeds reached in competition flights (full circles) by a glider with handicap 124 are shown.*

meteorological products or when thinking about instruments that indicate current cross-country speed [6] during climbing and gliding.

CONCLUSION

Handicap figures can be used for calculating cross-country speeds with reasonable accuracy for all gliders. Use of either *universal* or *empirical* polars is possible. *Universal* polars require only two parameters, *empirical* polars one more. Cross-country speed is obtained analytically from *empirical* polars and numerically from *universal* polars. Machines may be reduced to their handicap figure for modern flight planning, be it gliders, hanggliders, paragliders, or eventually even migrating soaring birds.

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