

The Impact of Wind Shear on Final Approach Glide Path

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Abstract

It is commonly recognized that wind shear on final approach in a glider can cause the glider to land short of the planned touchdown point. This paper examines the impact of different wind shear profiles, and shows that the reason for this is determined not as much by the wind shear itself as by how the pilot reacts to the wind shear. Alternative strategies are presented that can enable the glider to reach or even pass the planned touchdown point. These strategies make use of varying airspeed as the glider encounters the shear.

Nomenclature

E	Energy referenced to the runway threshold
g	Gravitational acceleration
h	Altitude above the runway threshold
L/D	Lift-to-drag ratio
m	Mass
R	Gliding range
t	Time
v_a	True airspeed
v_g	Ground speed
v_w	Wind speed

Introduction

Wind shear on final approach has long been recognized as a hazard for all aircraft, and it has been studied extensively for powered aircraft (e.g., Refs. 1–4). It can be particularly hazardous for gliders, as they cannot resort to an engine to power their way out of trouble. A common belief is that the presence of wind shear on descent always leads to a shortening of the glide distance to touchdown (range) versus flight in a steady and constant headwind. For example, Solies and Bögershausen [5] examine the case where a medium performance glider is flying at a true airspeed of 60 kts into a 20 kt headwind. They consider two conditions, first where the headwind is constant to the ground (no wind shear) and, second, where, beginning at 200 feet above the ground, the headwind reduces linearly to zero at the ground (wind shear). Assuming that the pilot maintains a constant true airspeed, they show that the range of the glider is significantly greater when wind shear is not present than in the case of wind shear, even though, in the case of wind shear, the headwind is reducing as the glider descends. It is not difficult to show that their conclusion will hold regardless of the strength of the wind shear or the airspeed flown. In fact, what may be a bit surprising is that the higher the airspeed at which the wind shear is encountered, the greater the shortening of the range to the beginning of

the flare.

Glider pilots are commonly taught to maintain a pre-selected airspeed on final. Thus, when encountering wind shear resulting in decreasing airspeed, they pitch down to increase airspeed back to the target airspeed. This results in an increase in ground speed, with an accompanying increase in kinetic energy, which must come from altitude loss. The increase in kinetic energy required to maintain airspeed through the shear layer increases with airspeed because kinetic energy is proportional to velocity squared. Thus, the faster a glider is going, the more altitude it must lose to regain a knot of airspeed. This is why increasing airspeed prior to an encounter with wind shear reduces range to the beginning of the flare.

While this explains the results of Solies and Bögershausen, the choice of the pilot to maintain a constant airspeed is purely arbitrary. There is no physical requirement that the pilot fly a constant airspeed trajectory to the ground. Hence, demanding the use of a constant-airspeed trajectory constitutes the imposition of a constraint on the flight path. If the pilot is seeking to maximize range to touchdown, this constraint can significantly penalize his objective. Thus, it would appear beneficial to examine alternative strategies for dealing with wind shear on final approach.

Energy

A popular approach to the analysis of flight trajectories is to integrate the equations of motion using Newtonian mechanics. Done accurately, this leads to a rather complex set of equations that, while precise, makes it difficult to gain an intuitive understanding of alternative flight trajectories. Instead, we will base our analysis on energy. Furthermore, we are not seeking extreme accuracy, as the effects we show are rather large and obvious. Accordingly, we make the following simplifying assumptions:

1. that the sailplane is a rigid body concentrated at its center of mass,

2. that the sailplane is not rotating about any axis,
3. we neglect changes in thermal energy of the sailplane, and
4. that the descent angle of the glider will be small such that, in an unaccelerated glide in calm conditions, the ratio of horizontal to vertical distance traveled equals the lift-to-drag ratio of the glider (L/D).

Further, we will assume that the L/D relative to the air remains constant over the full range of airspeeds analyzed (we do, of course, correct glide distance to accommodate winds). While this is clearly not the case, the trends of the results will, nonetheless, be informative. These assumptions impact the accuracy of what follows, but not enough to invalidate the results. Under these assumptions, the total energy of a sailplane is the sum of its potential and kinetic energies:

$$E_{total} = E_{potential} + E_{kinetic} \quad (1)$$

where

$$E_{potential} = mgh \quad (2)$$

and

$$E_{kinetic} = \frac{1}{2}mv^2 \quad (3)$$

Note that m refers to mass (slugs in the English system of units, kilograms in the metric system), and not weight. We shall take as our reference frame the earth at the threshold of the runway.

