# Total Energy – Part 1: Exchange of Energy between a Glider and the Atmosphere

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## Abstract

The Total Energy variometer of a glider takes into account the dynamic energy exchanges with the atmosphere. Unfortunately glider pilots take them erroneously as resulting from an effective vertical speed of the air. Furthermore the present day TE variometers measure the speed, and accordingly the kinetic energy of the gliders, in relation to the surrounding air, and not in relation to a constant speed inertial referential as they should do. Consequently, their indications of dynamic energy exchanges are gathered together and delayed. As a result, both effects mislead the accurate information necessary for an optimal location of thermals. Using modern gyrolasers, accelerometers and GPS techniques, it is now affordable to improve our TE variometers.

## Nomenclature

| | |
|---|---|
| $\vec{z}$ | Vertical unit, vector positive upwards |
| $\Delta H_{dyn}$ | Equivalent dynamic altitude |
| $\vec{V}$ | Glider inertial speed vector |
| $\overrightarrow{V_{air}}$ | Glider airspeed vector |
| $\vec{W}$ | Inertial wind vector |
| $W_z$ | Vertical inertial wind component |
| $v_{zt}$ | Glider's total vertical speed |
| $v_z$ | Glider's vertical speed representing variation in potential energy |
| $v_{zk}$ | Glider's vertical speed representing variation in kinetic energy |
| $v_{zdyn}$ | Glider's dynamic equivalent vertical speed |
| $v$ | Projection of the inertial speed on the direction of the inertial wind. |
| $\vec{\Gamma}$ | Acceleration vector of the glider |
| $\vec{\gamma}$ | Relative acceleration vector of the glider |
| $g$ | Acceleration of gravity |
| $m$ | Glider mass |
| $\vec{R}$ | Aerodynamic force vector |

## Introduction

This paper is a revision of my 1981 OSTIV Congress lecture, entitled 'Unsuitability of present variometers for dynamic soaring'[1]. It is divided in two parts. The first part is this paper. The second part follows this paper.

Total energy of a glider is the sum of its potential energy, proportional to its height, and its kinetic energy, proportional to the square of its speed. The definition of its height is clear, but in order to define its speed, the reference to be taken into account needs to be known.

Because the kinetic energy has to be independent of the heading of the glider even when there is some wind, the reference determining the speed must move at the horizontal speed of the air mass surrounding the glider.

But this air mass is made up of many particles whose speed vectors are different from each other, and we have to determine only one speed vector, valid for all the particles of this air mass. Fortunately, in accordance with the mechanical law of momentum conservation, the weighted average speed vector of air particles within a given air mass is constant, as well as consequently its weighted average horizontal speed vector, (the influence of the glider can be considered as negligible)

*Therefore, we choose this constant weighted average horizontal speed vector of the air mass surrounding the glider as being the speed of the inertial referential to be used for our classical calculations*. The analytic writing of both the potential and the kinetic energies in relation to this referential is then as simple as possible, as well as their derivatives in relation to the time.

Note : The potential and kinetic energies in relation to this inertial referential then are identical to what they would be in relation to the ground if the average horizontal wind speed was nil.

**Figure 1 description**

*The inertial wind vector* $\vec{W}$, *speed vector of any air particle at any given moment in relation to our inertial referential, is the sum of the vertical speed vector of this air particle (including the vertical gust) and its horizontal gust speed vector. It is also the sum of the average vertical speed and the 3D gust speed vectors*.

The inertial wind speed vector is supposed to be constant in the small volume occupied by the glider.

The glider can exchange energy with the atmosphere either by opposing its weight to the vertical speed of air, or by opposing its inertia to the 3D wind gust (vertical + horizontal). Doing so, it then can extract energy from the atmosphere when decreasing the inertial speed of air particles (rising air and gusts).

**Figure 2 description**

The total vertical speed $v_{zt}$ to be indicated on a Total Energy variometer is the sum of the glider's vertical speed $v_z$, representing the variation in its potential energy, and the glider's vertical speed $v_{zk}$ equivalent to the variation in its kinetic energy.

In relation to the above chosen referential, its expression can be developed as follows:

$$v_{zt} = v_z + v_{zk} = \vec{z}\vec{V} + (d\frac{1}{2}m\,V^2/dt)/mg = \vec{z}\vec{V} + dV^2/2gdt = \vec{z}\vec{V} + \vec{\Gamma}\vec{V}/g = \vec{z}\vec{V} + \vec{\gamma}\vec{V}$$

where $\vec{\gamma} = \vec{\Gamma}/g$

That is $$v_{zt} = (\vec{z} + \vec{\gamma})\vec{V}$$

The total vertical speed of the glider is the dot (scalar) product of its inertial speed vector $\vec{V}$ by the sum of the vertical unit vector $\vec{z}$ and its relative acceleration vector $\vec{\gamma}$.

**Figure 3 description**

Aerodynamic force $\vec{R}$+Weight $\overrightarrow{mg}$+ Inertia $\vec{I}$= 0 (in vectors)

Aerodynamic force $\vec{R}$= Sustentation $\vec{S}$+ Acceleration $\vec{\Gamma}$ (in vectors)

where vectors Sustentation and Acceleration are opposite to vectors Weight and Inertia. Dividing those vectors by mg results in

$$\vec{n} = \vec{z} + \vec{\gamma}$$

or the load factor vector $\vec{n}$ equals the sum of the vertical unit vector $\vec{z}$ and the relative acceleration vector $\vec{\gamma}$. Thus, the already well known result

$$v_{zt} = \vec{n}\vec{V}$$

or the total vertical speed is equal to the dot (scalar) product of the load factor vector and the inertial speed vector. This is consistent with the following general law: variation of internal energy of a body is equal to the energy provided by the external forces.

