Wind Energy Extraction by Birds and Flight Vehicles

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Abstract

When a bird or flight vehicle is oriented with a component of its lift vector aligned with the natural wind work is done on the flight system. Consequently, by suitable maneuvers, variations in wind speed can be used to add energy to the system. These procedures are used by albatrosses and many other birds. The equations of motion are simplified by normalizing using the minimum drag speed and integrating numerically for the control cycles involving angles of attack and bank. An energy neutral cycle by which an un-powered vehicle returns to initial velocity and height depends only upon the maximum lift/drag ratio of the vehicle and the wind speed variation. The minimum speed difference for a neutral energy cycle occurs for a vertical or horizontal step in wind speed. For a continuous wind profile a variational method is used to find the minimum gradient for a neutral energy cycle. Simple expressions are derived for the minimum wind variations for these two cases. The oceanic boundary layer and the shear layer downwind of a ridge are studied, and neutral energy wind criteria derived for them. Birds and small UAVs, with flight speeds comparable to atmospheric wind variations, can profit from wind energy extraction.

Nomenclature

a,b	=	constants for fixed bank trajectory
D'	=	drag
D^*_0	=	profile drag at cruise speed
D^*_i	=	induced drag at cruise speed
$\mathbf{d}$	=	unit vector in direction of relative wind
F	=	normalized distance along flight path
G	=	maximum value of L/D
g	=	acceleration due to gravity
H	=	normalized height above reference
H_{max}	=	maximum circuit height
H_o	=	height of wind shadow in lee of ridge
H_r	=	reference height for ridge wind flow
$\mathbf{i}$	=	unit vector in horizontal direction
$\mathbf{k}$	=	unit vector in vertical direction
L	=	ratio of lift coefficient to that for maximum L/D
L'	=	lift
L/D	=	lift to drag ratio
$\mathbf{l}$	=	unit vector in direction of lift
M	=	vehicle mass
N	=	normal acceleration, normalized
Q	=	dynamic pressure normalized to that for max. L/D
Q_o	=	initial value of Q
Q_F	=	value of Q at F
Q_β	=	value of Q at β
q^*	=	dynamic pressure at cruise speed.
S	=	normalized wind shear gradient
V	=	inertial speed
$\mathbf{V}$	=	normalized inertial velocity vector
V^*	=	cruise speed

U,V,W	=	normalized components of velocity in inertial space
$\mathbf{v}$	=	velocity vector
W^*	=	normalized wind speed
W_o	=	wind speed at reference height
W_{max}	=	wind speed at maximum circuit height
X	=	normalized horizontal distance in U direction
Y	=	normalized horizontal distance in V direction
ΔW	=	change in wind speed
ΔV_D	=	change in flight speed due to drag
α	=	inclination of drag force to the horizontal
χ	=	azimuthal angle
ϕ	=	angle of bank about axis parallel to wind

Fundamentals of energy extraction and dissipation

An unpowered vehicle (bird or sailplane) operating with an inertial speed, V, and height, h, with respect to some ground reference is considered. In zero wind conditions it experiences an airspeed equal and opposite to V. Its total energy per unit mass with respect to the ground reference can be expressed as an equivalent height, H_e, defined as $h + V^2/2g$. It can climb, turn, dive and execute maneuvers, although these will be performed at the expense of total energy, H_e. Drag acts in the direction of, and in opposition to, the motion, while lift is normal to the motion. As a result, drag always does work on the vehicle while lift forces cannot. Gravitational forces perform conservative work on the vehicle, so that if the potential energy, h, is included in the total energy then H_e is always reduced by drag forces, or dissipated, regardless of the vehicle motion, while lift forces cannot change H_e. This is a fundamental consideration.

If the vehicle operates in a wind field so that the airmass has a speed relative to the ground reference, then the aerodynamic forces are a function of the local airspeed, the vector sum of the inertial speed and the local wind speed. The vehicle response is a function only of the aerodynamic forces but changes only the inertial speed. It is now possible for the wind to do work on the vehicle, increasing its total energy, H_e, as described below.

For the wings level case of climb into wind, the inclination of the relative wind to the horizontal is shallower than the actual climb angle. As a consequence, the lift is not orthogonal to the flight path, but inclined forwards, so it does work on the vehicle, increasing its energy. For a downwind dive the same mechanism does work on the vehicle and again adds to its energy.

A similar situation occurs in level flight during a banked downwind turn ("downwind turn" is defined as one in which flight direction changes from upwind to downwind). The lift vector is inclined towards the center of the flight path, and a component of the wind also acts in this direction, so does work on the vehicle.

In the case of a level upwind turn, however, negative work is done on the vehicle. So, in the course of a general maneuver cycle, consisting of an upwind climb, downwind turn, downwind dive and an upwind turn to complete the cycle work is done by the wind during the first three legs of the trajectory. A simple rule is: "belly to the breeze", indicating that any orientation of the vehicle in which the lift vector is inclined in the direction the wind is blowing will enable the wind to do work on the vehicle.