The Energy Equations

We shall use the variables defined in **Nomenclature**. The rate of change of altitude is the sum of the loss of altitude resulting from energy loss from drag plus the loss resulting from acceleration:

$$\frac{dh}{dt} = \left(\frac{dh}{dt}\right)_{drag} + \left(\frac{dh}{dt}\right)_{acceleration} = -\frac{v_a}{(L/D)} - \frac{v_g}{g} \frac{dv_g}{dt} \quad (4)$$

where

$$v_a = v_g + v_w \quad (5)$$

and, for range,

$$\frac{dR}{dt} = v_g \quad (6)$$

Combining equations (4) and (6),

$$\frac{dR}{dh} = \frac{dR/dt}{dh/dt} = -v_g \left(\frac{v_a}{(L/D)} + \frac{v_g}{g} \frac{dv_g}{dt} \right)^{-1} \quad (7)$$

Likewise,

$$\frac{dv_g}{dt} = \frac{dv_g}{dh} \frac{dh}{dt} = -\frac{dv_g}{dh} \left(\frac{v_a}{(L/D)} + \frac{v_g}{g} \frac{dv_g}{dt} \right) \quad (8)$$

Combining equations (7) and (8),

$$\frac{dR}{dh} = -\frac{v_g(L/D)}{v_a} \left(1 + \frac{v_g}{g} \frac{dv_g}{dh} \right) \quad (9)$$

This equation can be integrated to obtain the gliding range from an initial set of conditions subject to conditions on airspeed. For example, if we constrain airspeed to remain constant throughout the approach, then

$$\frac{dv_a}{dh} = \frac{dv_g}{dh} + \frac{dv_w}{dh} = 0 \quad (10)$$

This formulation for gliding range is in close agreement with the results of Solies and Bögershausen. They present a case where the initial altitude is 200 feet, the airspeed is held constant through the descent at 101.2 ft/sec, and the wind is 33.8 ft/sec at 200 feet diminishing linearly to calm at the ground. The range they compute is 2784 feet, compared to 2798 feet as computed using the above formulation. These results agree to within about half a percent, which can be accounted for by the optimistic assumption on L/D being equal to ground speed divided by vertical speed, and by the rather crude integration method used to obtain the results presented here.

Example Cases

We now present three sets of example cases that illustrate both agreement with the results of Solies and Bögershausen and show that, in every case, there exist alternative descent trajectories that lengthen the gliding range, many of which lead to glide ranges that are greater than the no-wind-shear case. Whereas Solies and Bögershausen constrain the airspeed to remain constant throughout the descent, we will choose a different, though also non-optimal rule, namely that ground speed remain constant as the descent through the shear begins until airspeed is reduced to a pre-selected minimum and, from there to the ground, airspeed remains constant. The results for three sets of initial altitude and wind speed are presented in Figs. 1, 2 and 3.

Figure 1 presents the case considered by Solies and Bögershausen, namely with the glider encountering wind shear at 200 feet above the ground, with a wind speed of 20 kts (33.8 ft/sec) diminishing linearly to zero at the ground. The plot gives gliding range as a function of true airspeed at the time of wind shear encounter at 200 feet altitude. The dashed line shows the gliding range given no wind shear, namely that the wind is a constant 20 kts from 200 feet down to the ground. Gliding distance without wind shear increases modestly with increased initial airspeed as expected. The lower solid line shows the range in wind shear given the rule that airspeed remain constant throughout the descent. Perhaps counterintuitively, under this rule, the gliding range actually decreases as the airspeed increases. This is because the energy required to increase the glider's ground speed by a given amount increases proportionally with ground speed. Thus, both the energy required per foot of descent and the rate of the descent increase with increasing ground speed.