**Figure 4 description**

Furthermore, the glider inertial speed vector is equal to the sum of the glider's airspeed vector $\overrightarrow{V_{air}}$ and the inertial wind speed vector $\vec{W}$:

$$\vec{V} = \overrightarrow{V_{air}} + \vec{W}$$

**Fig 5 description**

Thus,

$$v_{zt} = \vec{n}\overrightarrow{V_{air}} + \vec{n}\vec{W} = v_{zaero} + v_{zsoaring}$$

or the total vertical speed of the glider indicated by an ideal accurate TE variometer is the sum of :

1) an equivalent negative "aerodynamic" vertical speed $v_{zaero} = \vec{n}\overrightarrow{V_{air}}$, quantifying the aerodynamic energy expenditure.

$$\vec{n}\overrightarrow{V_{air}} = \vec{z}\overrightarrow{V_{air}} + \vec{\gamma}\overrightarrow{V_{air}}$$

This equivalent aerodynamic vertical speed is the sum of an equivalent potential aerodynamic vertical speed and an equivalent kinetic aerodynamic vertical speed

2) and, as shown in Fig 6, an equivalent "soaring" vertical speed $Vz_{Soaring} = \vec{n}\vec{W}$, quantifying the exchange of soaring energy between the glider and the atmosphere by opposing its apparent weight to the inertial wind, the apparent weight being the opposite of the aerodynamic force, equal to the sum of its weight and its inertia.

$$\vec{n}\vec{W} = \vec{z}\vec{W} + \vec{\gamma}\vec{W} = W_z + \vec{\gamma}\vec{W}$$

This equivalent soaring vertical speed is made up of two terms:

a) An equivalent "static" vertical speed $\vec{z}\vec{W}$ that is equal to the vertical air speed component $W_z$. The glider exchanges energy with the atmosphere by opposing its weight to the vertically moving air. It depends solely on the presence of the glider in such vertically moving air.

b) An equivalent "dynamic" vertical speed $\vec{\gamma}\vec{W}$ that is equal to the dot (scalar) product of the relative acceleration vector by the inertial wind speed vector. The glider exchanges energy with the atmosphere by opposing its inertia to the inertial wind.

Only the acceleration component on the direction of the inertial wind results in a dynamic energy exchange. The phenomenon is uni-directional. The instantaneous value of the dynamic energy exchange (power) is independent of the speed of the glider.

Note : The equivalent static vertical speed depends on the vertical speed of the glider. The equivalent dynamic vertical speed depends on its acceleration. They are of different nature.

A given vertical gust can bring both potential and dynamic energies if the glider produces a suitable simultaneous acceleration.

The previous calculations do not take into account the transitory aerodynamic phenomena, and assume that the characteristics of the gliders in transition flow (unstationary) are identical to those in permanent flow (stationary). Energies of rotation are considered as negligible.

**Figure 7, constant speed inertial wind W**

When the speed of the inertial wind is constant, it is possible to calculate the equivalent dynamic altitude $\Delta H_{dyn}$ exchanged between two points A and B of the trajectory, because it is easy to integrate the equivalent dynamic vertical speed of the glider $v_{zdyn}$.

$$\frac{dH_{dyn}}{dt} = v_{zdyn} = \frac{\vec{\Gamma}}{g}\vec{W}$$

$$\Delta H_{dyn} = \int_A^B v_{zdyn}\, dt = \int_A^B (\frac{\vec{\Gamma}}{g})\,\vec{W} dt = \frac{\vec{W}}{g}\int_A^B d\vec{V}$$

$$\Delta H_{dyn} = \frac{\vec{W}}{g}(\vec{V_B} - \vec{V_A}) = \frac{W}{g}(v_B - v_A)$$

$v_A$ and $v_B$ being the projection on the direction of the inertial wind, of the inertial speeds of the glider in A and B (or of its airspeeds in A and B). Hence, Fig. 8, the *projections theorem*.

**The projections theorem**

The equivalent dynamic altitude exchanged between two points A and B of the trajectory in a constant speed inertial wind is equal to the dot product, divided by g, of the inertial wind vector $\vec{W}$ (vertical speed of air + horizontal gust) by the difference between the inertial speed vectors $\vec{V}$ of the glider in B and A (or by the difference between the airspeed vectors $\overrightarrow{V_{air}}$ in B and A, which is the same).

Thus, it is also equal to the product, divided by g, of the speed of the inertial wind W and the difference $v_B - v_A$ between the projections of the inertial speeds of the glider $\vec{V_B}$ in B and $\vec{V_A}$ in A on the direction of the inertial wind (or the difference between the projections of the airspeeds $V_{air}$ in A and B). It depends solely on the position of the glider when entering and when leaving the inertial wind area. It does not depend on the manoeuvres carried out by the glider in between.

The application of the projections theorem, for example in the case of a vertical upwards acceleration, makes it possible to calculate immediately the dynamic energy gain

$$\Delta H_{dyn} = \vec{W}\Delta\vec{V}/g$$

this being added to the static gain.

Inertial winds (vertical speed of air + horizontal gust), in fact, establish themselves gradually. The dynamic energy exchange (which is proportional to the intensity of the inertial wind) will be maximal if the acceleration is made at the same moment as when the inertial wind speed is maximal.