In order to extract energy from the wind in a circuit the vehicle must fly in regions of different wind speed during this cycle. A wind shear profile with wind speed increasing with height is not necessary, although this is what usually occurs in the earth boundary layer.

The simplest manifestation of wind variation for dynamic soaring is that of an infinite gradient, a horizontal shear layer above (or below) the vehicle so that the vehicle can enter the high-speed flow with negligible change in height. To exploit this, the vehicle enters the wind flow from the calm, flying upwind, executes a 180^0 downwind turn in the wind, drops out of the wind back into the calm and then makes an upwind 180^0 to return to the start with no significant change in height. This maneuver can be flown by a vehicle operating under (or above) a shear layer of very small thickness so that by small changes in height it can immerse itself in either stream. This is the most efficient way of extracting energy because no energy is lost in climbing or diving to seek out the energetic wind flow. The only parameters involved in the flight mechanics are the drag and ΔW, the wind speed step.

The energy increase is estimated as follows. The vehicle, with an initial inertial speed of V in calm air enters a head wind of speed ΔW. The airspeed now becomes $V+\Delta W$. The vehicle completes a 180^0 level turn to finish flying downwind at an airspeed of $V+\Delta W - \Delta V_D$, where ΔV_D is the speed loss due to drag. It re-enters the calm with an airspeed and inertial

speed of $V+2\Delta W-\Delta V_D$. It entered the headwind flow with an inertial speed of V, and departed with an inertial speed of $V + 2\Delta W- \Delta V_D$. The difference in kinetic energy between these states is the work done on the vehicle by the wind minus that due to drag dissipation. For the return leg of the upwind turn, the loss is again approximately ΔV_D so that the final speed change in the cycle is $2(\Delta W-\Delta V_D)$, illustrating that the work done by the wind must balance that lost in drag for no energy change in the cycle.

Another energy extraction case due to flow speed differences occurs when there is a region of higher speed flow horizontally adjacent to the calm with a thin shear layer in the vertical plane separating the airmasses. This is a vertical shear layer, such as occurs downwind of the sides of an obstacle at the edge of the wake. In this case the vehicle flying into the wind from the wind shadow, drifts laterally to enter the headwind, climbs, gaining energy and then moves laterally back into the adjacent calm where it can execute 360^0 turn and dive to return to the original state with increased energy.

A common natural flow state is that of the planetary boundary layer (PBL), where wind speed increases with height. In order to enter the energetic higher speed flow, the vehicle must climb, which involves penalties in increased drag and length of flight path. The simplest model is to postulate a linear wind profile with a uniform shear as the driving parameter and the maximum lift/drag ratio of the vehicle as the energy loss parameter and to determine the minimum shear for a given G. The least restrictive cycle constraint is to require only that the original velocity vector and height are regained. Normally the vehicle will not return to its inertial space starting point at the end of a cycle. In this case, because the cycle is not closed, we refer to it as a "loop".

Normalization of equations of motion

An unpowered vehicle is subject to aerodynamic and gravitational forces. The aerodynamic forces are a function of the angle of attack and airspeed. The response following Newton's Second Law can be written:

$$Mdv/dt = D'\mathbf{d} + L'\mathbf{l} + Mg\mathbf{k} \qquad (1)$$

where M is the mass, $\mathbf{v}$ the vector velocity in an inertial frame, D' the drag parallel to the relative wind with $\mathbf{d}$ the unit vector in the direction of airspeed relative wind and L' the lift normal to the relative wind and in the plane of symmetry of the vehicle, with $\mathbf{l}$ the unit vector in this direction. The unit vector $\mathbf{k}$ is vertical.

The equations can be normalized by the cruise speed, V^*, defined as the speed at which the vehicle has its minimum glide angle, or maximum L/D. This maximum is called the glide ratio, and denoted by G. Dynamic pressure for this state is q^* and at this dynamic pressure the profile drag and induced drag are given by D^*_o, D^*_i with $D^*_o = D^*_i$. The minimum drag is given by $D^*_o + D^*_i$. The ratio of dynamic pressure to that at minimum drag, q^*, is defined by Q $(= q/q^*)$, while the

ratio of lift coefficient to that at minimum drag is given by L. Time is normalized by $V*/g$, and length by, $V*^2/g$. Dividing by weight, $W (= Mg)$, and normalizing provides for the non-dimensional vector acceleration:

$$d\mathbf{V}/dT = \{D'/W\}\mathbf{d} + \{L'/W\}\mathbf{l} + \mathbf{k} \qquad (2)$$

The drag, D', consists of profile drag, proportional to dynamic pressure, and given by $W\{D*_0 Q\}$, plus induced drag, proportional to dynamic pressure and lift coefficient squared, and given by $W\{D*_i L^2 Q\}$. The lift L', is proportional to dynamic pressure and lift coefficient and given by $W\{LQ\}$. Substituting the above provides:

$$d\mathbf{V}/dT = Q[\{(1+L^2)/2G\}\mathbf{d}+L\mathbf{l}] + \mathbf{k} \qquad (3)$$

Equation (3) defines the vehicle motion, and it is seen that the only parameter is G, the maximum lift-drag ratio. Controls are exerted by L, which is a function of angle of attack, and controlled by the elevator (or similar tail aerodynamic mechanism for birds) and ϕ, the inclination of the lift vector, controlled by ailerons or wing twist, which dictate rate of change of the bank angle. The normalized relative wind, $\mathbf{W}_r$ is given by $\mathbf{W}_r = \mathbf{V} + W\mathbf{i}$ assuming the normalized inertial speed of the wind is W and its direction is in the horizontal, $\mathbf{i}$ direction, that is orthogonal to the vertical denoted by $\mathbf{k}$. From this vector the direction and magnitude of Q can be calculated.

