

# A Metric for Determining the Efficiency of Gust Energy Extraction

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## Abstract

**A metric for the efficiency with which an aircraft extracts energy from a vertical gust is derived using an energy analysis. The energy analysis provides the maximum possible energy available in the gust for extraction by the aircraft, and the efficiency is the percentage of that theoretical maximum extracted. The efficiency factor allows for the comparison of the gust-energy extraction potential of a wide range of aircraft configurations and gust profiles. Examination of the equation for maximum possible energy extraction also revealed ways of tailoring the design of the aircraft for increased extraction potential and the sensitivity of the maximum to various aircraft and gust parameters. Two example cases from the literature were analyzed to determine their respective energy extraction efficiencies.**

## Nomenclature

$a$	Wing three-dimensional lift curve slope
$AR$	Wing aspect ratio
$b$	Wing span
$\Delta D$	Change in drag through the gust
$e$	Span efficiency
$\Delta E$	Change in energy through the gust
$\Delta E_{sym}$	Change in energy through a symmetric gust
$L_G$	Gust length
$m_t$	Total aircraft mass
$q$	Dynamic pressure
$S$	Wing area
$T$	Induced thrust due to gust
$U_0$	Aircraft velocity before gust
$U_f$	Aircraft velocity after gust
$U_\infty$	Freestream velocity
$W$	Work performed by gust
$w_g$	Gust amplitude
$z_0$	Aircraft altitude before gust
$z_f$	Aircraft altitude after gust
$\Delta z_e$	Change in energy altitude
$\alpha$	Angle of attack
$\eta$	Gust energy extraction efficiency

## Introduction

Operation of aircraft within atmospheric gusts has been shown to provide noticeable performance benefits such as re-

duced drag and, in the case of high-performance sailplanes, increased cross-country speed [1, 2]. As a result, there is interest in design of aircraft for gust energy harvesting using both active [3–7] and passive approaches [2, 8–12]. The performance improvement of an aircraft due to interaction with a gust may be quantified in terms of the energy gained by the aircraft translated to a notional change in altitude, referred to as “energy altitude” [2, 13, 14].

Prior to further discussion on the energy gains possible, it is worthwhile to expand on what is meant by atmospheric gusts in general and in the context of this study. Atmospheric gusts can be generally defined as random wind-speed variations in all three dimensions. Although the most realistic gust modeling comes in the form of the Dryden and von Kármán spectral functions in three-dimensions [15], this work focuses on the influence of vertical gusts and uses simplified profiles (e.g. sinusoidal gusts) as examples. The influence of streamwise and spanwise variations in velocity are neglected. Still, the formulation and results of the work are applicable with respect to vertical air-mass motion, regardless of the gust model used. Finally, gusts are assumed to have a maximum magnitude of 15% or less. This assumption covers most of the gust magnitudes of interest (e.g. at a freestream of 40 m/s, this represents a gust of 6 m/s) while allowing for the application of small angle assumptions (e.g.  $\tan(\Delta\alpha) \sim \Delta\alpha$ ) to simplify the forthcoming derivation and analysis.

Previous literature [1–14] has quantified the energy gains of individual aircraft using methods of varying fidelity. In contrast,

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the objective of this study is to answer the questions:

1. How does one quantify the efficiency with which an aircraft extracts energy from a vertical gust?
2. What is the theoretical maximum energy that can be extracted by an aircraft from a vertical gust?
3. How does the theoretical maximum energy gain depend on the geometry of the airplane and details of the gust?

For this purpose, a gust efficiency factor is proposed that allows the comparison of the gust-energy extraction of different configurations. This gust efficiency factor is comparable to the span efficiency factors that is often used to evaluate the efficiency of a wing planform.