## The mechanism of dynamic energy exchanges

Dynamic energy exchanges between a glider and the air mass occur if the glider produces acceleration when being in an inertial wind (vertical speed of air + horizontal gust). The glider exchanges a certain amount of energy with the moving air mass (measured in relation to the moving air mass), this being different from the amount of energy actually exchanged with the atmosphere (measured in relation to the inertial referential), because for a specific force applied to the glider, the speed vectors of the glider in relation to the inertial referential and in relation to the air mass are different. The difference between these two quantities of energy constitutes the dynamic energy exchange.

For example, in the oversimplified case, where the glide ratio of the glider is assumed to be infinite and where the glider's speed in relation to the air mass is horizontal, the acceleration vector, if any, is perpendicular to the speed of the air and, then, the absolute value of the airspeed of the glider is not modified instantaneously. There is no exchange of energy with the moving air mass. However, this same acceleration is not perpendicular to the inertial speed. It modifies the absolute value of it, just as it modifies the glider's kinetic energy. The variation of kinetic energy constitutes the exchange of dynamic energy.

But in the general case where the glide ratio of the glider is taken onto account and its attitude is random, the situation is complex. The acceleration of the glider is not perpendicular to its airspeed and the absolute value of this airspeed varies for different reasons. This implies an instantaneous exchange of energy between the glider and the moving air mass, which is both potential and kinetic. The acceleration also modifies the glider's inertial speed vector, resulting in an energy exchange with the atmosphere that is different from the preceding one. The difference between these two energy exchanges constitutes the dynamic energy exchange.

**Fig 9 discussion**

The most simple and theoretical example, in which only one dimension in space is concerned, involves an engined aircraft without drag, accelerating horizontally within a tailwind gust. Here after the specific calculation: the power spent as an exchange of energy with the moving air mass is equal to the product of the traction $m\Gamma$ of the propeller by the aircraft's airspeed $V - W$, i.e. $m\Gamma(V - W)$ whereas the recovered power is equal to the product of this same traction $m\Gamma$ by the inertial speed $V$ of the aircraft, i.e. $m\Gamma V$ the difference is the dynamic power gain. It is equal to the product $m\Gamma W$ of the propeller traction $m\Gamma$ by the inertial wind $W$. This value is consistent with the theoretical result shown above. It is equivalent to a total vertical speed of $m\Gamma\, W/mg = \gamma W$.

Hereafter follow three realistic and significant examples involving two dimensions in space (In order to make things simpler, the average horizontal wind speed is taken as nil and the glide ratio of the glider as infinite):

1) The glider is flying in air rising at the speed $W_z$ (Fig. 10)

The glider's airspeed is supposed to be horizontal. The glider produces a positive vertical acceleration, given its ideal position. During the lapse in time dt, the glider's airspeed vector increases by the vertical vector $\overrightarrow{dV}$, perpendicular to it. Thus, the absolute value of this air speed is constant. The glider's inertial speed vector is increased by the same vector $\overrightarrow{dV}$, which is not perpendicular to it. Consequently, the absolute value of the glider's inertial speed $V$ is increased by $|\overrightarrow{dV}|$, this resulting in an increase in kinetic energy, that constitutes a dynamic energy. The equivalent dynamic vertical speed is equal to the product $\gamma_z W_z$ and the total vertical speed is

$$v_{zt} = W_z + \gamma_z W_z = (1+\gamma_z)W_z = n_z W_z$$

On this example it is easy to calculate directly the dynamic equivalent vertical speed

$$v_{zdyn} = \frac{d}{dt}\frac{V^2}{2g} = V\frac{dV}{gdt} = V\frac{(|\overrightarrow{dV}|\sin\alpha)}{gdt} = \frac{(V|\overrightarrow{dV}|)}{gdt}\frac{W}{V} = \gamma W$$

This result, valid in this peculiar case, is consistent with the above mentioned general result.

*It is in the pilot's interest to make an upward acceleration whenever the air mass is rising and a downward acceleration whenever the air mass is descending*. In this last case, the glider loses energy, although paradoxically it gains some dynamic energy. The static loss resulting from the descending air mass is higher than the dynamic gain resulting from a moderate downward acceleration. The sum of the gravity and dynamic accelerations is lower than one g. The apparent weight is still positive but lower than the real weight.

If the downward acceleration is as large as – 1g, the glider falls like a stone and the total exchange of energy is nil, (the residual drag being assumed to be nil), the dynamic gain being equal to the static loss. That means that it would be possible to cross a descending air mass without losing energy. Unfortunately in that case the speed of the glider increases so rapidly that productive time is not long enough to justify such an aerobatic manoeuvre.

The duration of an upward acceleration is limited by the glider's minimal airspeed. Keeping this airspeed within reasonable boundaries implies that these accelerations vary alternately upwards and downwards, depending on the vertical direction of air. Significant increases in dynamic energy can only be achieved when well synchronized with the vertical direction of the air.

The situation is relatively difficult to understand because the vertical speed of the glider can be the sum of three effects : static effect (vertical speed of air), possible part of dynamic effect resulting from an acceleration ($\vec{\gamma}\overrightarrow{W}$) and exchange of potential and kinetic energies (nil in our example because the airspeed has been chosen horizontal).

For a relative acceleration $\gamma_z$ = 0.4 and a vertical speed of air of 3m/s, the equivalent dynamic vertical speed taken into account by an ideal accurate TE variometer would be 1.2 m/s, and the total vertical speed 4.2 m/s. If the vertical acceleration lasts for 2 seconds, the dynamic gain is 2.5 metres.

Figures 11 and 12 show the energy exchanges in the case where the airspeed of the glider is not horizontal.