Generally, practical limits are imposed on Q by structural factors relating to the never-exceed speed of the vehicle (about 2.5 for modern sailplanes), on L by aerodynamic factors relating to maximum lift (about 2.0 for modern sailplanes), and on N (=LQ) by structural factors relating to wing bending (about 5 for modern sailplanes). It is likely that all these limits are lower for birds than for those of mechanical aircraft.

Fundamental Sachs solution

Sachs[2] has solved the fundamental problem of determining the minimum linear shear required for a vehicle to fly a cycle and return to the same height and speed vector, but not the same inertial location. This is called an "energy neutral" loop. A variational procedure was employed using the exact equations of motion. This produced an optimal bank and lift routine as a function of wing loading and G. This energy neutral trajectory requires extreme maneuvers – dynamic pressures vary by a factor of 16, G loads, defined by N, vary by a factor of 8, drag coefficients by about 4, while bank angles go from zero to 80°. This loop is of importance in establishing an exact lower bound for energy neutral cycle operating in a linear shear. As noted in the following paragraphs, although this loop is optimal in requiring the minimum gradient, the cycle requires a long time to execute, a lengthy flight distance, large height gain at the upwind end and a significant downwind drift for each loop.

Development of equations of motion

General equations in inertial frame

The equations derived above in vector form are re-written in scalar form in an inertial orthogonal Cartesian coordinate system fixed in the earth. The normalized inertial horizontal, lateral and vertical velocities, U, V, W, modified for various inclinations, become:

$$\frac{dU}{dT} = -\frac{1}{2G}\left(Q + N^2/Q\right)\cos a \cos \chi - N(\sin\alpha\cos\chi\cos\varphi + \sin\chi\sin\phi) \qquad (4a)$$

$$\frac{dV}{dT} = -\frac{1}{2G}\left(Q + N^2/Q\right)\cos a \sin \chi - N(\sin\alpha\sin\chi\cos\varphi - \cos\chi\sin\phi) \qquad (4b)$$

$$\frac{dW}{dT} = \frac{1}{2G}\left(Q + N^2/Q\right)\sin a - N\cos a\cos\phi + 1 \qquad (4c)$$

The non-dimensional airspeed squared, Q, is defined as:

$$Q = (U + W*)^2 + V^2 + W^2 \qquad (5)$$

where $W*$ is the normalized wind at the appropriate height.

The orientation of the vehicle is in general three dimensional and defined by three angles, α, χ and ϕ. The angle α, the inclination of the drag force to the horizontal and identical to the angle of the relative wind to the horizontal, is defined by:

$$\sin\alpha = W/\sqrt{Q} \qquad (6)$$

The azimuthal angle, χ, is given by:

$$\tan\chi = V/(U + W*) \qquad (7)$$

while the bank angle, ϕ, is defined as roll about an axis parallel to the relative wind with zero being wings level. The bank is positive in the clockwise sense viewed from the rear (right wing down) using a right hand screw rule oriented with this axis.

The inertial position, X, Y, H is defined:

$$dX/dT = U, \quad dY/dT = V, \quad dH/dT = -W \qquad (8)$$

where H represents height measured positive upwards, in the opposite direction to Z, which is taken downwards for this coordinate system. The above equations, 4a, b, c, are compact expressions of the dimensional equations used by Sachs.

The above set was solved, using fourth order Kutta-Runge, for an arbitrary lift ratio, L, and bank, ϕ, schedule. The lift and bank controls were specified at twenty equidistant time intervals on the cycle and the integration was performed using 400 equal time intervals per cycle. It is useful to compute the total energy, H_e, at each step to check the accuracy of the numerics. Setting the wind speed to zero for any time step must give a negative change in H_e

Analytic solutions

The equations above can be integrated exactly in the case of no wind for some special cases. Two particular cases are those of wings level, constant angle climb or dive and of constant bank angle, constant height turn. These are useful for checking numerical methods to determine step size in the numerical procedure for acceptable accuracy. They can also be patched together as approximations of smooth trajectories. The solutions are listed below.