### Derivation of Efficiency

The physical mechanism for the transfer of energy from an atmospheric gust to an aircraft can be explained in terms of the basic aerodynamic quantities of lift, drag, and thrust. Consider an aircraft traversing through a vertical gust for example. The relative velocities of the air with respect to the wing are shown in Fig. 1, where  $U_0$  is the airspeed of the linear trajectory through the gust in the aircraft-fixed reference frame and  $w_g$  represents the gust velocity that is assumed to be perpendicular with respect to the flight path.  $U_0$  can be inclined to represent gliding flight but is made horizontal for simplicity. The sum of the gust velocity and flight path velocity results in the effective velocity ( $U_{eff}$ ) that the aircraft experiences. The upwards gust tilts the lift vector forward as shown in Fig. 1a, creating a component of the lift vector that acts parallel to the flight direction. This lift component in the aircraft frame acts as a thrust term and performs positive work on the aircraft, adding to the overall energy state of the aircraft. For a downwards gust, the lift vector is tilted backwards

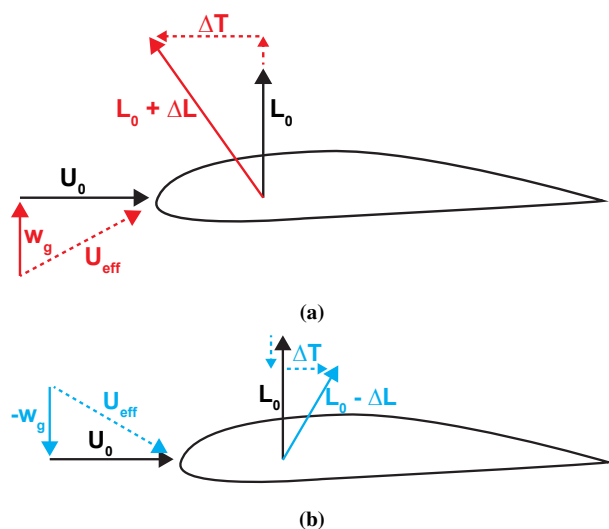


Fig. 1: Gust energy extraction mechanisms due to an (a) upward and (b) downward gust.

as shown in Fig. 1b, acting as an additional drag, and reducing the energy state of the aircraft. These mechanisms were identified and quantified experimentally by Katzmayr [16] and Jones et al. [17] in two-dimensions and numerically by Melville [14].

The total energy gain from a gust encounter can be characterized by the change in the energy altitude of the aircraft. The energy altitude of the aircraft,  $z_e$ , is derived from the total energy state of the aircraft:

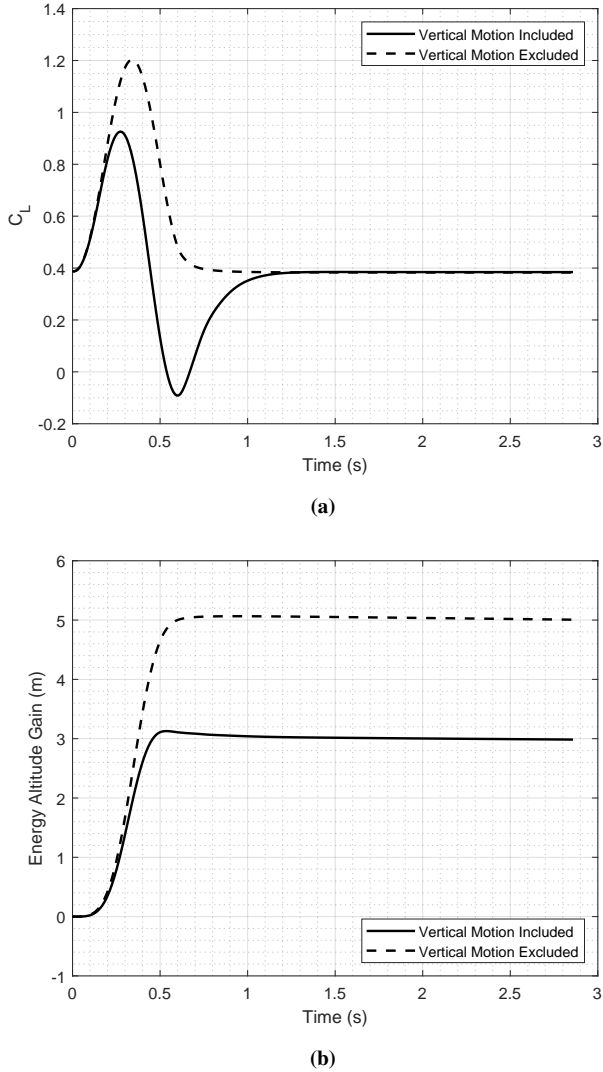
$$E_{tot} = \frac{1}{2}mU^2 + mgz = mgz_e \quad (1)$$