The upward sloped solid lines denote the gliding range given that the glider's airspeed is allowed to decrease so as to maintain a constant ground speed until a minimum true airspeed is obtained at which point the true airspeed is held constant. This is a flight trajectory that should not be difficult to fly by a trained

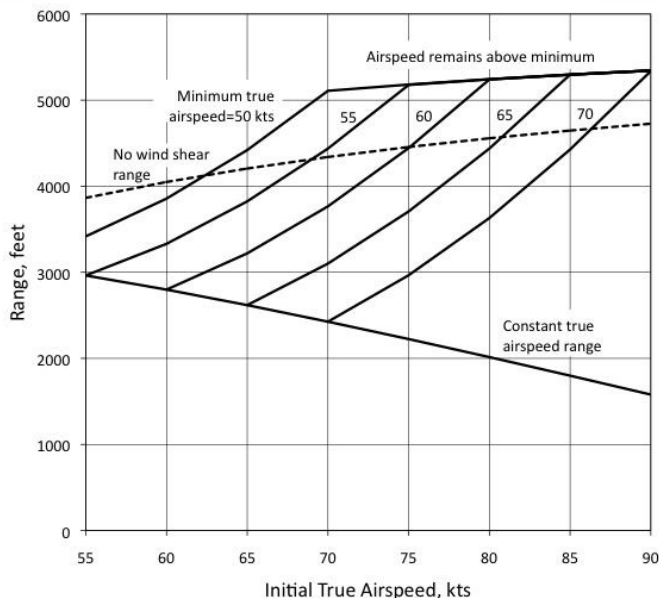


Fig. 1: Gliding range with a headwind of 20 kts at 200 feet diminishing linearly to 0 kts at 0 feet, maintaining a constant ground speed to the selected minimum true airspeed, then constant true airspeed.

pilot. For all cases where the initial true airspeed is adequate, a set of trajectories can be found that not only match but exceed the no-wind-shear case.

Finally, when the initial airspeed is high enough, the minimum airspeed constraint is not encountered throughout the descent. These initial airspeeds correspond to the top solid line that slopes gently upward with initial airspeed.

Figure 2 shows a case where the wind shear is not as strong, beginning with a 10 kt wind at 200 feet, diminishing linearly to zero at the ground. Whereas in the case of a 20 kt headwind, where the pilot had to sacrifice about 13 kts of airspeed to achieve a glide range equal to the no-wind-shear case, with a 10 kt headwind, the pilot need sacrifice only about 7 kts. Figure 3 shows a case where the wind shear begins at an altitude of 100 feet above the ground with a 10 kt headwind.

The key insight to be gained from these plots is that, whereas holding a constant airspeed throughout the approach always shortens the gliding range, with gliding range actually decreasing with initial airspeed, approach trajectories that sacrifice a modest amount of airspeed during the descent can take advantage of increased initial airspeed, and trajectories can always be found that do not shorten or even lengthen the gliding range through the shear layer.

Optimum Descent Trajectories

It is important to emphasize the impact of constraints on an optimization problem (Hazelrigg [6]). Optimization is the process of doing the best that can be done under a given set of cir-

cumstances. In the case of a glider approach through wind shear, a reasonable goal is to maximize gliding distance so as not to land short of the runway. The imposition of constraints never improves an optimal solution. We refer to constraints as either active or inactive. An active constraint is one that restricts the freedom of choice in selection of a control action, and it always penalizes the result. An inactive constraint is one that does not impact the freedom of choice in the selection of a control action and therefore does not penalize the result. In the case considered by Solies and Bögershausen, which is the case commonly taught to pilots of all aircraft, the rule *maintain-constant-airspeed* constitutes an active constraint that inflicts a considerable penalty on the objective of maximizing gliding range. Worse yet, it is a constraint that inflicts a penalty that increases with increasing initial airspeed, rendering the normal pilot's reaction of increasing airspeed to cope with an expected wind shear a self-defeating strategy.

Whereas the trajectories presented by Solies and Bögershausen are clearly constrained cases, so also are the cases presented in Figs. 1, 2 and 3. What these figures do is to present cases of differing constraints, showing that it is easy to find constrained trajectories that give clearly longer gliding ranges by altering the constraints. An interesting question is, what is the gliding range and control law of an optimal trajectory? This is a problem in the calculus of variations. The results are informative.