Wings level, constant N

For the case of an unpowered wings level climb or dive at an angle of θ, the normal acceleration, N, is constant and given by $N = \cos\theta$. The equations of motion can be integrated analytically to provide the dynamic pressure Q_F after a distance F from the initial state of Q_0 by the expression:

$$(Q_F - a)^a / (Q_F - b)^b = (Q_0 - a)^a / (Q_0 - b)^b$$
$$e^{-(a-b)F/G} \qquad (9)$$

Here a, b are defined as the roots:

$$a, b = -G\sin\theta \pm \sqrt{\{G\sin\theta\}^2 - \cos\theta^2} \qquad (10)$$

For the level flight case (θ =0) the above expression degenerates, but using the basic equations gives the final dynamic pressure as:

$$(Q_F^2 + 1) = (Q_0^2 + 1)e^{-2F/G} \qquad (11)$$

The final dynamic pressure is a function only of flight path angle, θ, distance flown along the flight path, F, and glide ratio, G.

Constant height, constant N

For the case of an unpowered constant height turn at constant bank angle, ϕ, and constant N the equation of motion is integrated analytically to provide the dynamic pressure, Qo after a turn through the azimuth angle, β, by the expression:

$$Q_\beta = N \tan\{[\operatorname{atan}(Q_0 / N)] - \beta/[G\sin\phi]\} \qquad (12)$$

Here Q_0 is the dynamic pressure ratio at θ = 0 and Q_β the value at β. The final dynamic pressure is a function only of normal acceleration, N, turn angle, β, and glide ratio, G.

Validation by comparison with Sachs open loop optimal case

The Sachs[2] result for optimal trajectory is not a closed circuit in inertial space. As shown in Fig. 1, it is a hairpin-like open loop where the vehicle starts flying cross wind horizontally, banks into wind climbing to maximum altitude, performs downwind 180[0] turn, dives towards original height and then banks to return to crosswind flight. This figure is drawn to scale for a reduced length scale of unity, and G of 45.

The vehicle returns to its original kinematic velocity state and height, but not the original position. It covers ground in a crosswind direction and drifts downwind. The case analyzed

by Sachs for a vehicle of 35 kg/m^2 wing loading and G = 45 maximum lift to drag ratio was repeated using the present formulation. The bank angle and lift ratio schedule quoted by Sachs was used to exercise and validate the present code. Sachs defined the vehicle by wing loading and G, and does not quote cruise speed. To check the present code cruise speed was varied until the loop matched that given by Sachs. A good match was obtained for a speed of 26.8 m/s. The shear gradient for an energy neutral loop was 0.0324 per second, normalized by cruise speed. This gives the non-dimensional shear, S, of 0.089. The control schedule was that used by Sachs and is shown as Fig. 2. The "clipped" top of the L schedule is caused by Sachs imposing a C_{Lmax} limitation on the control. Principal characteristics of the loop are a circuit period of 8.9 time units, maximum speed of 3.0 speed units, a maximum bank of 68^0, a total climb of 4.5 units, a crosswind displacement of 4.5 units and a downwind drift of 1.8 units

As a consequence of the normalization, the critical shear, S, can be a function only of G, so that S = S(G). It is of interest that by using this normalization all of Sach's data, for all vehicles in all linear shear flows, can be approximately collapsed to the simple form expressed by the equation:

$$S = 4.00/G \qquad (13)$$

Sachs results are for G = 20 – 80. The above expression is a good approximation for G within this range. A more precise determination of the function S(G) could be made by using the variational method to compute S for G values below 20, and refining Eq. (13).

Open loop, G = 25, linear wind shear

Modern sailplanes have glide ratios in excess of 40. No bird performs as well. A reasonable G value for an albatross is about 25 with a cruise speed of 15 m/s. This G value is used in all following calculations so that they will be applicable for comparison to observed performance of large birds. For the open loop G = 25 case the bank and lift ratio schedule developed by Sachs for G = 45 was used. Surprisingly, even for G = 25, this gives an adequate control schedule for an energy neutral open loop at an S value of 0.16, as predicted by the above equation. The loop has a shape similar to the G = 45 case. Principal characteristics for this glide ratio, (with the G = 45 result shown in parenthesis) are a circuit period of 9.0 (8.9) time units, maximum speed of 3.0 speed units, a total climb of 4.8 (4.5) units, a crosswind displacement of 5.8 (4.5) units and a downwind drift of 3.2 (1.8).

The control schedule, shown in Fig. 3 can be a function only of G. It is, in fact, almost independent of G, as shown in numerical results quoted by Sachs for different G cases. The optimal control schedule is not very sensitive to small changes. An ad hoc sensitivity study was made to determine the precision required for the bank and lift schedule. It is found that a simple, but smooth, schedule, arbitrarily selected to consist of Fourier cosine series with only a few terms (three for

lift ratio, L, two for bank angle, φ) gives almost identical final performance. The two schedules are shown in Fig. 3.

The ground tracks for two open loop canonical cases, G = 45, 25, with a linear profile are shown in Fig. 4. The loop satisfies the energy neutral condition, but is not the optimal control routine.