The change in energy altitude is thus, the change in total kinetic and potential energy per unit weight of the aircraft from the beginning to end of the gust, given by:

$$\Delta z_e = \frac{1}{2g}(U_f^2 - U_0^2) + (z_f - z_0) \quad (2)$$

where  $U_0$  and  $U_f$  are the aircraft velocities at the beginning and end of the gust,  $z_0$  and  $z_f$  are the altitudes at the beginning and end of the gust, and  $g$  is the gravitational acceleration. Not surprisingly, the energy-altitude gain of Eq. 2 can vary significantly due to the gust profile, for example its maximum amplitude and wave length. Furthermore, each aircraft configuration (weight, wing area, structure, etc.) will harvest a different amount of energy based on complex interactions between aerodynamics, vehicle dynamics, and structural dynamics. As a result, it is difficult to compare designs in a meaningful way. In order to compare the effectiveness with which different aircraft designs are able to extract energy from different gust profiles, an energy extraction efficiency is introduced.

The theoretical maximum energy altitude gain by an aircraft traversing a gust occurs when 100% of the work done by the thrust component transfers into the overall energy state of the aircraft. To calculate this theoretical maximum, several assumptions are implemented. The aircraft is assumed to traverse the gust on a linear trajectory, thus vertical dynamics of the vehicle due to changes in lift through the gust are neglected. This assumption allows for the maximum extraction of energy through the gust (the idealized case) as including vertical motion of the aircraft reduces the effective angle of attack created by the gust, and thus the “thrust” applied to the aircraft. As proof of this phenomenon, a higher-order lifting line aerodynamic approach with full flight dynamics [18] was used to model a Discus-2c at 40 m/s through a 25 m long 1-cosine gust with a magnitude of 6 m/s (15% of the freestream velocity) with and without vertical motion of the aircraft. The results in terms of the lift coefficient and energy altitude gain as a function of time are provided in Fig. 2. When full vehicle dynamics are included, the effective angle of attack of the aircraft is reduced due to upward motion of the aircraft as is visible in the lift coefficients shown in Fig. 2a. Although the vertical motion does increase the potential energy of the aircraft, the increase is negligible in comparison with the losses due to the reduced effective angle of attack and thus the energy altitude gain is larger without including the vertical



**Fig. 2: Comparison of lift coefficient response (a) and energy altitude gains (b) when including/excluding vertical aircraft motion.**

motion of the aircraft as shown in Fig. 2b. To further clarify, the authors are not claiming that the effect of vertical motion of the vehicle is negligible in practice. On the contrary, the previously cited example proves that the vertical motion of the vehicle can and does have a significant influence for large enough gusts. Neglecting the vertical dynamics of the vehicle in this analysis allows for an assessment of the maximum possible energy extraction from the gust.

When considering the energy balance of the aircraft, inviscid sources of drag (including vortex-induced drag and the drag due to the gust) must be included to remain consistent, but viscous effects may be neglected. This is defensible both because in the ideal case viscous effects are negligible, and in reality they have been shown to have very limited impact on the change in

energy due to the gust [19]. The wing itself is assumed to be rigid and retain a constant value of inviscid span efficiency as it traverses the gust. While deflection of the wing within the gust may in theory increase the span efficiency of the wing slightly due to non-planar effects, it will also reduce the effective span. Inclusion of wing deflection would also add in an elastic energy term and a structural dissipation term during the gust itself. It is assumed in the following derivation that these effects when combined are of a higher-order nature and are thus neglected.