2) The glider is flying at a constant altitude and speed, banking left in a horizontal gust coming from the right (Fig. 13)

The acceleration to the left leads to the air speed of the glider veering left, at a constant horizontal absolute value. During the lapse in time $dt$, the glider's airspeed vector increases by $\overrightarrow{dV}$ perpendicular to it, without energy exchange. The inertial glider's speed vector increases by the same vector $\overrightarrow{dV}$, which is not perpendicular to it, and increases the absolute value of the inertial speed by $|dV|$. Consequently, the kinetic energy increases, which constitutes a gain in dynamic energy $v_{zdyn}$.

$$v_{zdyn} = \vec{\gamma}\overrightarrow{W} = \gamma W\cos a$$

α being the angle between the direction of the acceleration and the direction of the gust.

For two given values of the gust and acceleration, the gain is maximal when their directions are identical (cos α = 1). As in the previous case, it is possible to calculate directly the equivalent dynamic vertical speed as being the product γWcosα, consistent with the above mentioned general result

$$v_{zdyn} = V\frac{|dV|}{(gdt)} = V\frac{(|\overrightarrow{dV}|\sin\beta)}{(gdt)}$$

$$\frac{(|\overrightarrow{dV}|)}{(gdt)} = \frac{\Gamma}{g} = \gamma$$
$$V\sin\beta = AH = W\cos\alpha$$
$$v_{zdyn} = \gamma W\cos\alpha$$

The achievement of the suitable bank angle may require a little extra time, which was not necessary in the preceding example. However, for the same load factor, the acceleration can be significantly higher (for a load factor of 1.4 , 1 instead of 0.4). For an angle of 45° and a gust of 3m/s, the gain is 3m/s, which is considerable. If the gust lasts several seconds, the total gain can exceed 10 metres.

3) The glider is flying horizontally in a horizontal headwind gust (Fig. 14)

The direction of an immediate acceleration is necessarily perpendicular to the gust, thus producing no dynamic effect. No gain is possible until after the glider has changed its attitude into one in which it can direct the acceleration in a suitable direction. For example, the manoeuvre could be a rise in altitude ΔH, leading to a decrease in airspeed of ΔV, during which the glider develops an acceleration having a component to the rear in the direction of the gust, thus achieving a dynamic gain.

Let us suppose that the speed of the gust is constant, in which case airspeed of the glider becomes V + W – ΔV and inertial speed V - ΔV. Then

$$mg\,\Delta H = \tfrac{1}{2} m\,[(V + W)^2 - (V + W - \Delta V)^2]$$
$$\Delta H = \tfrac{1}{2}\,[(V + W)^2 - (V + W - \Delta V)^2] / g$$

The glider will have achieved a total altitude gain consisting of dynamic gain only

$$\Delta H_t = \Delta H + [\tfrac{1}{2}(V - \Delta V)^2 - \tfrac{1}{2}V^2] / g \text{ (potential + kinetic)}$$

i.e. after calculation

$$\Delta H_t = \Delta H_{dyn} = W\,\Delta V / g.$$

This result could have been immediately obtained using the projections theorem: The gain is proportional to the speed of the gust and to the magnitude of the manoeuvre. For a gust of 3 m/s, and a reduction in speed of 3 m/s, the gain will only be 1 metre. A headwind gust is not compatible with significant dynamic energy exchanges.

Logically, the most interesting inertial winds are, in order, a side wind gust, rising air and a headwind gust.

**Figure 15 description**

Let us recall an interesting dynamic energy exchange situation already analysed by Goodhart[2] of a glider circling horizontally in a radial field of flow. If the dynamic gain is equal to the aerodynamic loss, the total exchange of energy being nil, the inertial horizontal speed vector $\vec{V}$ of the glider, tangent to the circular trajectory, is the sum of its air speed vector and the convergent horizontal speed of the air. The aerodynamic force pointing in the direction of the axis of rotation is perpendicular to the inertial speed. The equivalent dynamic vertical speed is $v_{zdyn} = \vec{\gamma}\vec{W}$.

Examples covering three dimensions in space are much more complex and will not be developed here.

## Conclusions

The main information contained in this first part of the paper is the determination of the algebraic expression of the dynamic energy exchange as its equivalent climb rate

$$v_{zdyn} = \vec{\gamma}\overrightarrow{W},$$

or the dot product of the relative acceleration vector (acceleration vector of the glider divided by the gravity acceleration) and the inertial wind vector. This conclusion was delivered in my 1981 lecture[1].

A finding from the gust analyses is it is in the pilot's interest to make an upward acceleration whenever the air mass is rising and a downward acceleration whenever the air mass is descending.

This paper is continued in Part 2: 'The unreliability of existing TE variometers in turbulent and vertically moving air'.

## References

[1]François Ragot. Unsuitability of existing TE variometers for dynamic soaring. OSTIV Congress 1981.

[2]Goodhart. Circling flight in a radial field of flow.

