GYROSCOPIC MOMENTS ON A GLIDER IN TURNING FLIGHT

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ABSTRACT

The magnitude of the gyroscopic rolling moment acting on a glider in a tight turn is estimated and compared with the aerodynamic rolling moment due to yawing velocity.

SUMMARY

The magnitude of the gyroscopic rolling moment acting on a glider in a tight turn is estimated and compared with the aerodynamic rolling moment due to yawing velocity. The value of the gyroscopic moment for the case studied, which is typical of a high-performance soaring glider, is 13.5 percent of the aerodynamic moment in a 45-degree bank. The gyroscopic moment is in a direction to oppose the aerodynamic moment. Some comments are given on the effects of other sources of rolling moment in turns.

In order to estimate the gyroscopic moment, the value of the radius of gyration in roll is required. An approximate analysis for estimating this quantity is included.

INTRODUCTION

Gyroscopic moments resulting from inertia forces on the parts of an airplane are important in determining the spinning characteristics, but are usually considered negligible in ordinary turning maneuvers. Because of the large wing span and high turning rate of a glider in tight turns, however, the inertial rolling moment may more important on gliders than on other types of airplanes. In this note the magnitude of this gyroscopic moment is estimated and compared with the aerodynamic rolling moment.

SYMBOLS

b	wing span
C_L	lift coefficient, L/qS
C_ℓ	rolling moment coefficient, $\frac{\mathcal{L}}{qSb}$
I_X, I_Y, I_Z	moments of inertia about body axes
K	ratio of semispan to extended semispan, $\frac{b/2}{ye}$

K_X	radius of gyration about X axis
k_x	ratio of radius of gyration to wing span, K_X/b
$\mathcal{L}$	rolling moment
m	mass of glider
m_w	mass of wings of glider
q	dynamic pressure, $\frac{\rho}{2} V^2$
r	yawing velocity about Z body axis
s_a	area of structural material in chordwise section of wing
s_{a_o}	value of S_a at wing root
V	airspeed
y	spanwise coordinate
y_e	extended semispan, measured to where leading and trailing edges, when extended, meet at a point
λ	taper ratio, tin chord/ root chord, 1-K
μ	relative density factor, $m/\rho Sb$
ρ	air density
w	density of structural material in wing
ϕ	angle of bank
ω	angular velocity about vertical axis

CALCULATION OF GYROSCOPIC ROLLING MOMENT

The gyroscopic rolling moment acting on a glider in a tight turn results from the tendency of all rotating bodies to aline themselves as closely as possible with the plane of rotation. In figure 1, the rolling moment acting on the wings of a glider tending to reduce the angle of bank is illustrated.

The gyroscopic rolling moment acting on a glider in a turn at approximately constant altitude is given by the formula (see reference 1).

$$\mathcal{L} = \frac{-\omega^2}{2}(I_Z - I_Y)\sin 2\phi \qquad (1)$$

If the weight of the glider is distributed primarily in the plane of the wings, as is usually the case, $I_Z - I_Y = I_X$. The rolling moment is then

$$\mathcal{L} = \frac{-\omega^2}{2} I_X \sin 2\phi$$

where ω is the angular velocity about a vertical axis caused by turning of the glider.

A formula given in reference 2 for the nondimensional yawing velocity about the Z body axis is

$$\frac{rb}{2V} = \frac{C_L}{8\mu}\sin 2\phi$$

Because this yawing velocity is a component of the total angular velocity about the vertical axis, the nondimensional angular velocity about the vertical axis is

$$\frac{\omega b}{2V} = \frac{C_L}{8\mu}\frac{\sin 2\phi}{\cos\phi} = \frac{C_L}{4\mu}\sin\phi$$

hence

$$\omega^2 = \frac{V^2}{b^2}\frac{{C_L}^2}{4\mu^2}\sin^2\phi$$

Substituting this value in formula (1) and placing the result in coefficient form gives for the rolling-moment coefficient:

$$C_\ell = \frac{C_L^2}{4\mu}\,\frac{m_w}{m}\,k_X^2 \sin^2\phi \sin 2\phi \qquad (2)$$

In order to use this formula, the value of k_X, the ratio of radius of gyration in roll to the wing span, must be known. Inasmuch as measured values of this quantity are not readily available, an estimate is made based on the moment of inertia of a simple geometric body. The wing of the glider, which contributes almost all of the inertia in roll, may be approximated as an elongated truncated pyramid. Calculations of the value of k_X are given in the appendix for two cases, one in which the weight of a chordwise element of the wing is proportional to the chord (corresponding to a constant thickness of skin and structural members) and one in which the weight of a chordwise element is proportional to the cross-sectional area of the airfoil (corresponding to a thickness of skin and structural members proportional to the chord). The wing of an actual glider is expected to fall between these conditions.

Values of the ratio of radius of gyration to wing span, k_X, and of k_X^2, are plotted as a function of taper ratio in figures 2 and 3. For a typical case for taper ratio, λ, of 0.5, the values of k_X for the two cases are

$k_X = .263$ (constant skin thickness)

$k_X = .239$ (skin thickness $\sim$ chord)

NUMERICAL EXAMPLE

Values of the gyroscopic rolling-moment coefficient have been calculated for the glider used as an example in reference 2. The characteristics of this glider are listed in table I. Additional values assumed in the calculations are a wing weight of 2225 N (500 lb) and a value of k_X of 0.241. If these values are substituted in formula (2), the values of rolling-moment coefficient in a 45° bank are found to be as follows:

C_L	0.95	1.6
$C_{\ell\,(\text{gyroscopic})}$	-.00252	-.00719
$C_{\ell\,(\text{aerodynamic})}$	.0188	.0533

DISCUSSION

The value of the gyroscopic rolling moment in this case is only 13.5 percent of the aerodynamic rolling moment due to yawing. The sign of the moment is opposite from that of the aerodynamic moment and would, therefore, result in a small reduction in the aileron deflection required for trim.