In its most general form, the work done by the gust on the aircraft is given by:

$$W = \int_0^{L_G} T(x) dx \geq m_t g \Delta z_e \quad (3)$$

where  $T(x)$  is the lift component projected onto the flight trajectory from the gust, with respect to the aircraft's position in the gust, and  $m_t$  is the aircraft mass. As indicated in Fig. 1,  $T(x)$  can take the form of either a positive or negative contribution, with a positive contribution being a net "thrust" experienced by the aircraft and a negative contribution being an increase in drag. The net-energy change from the gust is equivalent to the difference in the work done by the gust on the aircraft and the work related to the changes in aircraft drag due to the gust, which are primarily due to changes in load factor as the gust is encountered. This change in energy,  $\Delta E$ , can thus be expanded as

$$\Delta E = \int_0^{L_G} (T(x) - \Delta D(x)) dx \quad (4)$$

$\Delta D(x)$  is the difference between the drag changes through the gust,  $D_g$ , and the drag experienced in steady-state flight,  $D_0$ , which are defined by

$$D_g(x) = \frac{1}{q\epsilon\pi} \left( \frac{qSa(\alpha + \Delta\alpha)}{b} \right)^2 \quad (5)$$

$$D_0(x) = \frac{1}{q\epsilon\pi} \left( \frac{qSa\alpha}{b} \right)^2 \quad (6)$$

The thrust term is defined as

$$T(x) = qSa(\alpha + \Delta\alpha) \tan(\Delta\alpha) \quad (7)$$

where the term  $\tan(\Delta\alpha)$  projects the lift force due to the gust into the horizontal flight direction.

Assuming viscous effects are negligible and the aircraft is rigid, the thrust and drag terms may be estimated through the lift curve slope,  $a$ , of the aircraft and the angle of attack changes,  $\Delta\alpha$ , through the gust:

$$\Delta E = \int_0^{L_G} qSa(\alpha + \Delta\alpha) \tan(\Delta\alpha) - \left( \left( \frac{qSa(\alpha + \Delta\alpha)}{b} \right)^2 - \left( \frac{qSa\alpha}{b} \right)^2 \right) \frac{1}{q\epsilon\pi} dx \quad (8)$$

where  $q$  is the dynamic pressure,  $S$  is the reference area,  $e$  is the span efficiency, and  $b$  is the wingspan. For a sinusoidal gust of length  $L_G$  and magnitude  $w_g$ , the angle of attack changes, ignoring secondary effects such as wake induction and apparent mass effects, with respect to the aircraft's location in the gust,  $x$ ,  $\Delta\alpha$  takes the following form:

$$\Delta\alpha(x) = \frac{w_g \sin(2\pi x \frac{L_G}{U_0})}{U_0} \quad (9)$$

The first term of the integrand in Eq. 8 represents the projection of the lift force due to the gust into the thrust direction; the second term is an estimation of the induced drag while passing through the gust. The third term is the steady-state vortex-induced drag that the aircraft would experience over the same length without encountering the gust. Using a small angle approximation for the change in angle of attack due to the gust and grouping like terms, Eq. 8, can be simplified to the following:

$$\Delta E = \int_0^{L_G} qSa \left( \alpha\Delta\alpha \left( 1 - \frac{2a}{\pi AR e} \right) + (\Delta\alpha)^2 \left( 1 - \frac{a}{\pi AR e} \right) \right) dx \quad (10)$$

In this formulation, the first term represents the first order response with respect to  $\Delta\alpha$ . This term's net contribution is zero in the case of a wing encountering a symmetric gust (e.g. a sinusoidal gust) where there are equal regions of upward and downward air mass motions. It should be noted, however, that gusts are often asymmetric in real world applications (e.g. thermals). The integral of the second term (the  $(\Delta\alpha)^2$  term) is non-zero, even for a symmetric gust.