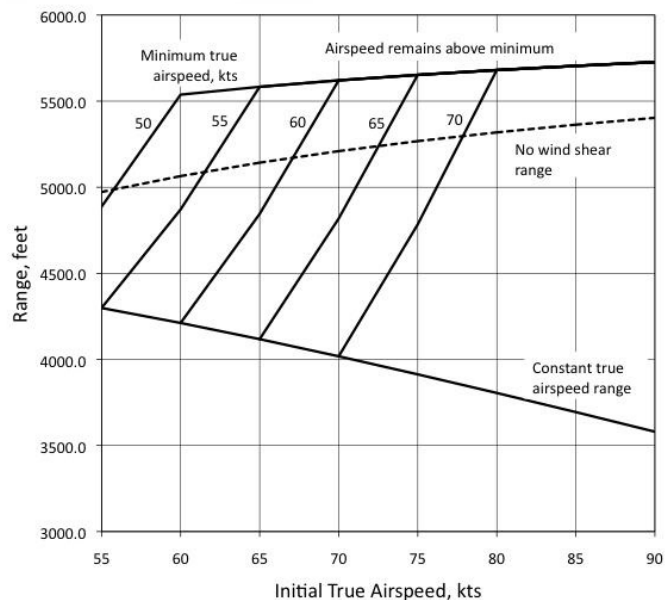


Fig. 2: Gliding range with a headwind of 10 kts at 200 feet diminishing linearly to 0 kts at 0 feet, maintaining a constant ground speed to the selected minimum true airspeed, then constant true airspeed.

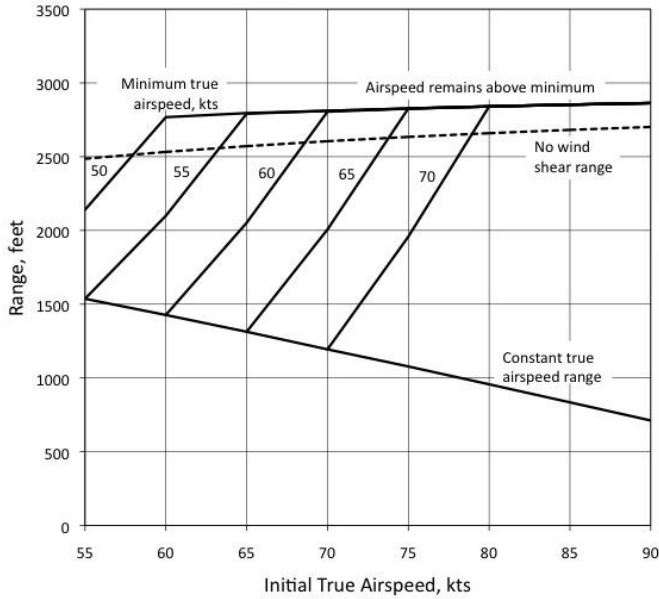


Fig. 3: Gliding range with a headwind of 10 kts at 100 feet diminishing linearly to 0 kts at 0 feet, maintaining a constant ground speed to the selected minimum true airspeed, then constant true airspeed.

The optimization problem can be stated as follows:

$$\text{Maximize } R = - \int_0^{h_0} f(v_g, \frac{dv_g}{dh}, h) dh \quad (11)$$

where

$$f(v_g, \frac{dv_g}{dh}, h) = \frac{v_g(L/D)}{v_g + v_w} + \frac{v_g^2(L/D)}{g(v_g + v_w)} \frac{dv_g}{dh} \quad (12)$$

Optimality is given by the Euler-Lagrange condition (Elsgolc [7]):

$$\frac{\partial f}{\partial v_g} - \frac{d}{dh} \left(\frac{\partial f}{\partial v_g'} \right) = 0 \quad (13)$$

where

$$v_g' = \frac{dv_g}{dh} \quad (14)$$

Solving this leads to the unconstrained optimal rule for the speed at which the descent through the shear layer should occur:

$$v_g(h) = \sqrt{\frac{g v_w(h)}{d v_w'(h)}} \quad (15)$$

where

$$v_w' = \frac{d v_w}{d h} \quad (16)$$

This rule says that v_g should be infinite if there is no wind gradient, but decreasing as the wind gradient increases. This actually makes intuitive sense recognizing that our assumption is

that L/D is a constant. Clearly, if the L/D does not decrease with increasing airspeed (obviously not a realistic case), then the optimal course of action is to go as fast as possible to minimize the impact of the headwind on the effective. On the other hand, in the case of a wind gradient, energy consumed by acceleration is minimized by minimizing ground speed.