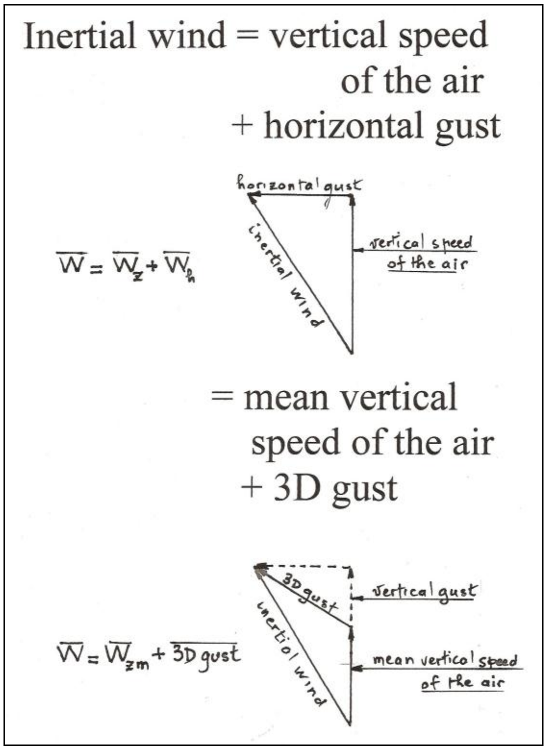


**Figure 1** Split of inertial wind

Total vertical speed
(potential + kinetic)

$$v_{zt} = v_{zp} + v_{zc} = v_z + [d\ ½\ m\ V^2 / dt]\ /mg$$

$$= \bar{z}\,\bar{V} + d\bar{V}^2 / 2g\ dt = \bar{z}\,\bar{V} + \bar{\Gamma}\,\bar{V} / g$$

$$v_{zt} = \bar{z}\,\bar{V} + \bar{\gamma}\,\bar{V}$$

$\bar{z}$ : vertical unit vector positive upwards

$\bar{V}$ : glider's speed vector

m : glider's mass

g : gravity acceleration

$\bar{\Gamma}$ : glider's acceleration vector

$\bar{\gamma}$ : glider's relative acceleration vector $\bar{\Gamma} / g$

$\bar{n}$ : load factor vector = aerodynamic force divided by the glider's weight mg

$\bar{n} = \bar{z} + \bar{\gamma}$ = sum of the vertical unit vector and the relative acceleration vector

$$v_{zt} = \bar{z}\,\bar{V} + \bar{\gamma}\,\bar{V} = \bar{n}\,\bar{V}$$

**Figure 2** Total vertical speed

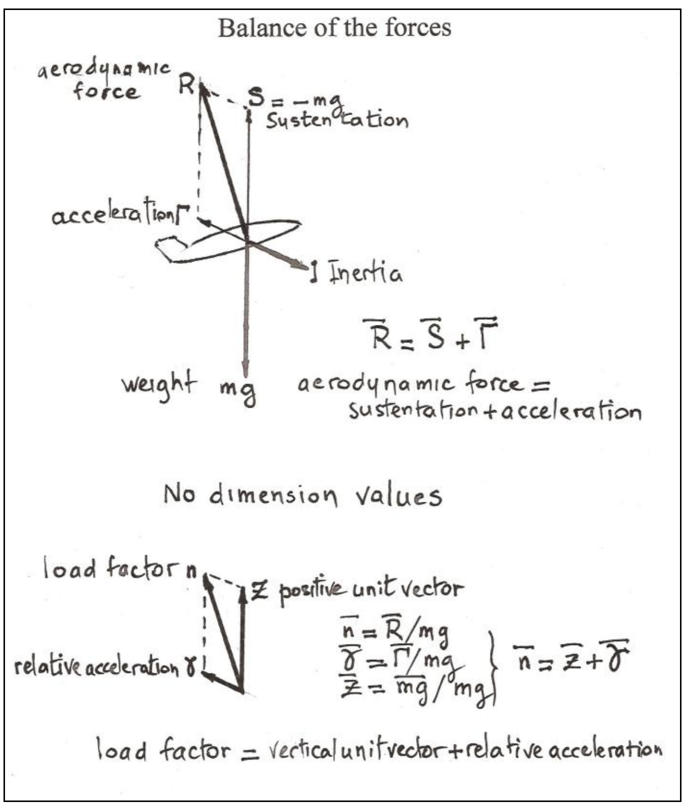


**Figure 3** Balance of forces

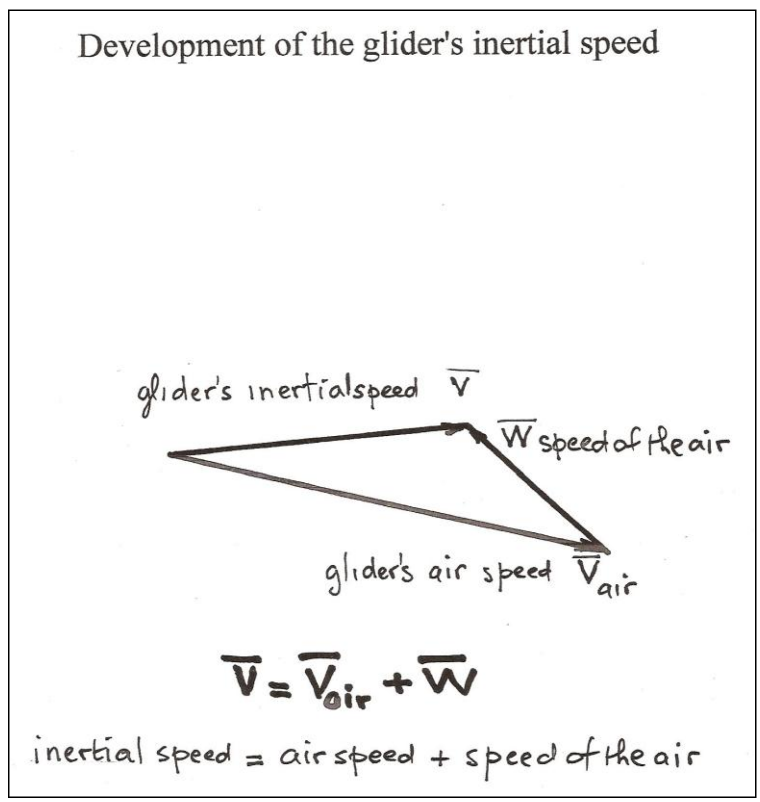


**Figure 4** Development of glider inertial speed

Development of the total vertical speed

$$v_{zt} = \bar{n}\,\bar{V} = \bar{z}\,\bar{V} + \bar{\gamma}\,\bar{V}$$

Total vertical speed $v_{zt}$ = sum of
vertical speed of the glider $\bar{z}\,\bar{V} = v_z$
+ equivalent kinetic vertical speed $\bar{\gamma}\,\bar{V}$