It is interesting to consider further the special case of a symmetric gust. In this case, the available energy due to the gust is given by:

$$\Delta E_{sym} = \int_0^{L_G} qSa (\Delta\alpha)^2 \left( 1 - \frac{a}{\pi AR e} \right) dx \quad (11)$$

This result is independent of angle of attack ( $\alpha$ ), which is useful from a modeling and experimental perspective. The energy potential of a symmetric airfoil at zero angle of attack traversing a symmetric gust can be assumed to be identical to that of a cambered airfoil at a steady-level-flight angle of attack, with the former being significantly easier to model and test. Within the bracketed term, the thrust force is represented by the "1" and the induced-drag penalty due to load-factor changes is represented by the fraction. While the thrust term is usually taken into account, the induced drag penalty is often overlooked. This term is important, however, because it indicates that if the aspect ratio is too low, the energy extraction capability is reduced significantly. For example, an elliptically loaded wing extracts 30% less energy from a symmetric gust with an aspect ratio of 5 due to this induced drag penalty. Even with an aspect ratio of 20 this penalty reduces the energy available for harvesting by approximately 10%.

Based on Eq. 10 and Eq. 11, the following recommendations can be made to maximize the overall energy exploitation in a gust through improving wing aerodynamics:

1. Reduce span loading ( $\frac{qSa\alpha}{b}$  term in Eq. 8)
2. Increase span efficiency ( $e$ )
3. Increase wing area ( $S$ )
4. Increase aspect ratio ( $AR$ ) to increase the lift-curve slope ( $a$ )

Independent from the design of the wing, a steeper gust gradient (increased  $\Delta\alpha$ ) also results in greater energy gains.

Recall that Equation 10 is an estimation of the maximum energy that can be extracted from a gust by an aircraft. The energy gain is in reference to the energy that would be lost due to the drag that the aircraft would experience in a steady-level flight over the same distance. This value can be converted into an ideal energy altitude as

$$\Delta z_{e,ideal} = \frac{\Delta E}{m_t g} \quad (12)$$

The efficiency of the aircraft with respect to gust harvesting is then given by:

$$\eta = \frac{\Delta z_e}{\Delta z_{e,ideal}} \quad (13)$$

The efficiency factor of Eq. 13 allows the comparison of efficiency of harvesting gust energy independently of aircraft configuration and gust profile. In essence, the ideal energy altitude gain computed in Eq. 12 provides a fixed benchmark for comparing different configurations and assessing their gust-energy extraction capabilities. The efficiency term from Eq. 13 then can be used in a similar vein to the span efficiency used for induced drag calculations, which is based on an idealized elliptical lift distribution. The application of the efficiency term is particularly important when trying to assess the effectiveness of different aircraft. The following section demonstrates this ability using the example of two different sailplane designs.

## Example Applications

To provide context and examples of application of the described approach, the efficiency of gust energy extraction of two sample cases from the literature were determined. Mai [2] estimated the energy gain from a rigid PIK-20 glider, whose geometry is described in Table 1, traversing a 50 m, 2 m/s 1-cosine gust, while flying at 40 m/s using a vehicle dynamics model and aerodynamics based on strip theory. The results from the comparison of the PIK-20 ideal and estimated energy altitude gains are summarized in Table 2. The maximum ideal energy altitude gain for this case is 1.75 m, while Mai estimates an energy altitude gain of 1.25 m, resulting in an efficiency of 71.4%. In a second case, Mai estimated the energy gain from a rigid ALCOR sailplane traversing the same gust at the same speed. The

Aircraft	Wingspan (m)	Planform Area (m <sup>2</sup> )	Aspect Ratio	Mass (kg)
ALCOR	20	14.3	28	350
PIK-20	15	10	22.5	350

Table 1: Summary of aircraft parameters.

Aircraft	Ideal Energy Altitude Gain (m)	Estimated Energy Altitude Gain (m)	Efficiency
ALCOR	2.13	1.25	58.6%
PIK-20	1.75	1.25	71.4%

Table 2: Ideal and estimated energy altitude gains for the PIK-20 and ALCOR sailplanes.

results for this configuration are also listed in Table 2. With the ALCOR aircraft parameters, described in Table 1, the maximum ideal energy gain is 2.13 m resulting in an efficiency of 58.6%. Thus, the PIK-20 aircraft is more efficient at extracting the available energy from the defined gust in this case. In order to understand why, it is helpful to consider the sensitivity of the maximum ideal energy to aircraft and gust parameters.