Closed circuit, G = 25, linear wind shear

As has been noted, an alternative, but more demanding, objective is that the cycle should return not only to the same inertial speed and height state but also to the same inertial position. We call these closed cycles "circuits". This is a more restrictive constraint and requires a stronger shear than the simple energy neutral "loop". The circuit yields more options for navigation in any arbitrary direction. For example, the circuit will permit orbiting about a given inertial position indefinitely, while the minimal energy neutral loop would involve a steady downwind drift.

A trial and error procedure was used to define a control schedule for such a circuit. The solution indicates that a shear value of 0.17 is required. The open loop requires 0.16. Principal characteristics of the circuit are a circuit period of 8.6 time units, maximum speed of 3.0 speed units, a total climb of 4.5 units, a crosswind displacement of 0.0 units and a downwind drift of 0.0. This cycle, which provides more range access for the bird, does not require a significantly higher shear than the minimal loop.

The control schedule was first approximated by adjusting the schedule by hand and observing the result. This gave a circuit which matched the closure conditions but involved a somewhat irregular schedule. A solver code with 40 unknowns (Lift ratio and bank at 20 stations) was then constructed and a solution obtained for eight conditions (U, V, W, X, Y, H, L, φ) returning to T = 0 state. Because there are many more variables than constraints, this can be satisfied by an infinity of schedules, but the apparently smoothest schedule was selected.

It is of interest small variations in the schedule will still give a satisfactory solution, as is the property of an extremal. The schedule selected cannot be the theoretical optimal, but must be close. The final smooth schedule, called Schedule 7, is used here. The differences between it and the first 'ad hoc' solution, called Schedule 1, are shown in Fig. 5. Both schedules produce energy neutral circuits.

The fact that an acceptable energy neutral circuit can be obtained from a fairly crudely controlled schedule is of interest in dynamic soaring by birds, suggesting that extreme precision, which could not be achieved by the bird in a natural, variable wind shear, is not necessary. Temporal variations in wind profile must be particularly trying for the bird to handle. The period of a circuit is about 10 so that for a bird with cruise speed of 15 m/s the period is about 15 sec., and it becomes necessary for the bird to control its trajectory anticipating a likely wind profile that will occur about 7 1/2 seconds in the future.

Closed circuit, G = 25, in step shear

A fundamental wind shear model is that of step shear, expressed as ΔW and discussed qualitatively in the first section. The existing codes were used to exactly compute a cycle of this nature to give quantitative results. The circuit analyzed here is not optimal, but it is a possible flight path and provides an upper bound of the ΔW required. The procedure was as follows.

The optimal bank was determined for a 180^0 constant height turn as a function of entry speed. Such an optimum exists because a shallow bank will expend less energy per unit distance, due to reduced induced drag, but will require a longer distance to complete the turn. The downwind turn takes place in the wind field, starting upwind. After the 180^0 turn the bird enters the zero wind field at the same inertial speed, and selects the optimal bank for a 180 upwind turn. A short straight and level segment is required to regain the ground lost during the downwind drift, and return to the starting point. The wind speed is selected so that the final speed returns to the initial value. This process was iterated over a range of flight speeds to determine the minimum wind step, ΔW, required for a complete circuit. For G = 25, the wind step, ΔW is calculated to be 0.19. For other levels of G the wind step, ΔW, was calculated numerically using the above procedure. The results can be expressed to a good approximation by:

$$\Delta W = 4.75/G \qquad (14)$$

Ground tracks for these loop and circuit trajectories are shown in Fig. 6. Examination of this figure shows that the length of the flight path and time of cycle is much shorter for the step shear case than those for the linear shear case.

Circuits with planetary boundary layer wind profiles

Because of mixing in the lower layers of the atmosphere, the planetary boundary layer (PBL) normally has a profile in which the speed increases with height while the shear gradient reduces. The actual natural profiles vary widely, depending on heat flux, surface roughness and wind speed itself. Consequently, there is no particular characteristic profile to choose for fundamental calculations. Two possible, but not general, profiles are chosen for illustration: ridge flow and oceanic flow.

Ridge flow is the profile created by an obstacle to the flow, where the vehicle emerges from a zero speed wind shadow into a typical separated PBL. This occurs naturally when a vehicle climbs from downwind and below a ridge into a normal wind flow, or in the separated flow in the lee of an ocean wave. This is a practical approximation of the ideal step shear case already discussed. The important feature of this type of flow is that extreme wind shears occur, and the vehicle can make its return path sheltered by the wind shadow, thus paying little penalty of downwind drift or of energy loss during the upwind turn.

This wind model has recently been studied by Sachs and Mayrhofer[3]. They used a wind profile of the form:

$$W = 0, \qquad\qquad H<H_0$$
$$W = W_r\{(H-H_0)/H_r\}^m, \qquad H>H_0 \qquad (15)$$

where m = 0.2 and H_0 = 0.097 and H_r = 0.097 was selected. H_0 represents the vertical extent of the calm below the ridge in which the vehicle makes the upwind turn. For the scale used by Sachs and Mayrhofer this is about 10 m. Results are shown in Table 1. For the high performance vehicle of G = 45 it is noted that there is a small discrepancy between the results of Sachs and Mayrhofer and those of the present paper. The reason for this is unknown, but may be due to numerical round-off errors. Sachs and Mayrhofer do not provide a solution for the G = 25 case, which has been calculated by the present method and is shown in Table 1.