furthermore $\bar{V} = \bar{V}_{air} + \bar{W}$

thus $v_{zt} = \bar{n}\,\bar{V} = \bar{n}\,\bar{V}_{air} + \bar{n}\,\bar{W}$ = Vzaero + Vzsoaring

Total vertical speed $v_{zt}$ = sum of
equivalent aerodynamic vertical speed $\bar{n}\,\bar{V}_{air} = v_{zaero}$
+ equivalent soaring vertical speed $\bar{n}\,\bar{W} = v_{zsoaring}$

$\bar{n}\,\bar{V}_{air} = \bar{z}\,\bar{V}_{air} + \bar{\gamma}\,\bar{V}_{air}$
equivalent aerodynamic vertical speed $\bar{n}\,\bar{V}_{air} = v_{zaero}$ = sum of
static aerodynamic vertical speed $\bar{z}\,\bar{V}_{air}$
+ equivalent dynamic aerodynamic vertical speed $\bar{\gamma}\,\bar{V}_{air}$

$\bar{n}\,\bar{W} = \bar{z}\,\bar{W} + \bar{\gamma}\,\bar{W}$
equivalent soaring vertical speed $\bar{n}\,\bar{W} = v_{zsoaring}$ = sum of
static soaring vertical speed $\bar{z}\,\bar{w}$
+ equivalent dynamic soaring vertical speed $\bar{\gamma}\,\bar{W}$

**Figure 5** Development of total vertical speed

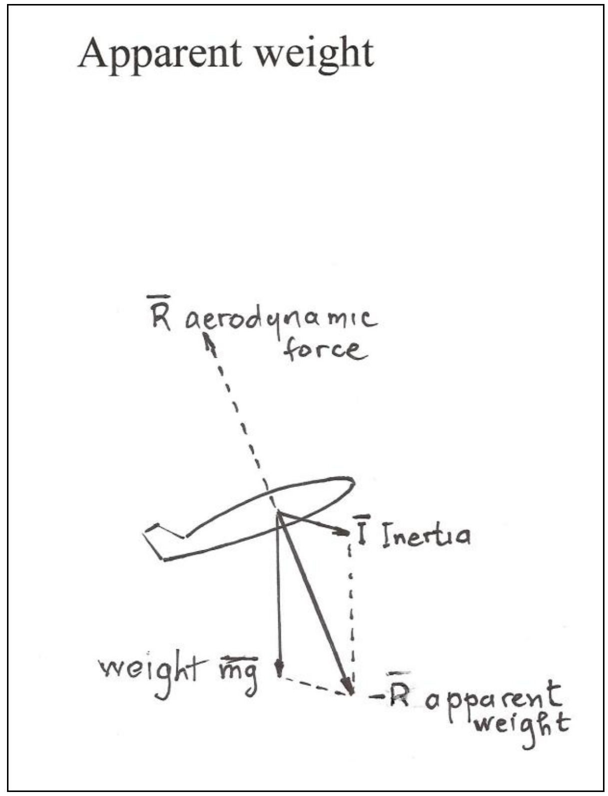


**Figure 6** Apparent weight

Projections theorem

$$dH_{dyn}/dt = v_{zdyn} = (\overline{\Gamma}/g)\,\overline{W}$$

$$\Delta H_{dyn} = \int_A^B (\overrightarrow{\Gamma}/g)\,\overrightarrow{W}\,dt = (\overrightarrow{W}/g)\int_A^B d\overrightarrow{V}$$

$$\Delta H_{dyn} = (\overrightarrow{W}/g)\,(\overrightarrow{V}_B - \overrightarrow{V}_A) = (W/g)\,(v_B - v_A)$$