### Sensitivity to Aircraft and Gust Parameters

Because Eq. 10 is straightforward to evaluate for a given aircraft and gust profile, it is possible to rapidly explore the sensitivity of the ideal energy altitude gain to various aircraft and gust parameters. Two such explorations are provided in terms of aircraft parameters in Fig. 3 and gust parameters in Fig. 4. These sweeps are based on a notional Discus-2c aircraft (18 m span, 11.36 m<sup>2</sup> wing area, 440 kg mass). The baseline conditions for the provided sweeps are an aircraft velocity of 40 m/s, wing loading of 38.7 kg/m<sup>2</sup>, a vertical gust magnitude of 1 m/s, and a gust length of 50 m.

The parameters used to define the aircraft and its operation influence the ideal maximum energy altitude in an intuitive manner, as shown in Fig. 3. The variation of energy altitude gain with aircraft forward velocity is shown in Fig. 3a. The length of the gust is assumed constant in this sweep, thus for the 1-cosine gust, the faster the aircraft traverses the gust, the less energy it is able to harvest. Alternatively, the sine gust has equal regions of gain and loss. The magnitude of the gain due to the thrust component in the upward gust is larger than the magnitude of the loss due to the downward gust, resulting in a net overall gain that is independent of velocity. The variation of energy altitude gain with aircraft wing loading is shown in Fig. 3b. These plots indicate that for both gust profiles, greater energy can be harvested with lower wing loading. If the energy in the gust is assumed to be independent of the aircraft, a lighter wing-loading aircraft has more energy to gain, in agreement with the findings by Phillips [1].

This trend is what explains the difference in efficiencies between the PIK-20 and ALCOR sailplanes. The PIK-20 wing loading is 35 kg/m<sup>2</sup> while the ALCOR is 24.5 kg/m<sup>2</sup>. Thus, the maximum ideal energy available to be extracted with the ALCOR is higher than that of the PIK-20. If the two aircraft are predicted to extract the same amount of energy, the PIK-20 must

be more efficient.

The parameters used to define the gust also influence the ideal maximum energy altitude in an intuitive manner, as shown in Fig. 3. The stronger the vertical gust, the more energy available to be harvested, as shown in Fig. 4a. This relationship is

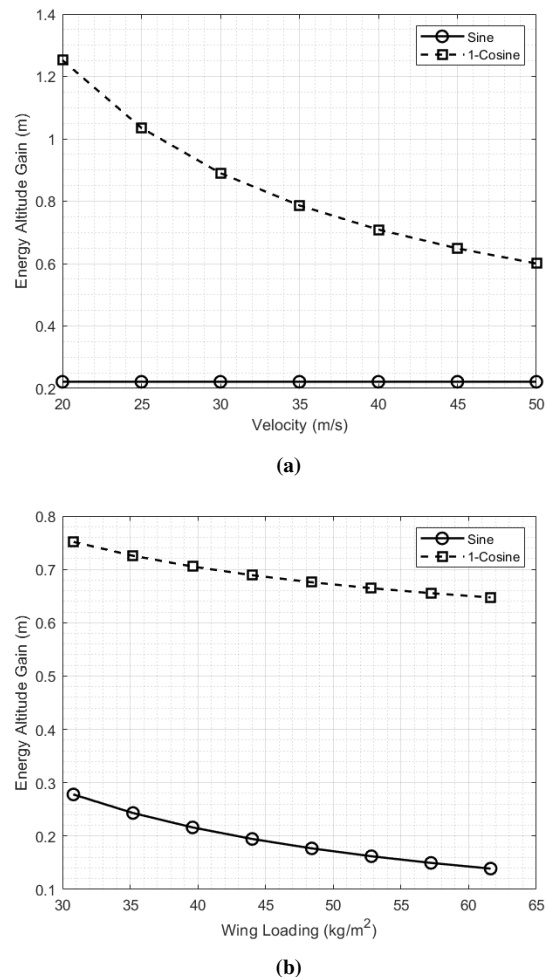
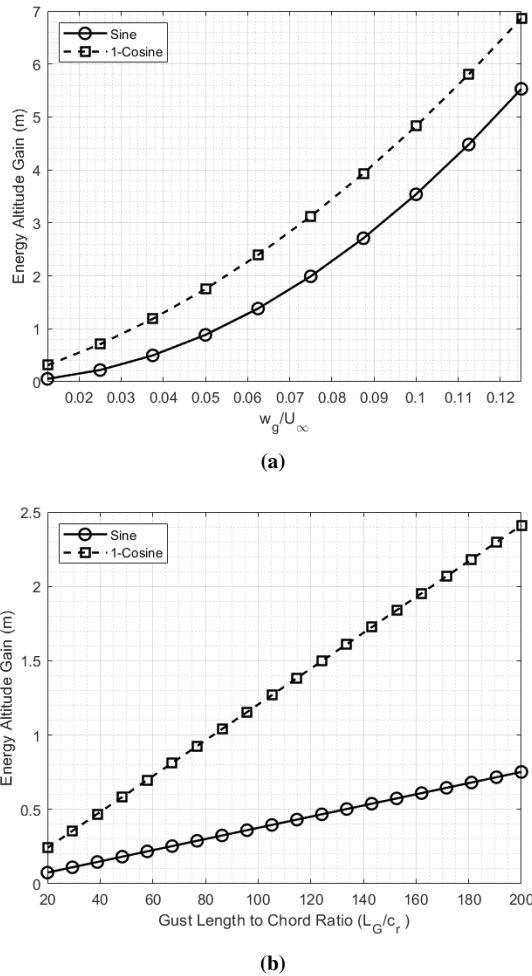