We will now solve for the unconstrained gliding range in the baseline case of a 20 kt headwind at 200 feet diminishing to zero at the ground, and where the true airspeed upon entry to the shear layer is the optimal descent speed, which for this case is 47.6 kts. Under the constrained case of constant true airspeed, the gliding range computes to 3170 feet. We can compute the unconstrained gliding range by assuming that the glider enters the shear at the optimal speed of 80.25 ft/sec (47.6 kts), decreasing to zero at the ground. Again, assuming that the glider does not stall and that the L/D remains constant at 30.4, comparable to the assumptions of Solies and Bögershausen, the gliding range is 4575 feet, a 44 percent improvement over the constrained case.

Suppose the initial airspeed is higher than the optimal airspeed for penetration of the shear layer. Then the optimal thing to do would be to dissipate airspeed at the upper boundary of the shear layer until the optimal shear penetration speed is reached. Given this strategy, the gliding range is equal to the gliding range through the shear layer as given above, plus the gliding range at the shear boundary obtained by allowing the airspeed to decay down to the optimal shear layer penetration speed. Ground speed decays at a rate proportional to the drag force, which under our assumptions is constant.

$$\frac{d v_g}{d t} = - \frac{g}{(L/D)} \quad (17)$$

Thus,

$$v_g = v_{g0} - \frac{g t}{(L/D)} \quad (18)$$

where v_{g0} is the ground speed at the boundary of the shear layer. Since v_g decays linearly with time, the added range obtained while the glider decelerates at the boundary of the shear layer is

$$\Delta R = \frac{v_{g0} + v_g}{2} t \quad (19)$$

We compute ΔR for the case where v_g is the optimal initial wind shear penetration speed, and the corresponding value of t is given by

$$t = \frac{(L/D)}{g} (v_{g0} - v_g) \quad (20)$$

In the case that the initial true airspeed is 60 kts, with a wind speed at 200 feet of 20 kts (33.8 ft/sec), $v_{g0} = 67.4$ ft/sec and $v_g = 46.45$ ft/sec. This yields $\Delta R = 1126$ feet. Thus, the total gliding range for an initial airspeed of 60 kts is 5701 feet. Not only is this more than twice the distance computed by Solies and Bögershausen, it is 41 percent greater than the no-wind-shear case.

Of course, this is all academic as no glider could fly a trajectory that has airspeed diminishing to zero. But now we have an

insight as to how we could fly a more nearly optimal trajectory under reasonable constraints. It is reasonable to constrain the minimum airspeed flown on the approach. With this constraint and noting that optimal shear penetration speeds are generally rather low, given an initial approach speed, upon entering a shear layer the pilot should hold altitude until the glider decelerates to the minimum desired approach speed, then penetrate the shear layer at the minimum desired airspeed.

Let us apply this rule to the case presented by Solies and Bögershausen. If the glider enters the shear layer at 60 kts, then slows at the entry altitude to 50 kts, we obtain a time to slow to 50 kts of 15.92 seconds, with a distance traveled of 936 feet. The descent through the shear layer at 50 kts yields a range of 3109 feet. Thus, the total gliding distance would be 4045 feet, which is approximately equal to the no-wind-shear case. This range is also 45 percent higher than the range obtained by Solies and Bögershausen.

Figure 4 gives the gliding range for the case where, when the shear layer is encountered, the pilot maintains altitude at the upper boundary of the shear layer until his true airspeed is reduced to the pre-selected minimum airspeed. The shear layer is then penetrated at the selected minimum airspeed. As we should expect, the results for this control law are better than the case of maintaining constant ground speed or selected minimum true airspeed, whichever is greater. It is also obvious that increasing airspeed under this law can greatly increase gliding range. It is interesting, however, to compare the results of this figure to