A possible PBL profile above the ocean for some flow states can be represented by the Sachs model with H_0 = 0. The present model was exercised for this for the case G = 25, and provided the loop having H_{max} = 0.80, with the other parameters shown in Table 2. This provides a trajectory and performance that is between that for the linear profile and the step shear, as is expected.

Maximum wind speed for basic profiles

Four types of profiles are modeled to provide basic flow cases: step shear, linear shear, ridge flow and oceanic flow. The first two are mathematically ideal cases, and the latter two represent idealizations of real flows. Energy considerations of these types are discussed below.

Energy addition always occurs through coupling with the wind speed during the downwind turn, and consequently it is the maximum wind speed that is the fundamental criterion for an energy neutral circuit. The vehicle must climb in order to reach this energy bearing flow. But drag energy is expended during the cycle so that the required magnitude of the upper level wind depends on how much height (and drag loss) must occur in reaching it.

Minimum upper level wind speed occurs for the case of a step change in speed, a vertical sharp-edged gust, ΔW. The next most favorable case, ridge flow, is similar, where the return loop occurs in calm conditions under the wind shadow of an obstruction. For a normal PBL, as experienced by oceanic birds, the situation is slightly less favorable, since part of the upwind turn is made in lower level wind. For the linear shear the most unfavorable condition occurs. These four cases have been calculated, all for G = 25 and for a neutral energy circuit. They are summarized in Table 3, which shows the height at which the wind flow starts, H_0, the maximum height, H_{max} and the maximum wind speed, W_{max}, required for an energy neutral circuit. The difference between the first two cases is not extreme. For linear shear a large height change and wind speed is required to extract the requisite energy. Oceanic flow is between these cases. Since operating conditions for birds and gliders more closely match the first

three cases, it is apparent that energy extraction from natural winds and ridge conditions requires less extreme profiles than are required using the linear model.

The wind profiles for the basic cases are shown in Fig. 7. It is seen that the cases of ridge flow and oceanic flow require quite modest wind speed changes, and small changes of height through a cycle. For example for an albatross of the characteristics specified to perform in an energy neutral cycle in an ocean boundary layer the wind speed would have to rise from zero at sea level to about 3.7 m/s at 18 m.

Conclusions

The basic dynamics of energy extraction from wind variations is described. The wind will do work on a vehicle if there is a component of the lift in the direction of the wind. For the most extreme wind gradient, a step, the required step magnitude, ΔW, is equal to the speed loss of the vehicle in a 180^0 constant height turn. The equations of motion are normalized using cruise speed (speed for minimum drag) and the acceleration of gravity. In the normalized form, the only parameter required to describe the vehicle drag is the maximum lift/drag ratio, G. The control variables are the lift coefficient and the bank angle, which, for the classical aircraft, may be selected by elevator and aileron and, in the case of birds, by their ornithic flight control system

For the case of a linear wind profile the trajectory for absolute minimum wind shear is an open loop, called an energy neutral circuit, which returns to original speed and height, but to an inertial position displaced crosswind and downwind. For this case the minimum normalized gradient, S, is given by 4.00/G, while for the step gradient the speed change, ΔW, is given by 4.75/G.

By testing various control schedules, it is found that the exact optimal schedule for minimum wind shear, as developed by Sachs, need only be approximated.

Other wind flows are studied, including that of a closed circuit where the vehicle returns to its original height and speed and inertial position. This requires a slightly higher gradient than the open loop. Other wind flow cases involve the energy neutral loop in a boundary layer-like profile typical of an oceanic flow. Another is characteristic of the flow downstream of a ridge. In terms of the maximum wind required for a energy neutral cycle the lowest wind speed case is the step, followed by the ridge, followed by the oceanic and finally by the linear shear, which requires about four times more wind speed variation than the step case.

The modest wind differentials and relatively crude control schedules required for the cases of ridge-like flow and oceanic flow suggest that these loops and circuits are a fairly simple maneuver routine for birds to use in extracting energy from the wind, and that natural wind flows of sufficient speed differential are ubiquitous.

Results from the above conclusions are summarized below:

1. Fundamentals of wind energy extraction by flight vehicles are described.

2. The equations may be normalized in terms of only maximum L/D, defined as G, and cruise speed, V*.

3. Minimum wind variation for energy neutral cycles, a vertical step in speed, ΔW, is given by: $\Delta W = 4.75/G$.

4. Minimum linear shear gradient for energy neutral cycles, S, is given by: $S = 4.00/G$

5. Solutions for natural wind profiles characteristic of oceanic and ridge flows are given.

6. Optimal control schedules have been developed, but performance is not sensitive to exact adherence to these schedules.

7. Oceanic wind profiles require quite modest wind speeds for birds with performance characteristics of the albatross, and these winds are common over the oceans.