**Figure 7** Projections theorem

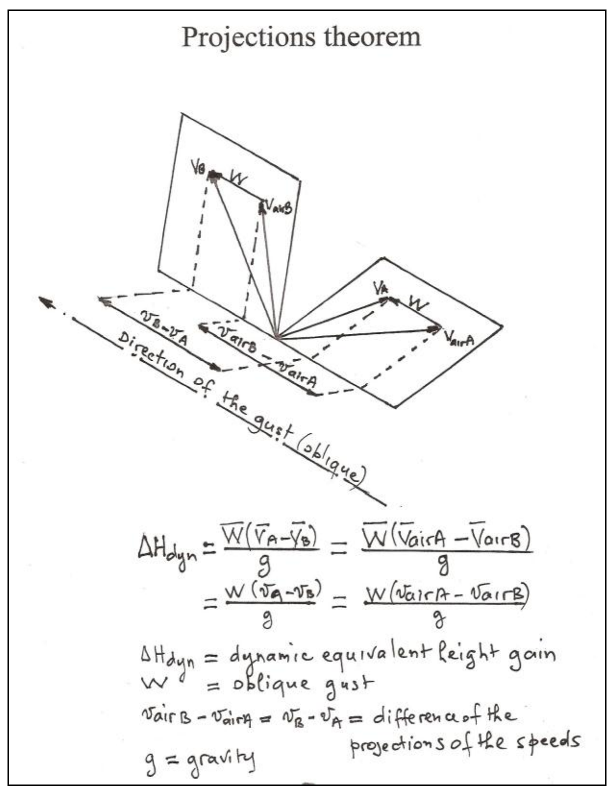


**Figure 8** Projections theorem

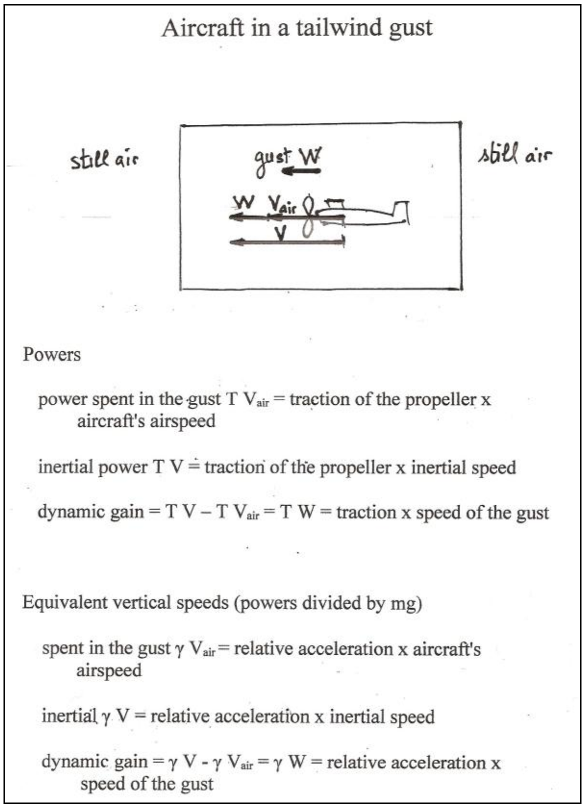


**Figure 9** Aircraft in tailwind gust

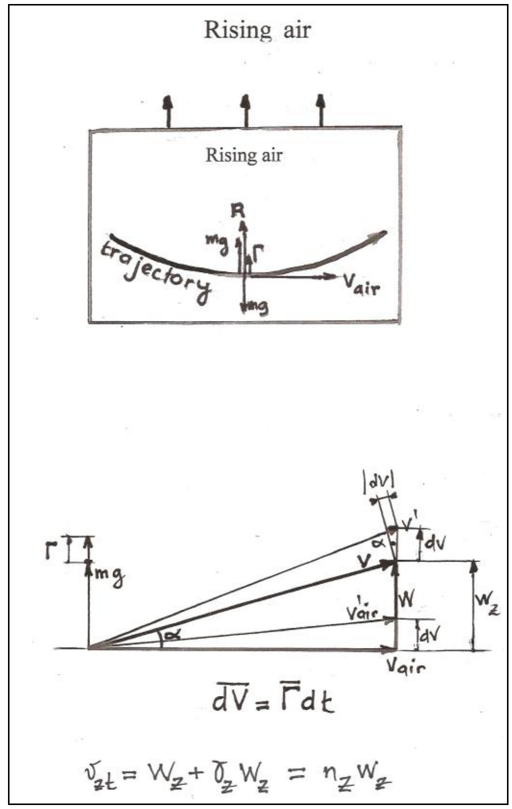


**Figure 10** Rising air

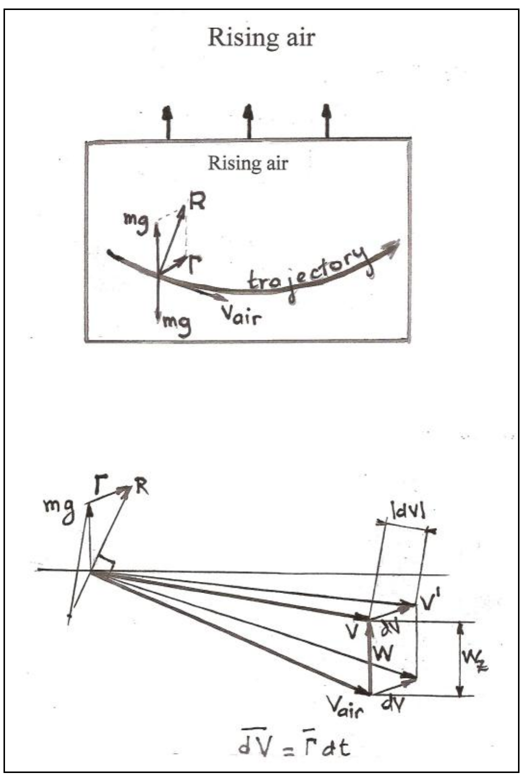