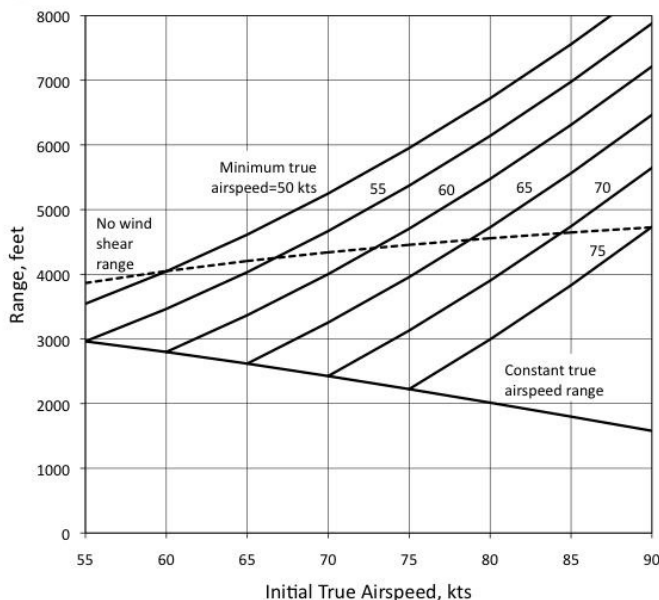


Fig. 4: Gliding range with a headwind of 20 kts at 200 feet diminishing linearly to 0 kts at 0 feet, allowing airspeed to decrease to the selected minimum true airspeed at the top of the shear layer, then penetrating the shear layer at the selected minimum true airspeed.

those of Fig. 1 and note that, up to the point where the airspeed through the shear layer remains above the minimum airspeed, the two control laws do not yield significantly different range. This is an indication that range is relatively insensitive to the precision by which the pilot flies the desired trajectory, provided only that he dissipates the excess speed as close to the top of the shear layer as practical.

Conclusions

The data presented in this work illustrate clearly that it is not wind shear itself that results in gliders landing short of the runway, but rather the way that we teach pilots to deal with wind shear. Pilots are commonly taught to maintain a constant airspeed during an approach to landing, and to increase their airspeed if they expect to encounter wind shear. Combined, these actions are counterproductive. In fact, in the presence of wind shear, both maintaining a constant airspeed and increasing the initial airspeed reduce gliding range. However, increasing airspeed can be quite effective at increasing gliding range in the presence of wind shear if the pilot is willing to sacrifice some of his initial airspeed. Certainly, if a pilot is comfortable flying an approach at 60 kts, he should be equally comfortable at allowing an initial airspeed of, say, 70 kts to decay to 60 kts upon encountering wind shear. This sacrifice of airspeed greatly increases gliding range, enabling the pilot to take advantage of the increased initial approach speed. A reasonably effective approach is to allow the airspeed to decay at a rate that maintains a constant ground speed as the descent into the shear layer begins, and then to maintain a constant minimum airspeed. However, an even more effective approach is to aggressively diminish the airspeed to the desired minimum airspeed as high in the shear layer as practicable. The data also show that gliding range is not particularly sensitive to the specific trajectory flown, provided that the pilot allows his airspeed to decrease as soon as practicable upon entry into the shear layer. This should not be a difficult strategy to learn, and with reasonable choice of initial and minimum true airspeeds, gliding range should not be significantly affected by the presence of moderate wind shear.

The flight profiles suggested by the above mathematics are in agreement with the wind shear recovery technique recommended by the Flight Safety Foundation [4] for large airplanes:

Allow airspeed to decrease to stick-shaker onset (intermittent stick-shaker activation) while monitoring airspeed trend; closely monitor airspeed, airspeed trend and flight path angle...; and when out of the wind shear, retract the landing gear, flaps and slats, then increase the airspeed when a positive climb is confirmed and establish a normal climb profile.

The recommendation is to penetrate the wind shear at the lowest safe speed, which minimizes the energy loss resulting from the wind shear.

The only remaining question is how to establish an expectation of wind shear prior to encountering it. A clear sign that wind

shear should be expected occurs when the tailwind on downwind leg of the approach differs from the headwind on the ground. Simply speaking, if the GPS shows a tailwind speed on downwind of 20 kts, and the AWOS notes a headwind on the ground of 5 kts, the pilot could expect a 15 kt wind shear at some point on the approach. A good way to prepare for this might be to add about two-thirds of the expected shear, 10 kts in this case, to the desired minimum approach speed of, say, 60 kts. He would then fly an initial approach speed of 70 kts and plan on sacrificing 10 kts upon encounter with the shear layer. This strategy would lead to a gliding range that is relatively unaffected by the wind shear.

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