References

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Table 1
Speed parameters for ridge flow

G	H_o	W_r	W_{max}	Source	Profile
45	0.097	0.097	0.114	Sachs & M	Ridge
45	0.097	0.097	0.104	Present	Ridge
25	0.097	0.133	0.198	Present	Ridge

Table 2
Speed parameters for oceanic flow

G	H_o	W_r	W_{max}	Source	Profile
25	0.000	0.163	0.249	Present	Oceanic

Table 3
Maximum wind speed and height for basic profiles

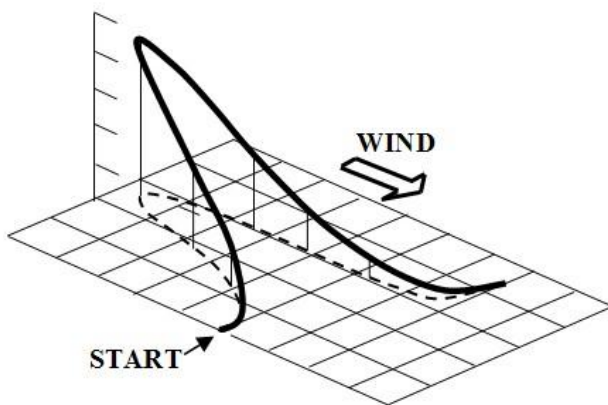| G	H_o	H_{max}	W_{max}	Profile
25	0.000	0.097	0.167	Step Shear
25	0.097	0.810	0.198	Ridge Wind
25	0.000	0.800	0.249	Oceanic Wind
25	0.000	4.500	0.757	Linear Shear

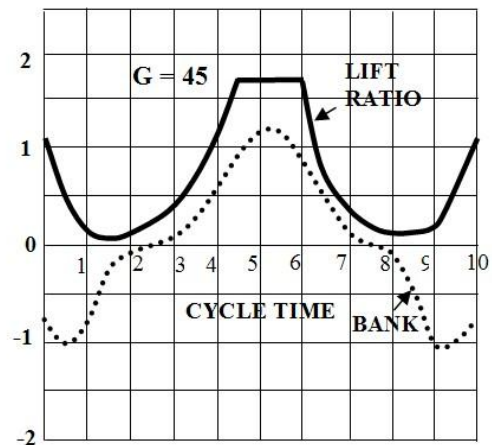**Figure 1** Typical neutral energy loop for linear wind shear.

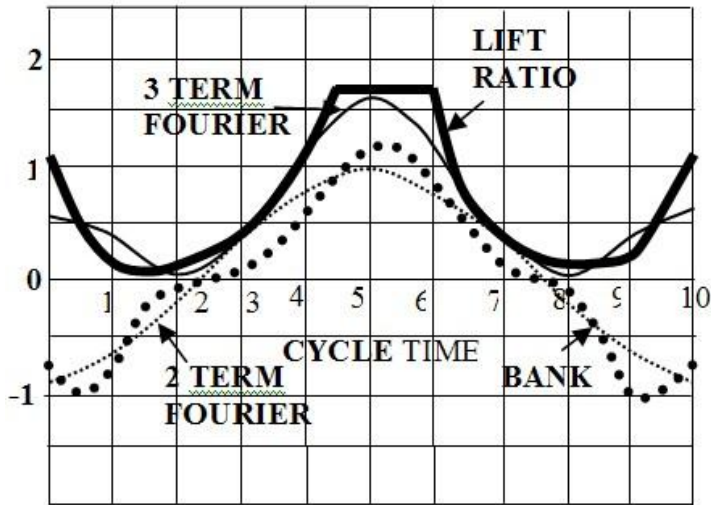**Figure 2** Control schedule developed by Sachs.

Figure 3 Optimal control schedule compared with simplified Fourier schedule.
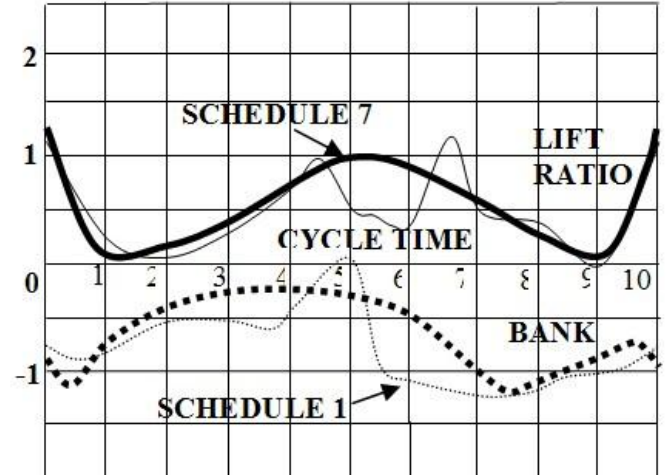