**Figure 11** Rising air

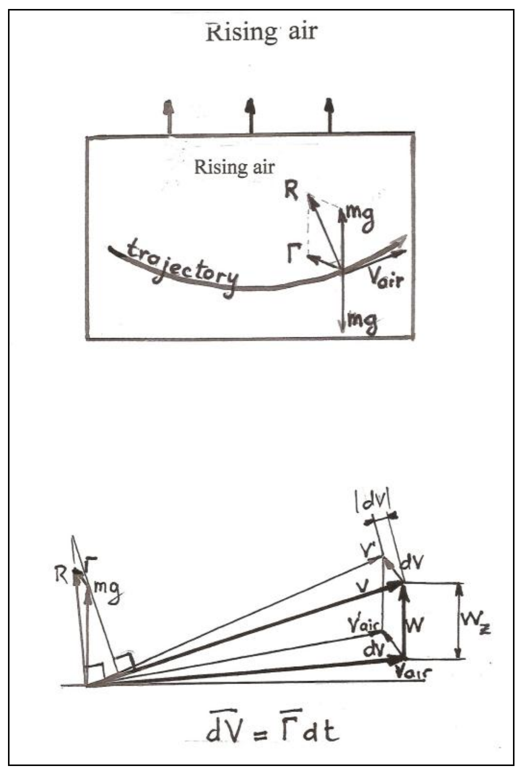


**Figure 12** Rising air

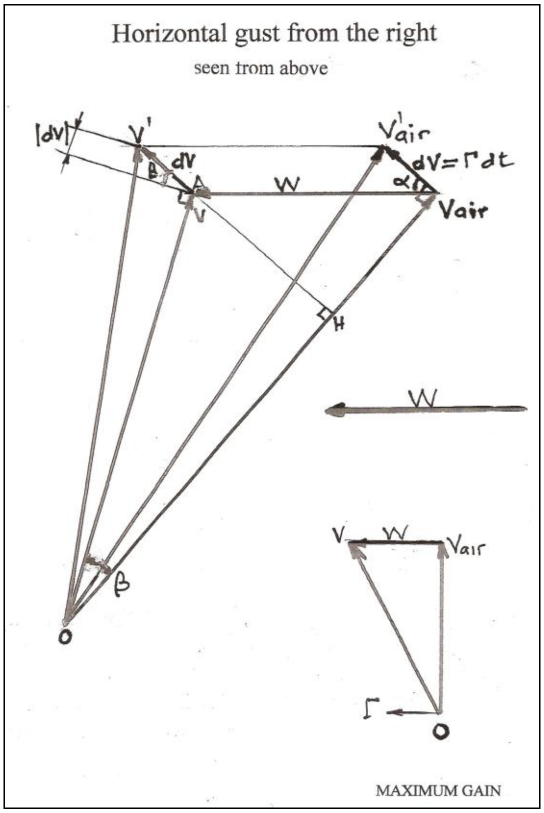


**Figure 13** Horizontal gust from the right seen from above

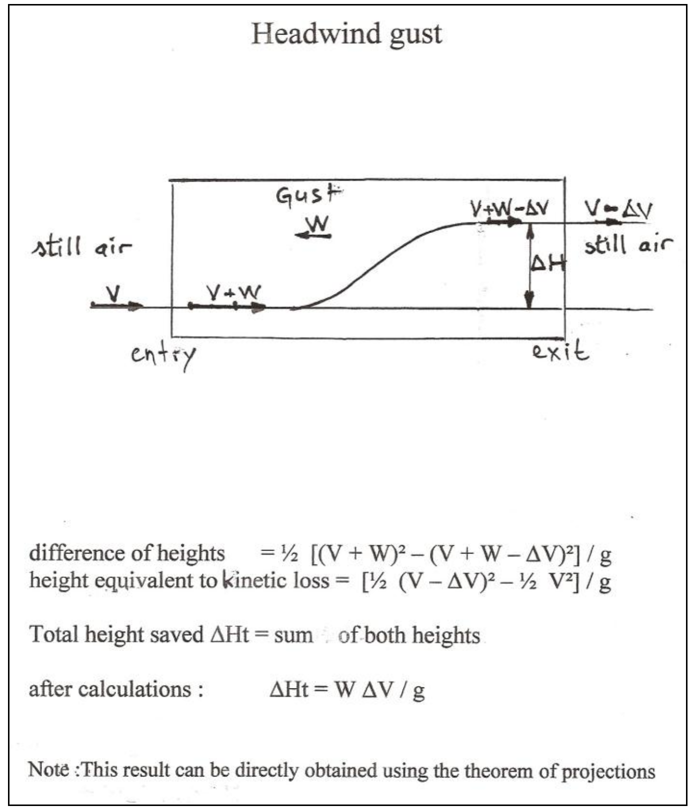


**Figure 14** Headwind gust

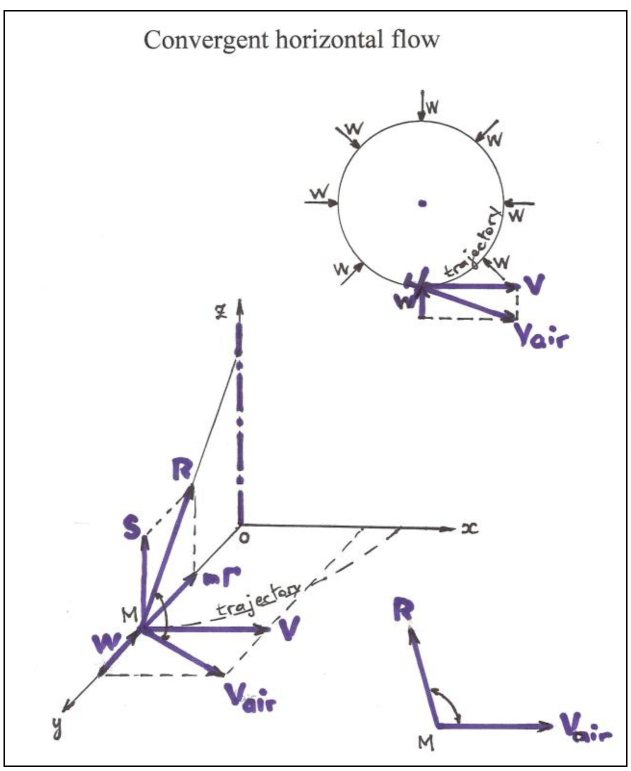


**Figure 15** Convergent horizontal flow