Fig. 3: Variation in altitude energy available as a function of (a) aircraft velocity and (b) aircraft wing loading varied by weight.



**Fig. 4: Variation in altitude energy available as a function of (a) vertical gust magnitude and (b) length of gust.**

non-linear, however, and so significantly more energy is available with increasing magnitude. Relating this specific case back to the discussion regarding Eq. 10, increasing the magnitude of the gust at a set velocity increases the velocity gradient. Alternatively, the influence of the length of the gust is fairly linear, as shown in Fig. 4b. This is akin to a variation in frequency of the gust, with lower frequency gusts occurring at longer gust lengths. Comparison of these two results indicates that in this case, amplitude has a more significant impact on the resulting energy gain. In general, high amplitude high wavelength gusts provide the most energy.

### Conclusion

A metric for the efficiency of aircraft gust energy extraction has been derived through determination of the theoretical maximum energy available in a gust. Based on this equation it was found that for a symmetric gust, induced drag penalties due to gust-induced changes in load factor are non-negligible (e.g.

on the order of 30% for a wing with an aspect ratio of 5 and nearly 10% for an aspect ratio of 20). Additional recommendations were made to maximize the energy extraction potential of a wing. The maximum energy then was used to determine the efficiency of gust energy extraction for a given wing and gust. This efficiency was calculated for two example cases found in the literature and the results were reasonable in magnitude. The maximum energy metric then was used to explore the influence of two aircraft parameters and two gust parameters to determine their influence on the energy available for extraction.

The presented metric operates on the simplifying assumptions that the aircraft maintains a horizontal flight trajectory through the gust and that changes in lift due to the gust do not alter the vertical dynamics of the aircraft. While in practice changes in lift will create vertical accelerations of the wing and fuselage, their vertical motion will only act to alleviate the effects of the gust, reducing the effective angle of attack on the aircraft. Thus, the horizontal flight trajectory represents the most ideal case for maximizing the energy extraction. The metric also assumes the aircraft span efficiency remains constant throughout the gust. This is an appropriate assumption for sufficiently stiff wing structures; however, it is not valid for more flexible geometries. Flexible geometries may be able to achieve higher span efficiencies as the wing deforms through the length of the gust. Nevertheless, the constant span efficiency provides a conservative estimate and is sufficient for the purposes of computing a gust energy gain to act as a benchmark for comparison.

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