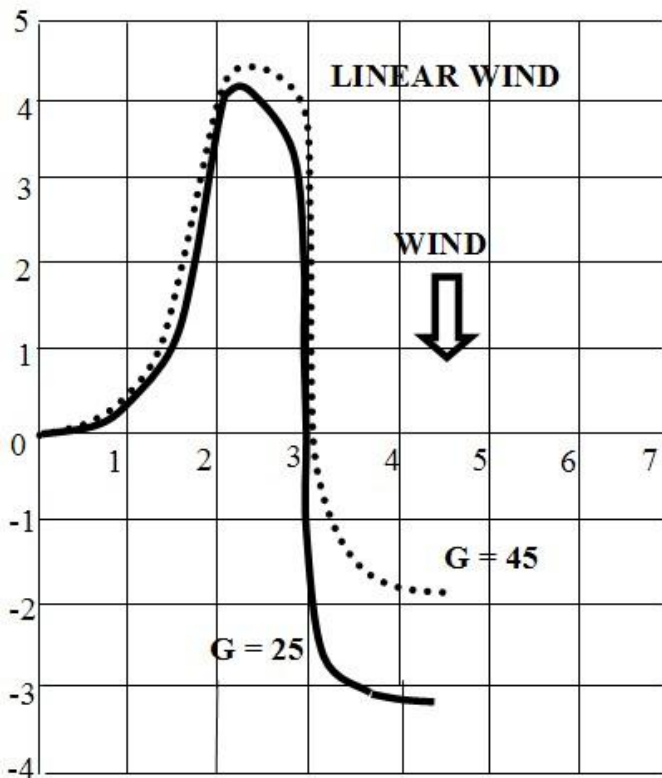**Figure 5** Comparison of control schedules for two energy neutral circuits.

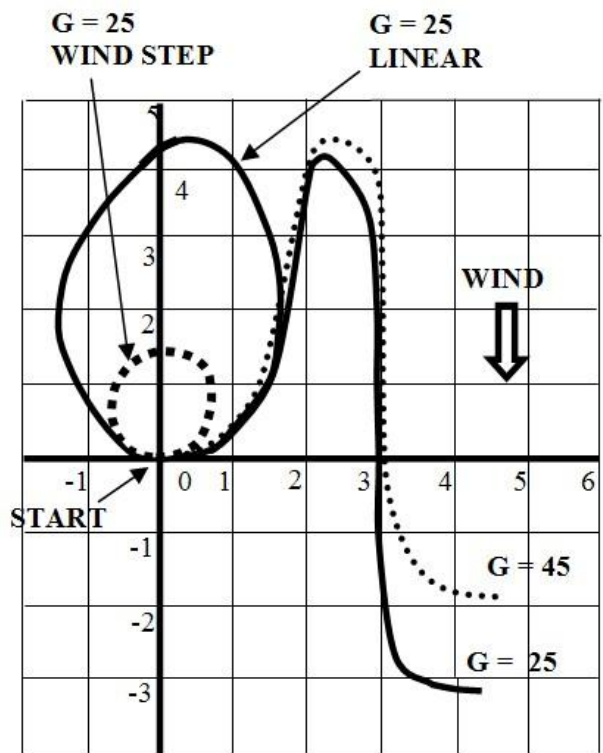**Figure 4** Ground tracks for open loop energy neutral trajectories.
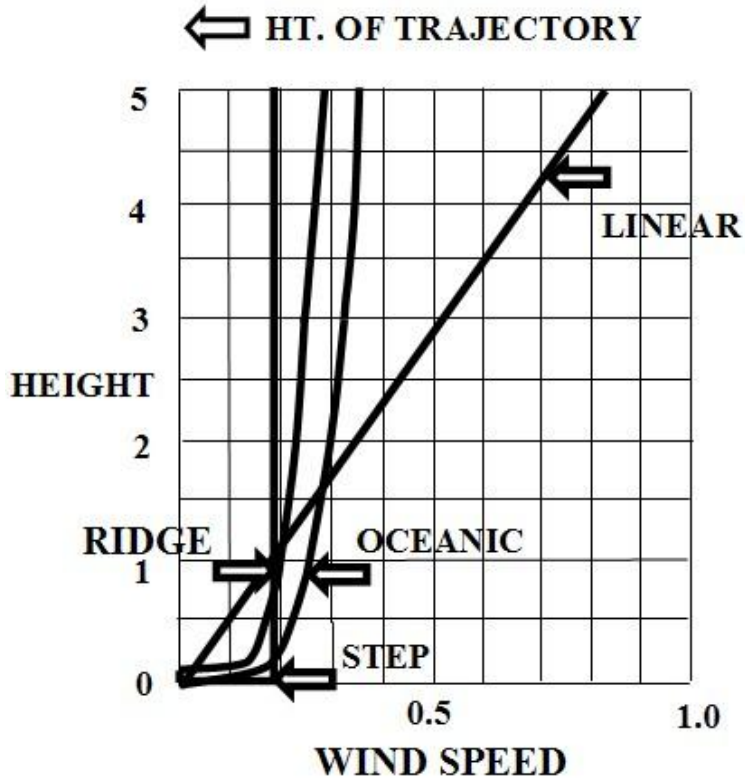

Figure 6 Ground tracks for fundamental open and closed trajectories.

Figure 7 Wind profiles for basic cases.