Inasmuch as both sources of moment vary with the factor C_L^2/μ the ratio of gyroscopic moment to aerodynamic moment for a given glider and bank angle remains the same at all values of lift coefficient or relative density factor.

The variations of the two types of moment with bank angle are shown in figure 4. These values are shown at a lift coefficient of 1.6. The aerodynamic rolling moment coefficient reaches a maximum at an angle of bank of 45°, whereas the gyroscopic rolling moment reaches a maximum at a bank angle of 60°.

The inertial rolling moment might be considered as a possible source of moment to reduce the aileron deflection required in turns. This method does not appear very useful, however, because it is relatively ineffective for bank angles less than 40° and because the increased inertia in roll required would be excessive. For example, the addition of weights of 289 N (65 pounds) to each tip of the glider used as an example would approximately double the moment of inertia in roll. This addition would increase the gyroscopic moment from 15 percent to 30 percent of the aerodynamic moment at a bank angle of 45°. The doubled moments of inertia in yaw and roll would be expected to make the glider undesirably sluggish in response to controls.

CONCLUDING REMARKS

The foregoing analysis provides an estimate of the gyroscopic rolling moment acting on a glider in a steady turn. Previous analyses and flight studies have given some insight into the various sources of rolling moment which may act on a glider in this maneuver. A brief summary of these effects may therefore be of interest.

The primary source of rolling moment acting on a glider in a turn is the aerodynamic rolling moment due to yawing velocity. For the glider used as an example in this report, about 12° deflection of each aileron is required to trim out this rolling moment near the maximum-lift coefficient. This deflection might be changed, however, by the following sources of rolling moment acting on the glider:

Factors tending to increase aileron deflection:

- flow separation on downward-deflected aileron
- wing twist

Factors tending to decrease aileron deflection:

- inward sideslip, combined with dihedral effect
- velocity gradient across span due to circling in thermal
- velocity gradient caused by influence of trailing vortices from previous turn
- gyroscopic rolling moment
- asymmetric loading of outboard wing

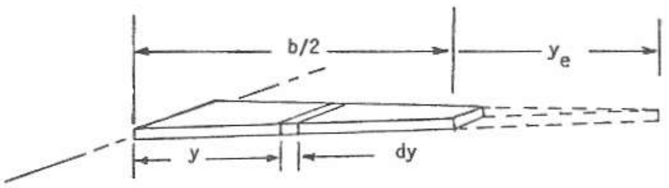
Of these factors, the effect of sideslip depends on piloting technique. The velocity gradient across the span caused by circling in a thermal may have a large effect in a small diameter thermal, but would obviously be absent in a large thermal or in still air. The effect of trailing vortices has been shown to be small provided the glider is descending with respect to the surrounding air at the rate corresponding to a steady turn. Finally, the gyroscopic rolling moment has been shown in the present report to be relatively small.

In view of the number of variables contributing to the aileron deflection required in turns, any attempts to compare measured and predicted control deflections should be performed with adequate instrumentation to determine the effects of these variables.

APPENDIX

Calculation of radius of gyration of a wing in roll

Case 1 - weight of each chordwise element proportional to chord



$$I_x = \int y^2 \, dm \qquad m_w = \int dm$$

$$dm = \rho_w \, s_a \, dy$$

$$s_a = s_{a_o} \left(1 - \frac{y}{y_e}\right)$$

hence

$$I_x = \rho_w \, s_{a_o} \int_o^{b/2} y^2 \left(1 - \frac{y}{y_e}\right) dy$$

let

$$b/2 = K \, y_e$$

$$I_x = \rho_w \, s_{a_o} \left[\frac{-y^3}{3} - \frac{-y^4}{3y_e}\right]^{Ky_e} = \rho_w \, s_{a_o} \, y_e^3 \left[\frac{K^3}{3} - \frac{K^4}{4}\right]$$

also

$$m_w = \rho_w s_{a_0} \int_0^{Ky_e} (1 - \frac{y}{y_e}) \cdot dy$$

$$m_w = \rho_w s_{a_0} \left[y - \frac{y^2}{2y_e} \right]_0^{Ky_e} = \rho_w s_{a_0} y_e \left[K - \frac{K^2}{2} \right]$$

$$I_X = m_w K_X^2$$

hence

$$K_X^2 = \frac{I_X}{m_w} = y_e^2 \left[\frac{\frac{K^3}{3} - \frac{K^4}{4}}{K - \frac{K^2}{2}} \right]$$

$$k_X^2 = \frac{K_X^2}{b^2} = \frac{1}{4K^2 y_e^2}$$

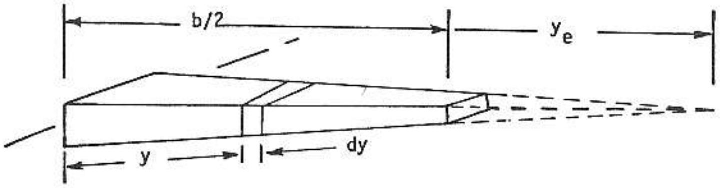
hence

$$k_X^2 = \frac{1}{4K^2} \left[\frac{\frac{K^3}{3} - \frac{K^4}{4}}{K - \frac{K^2}{2}} \right] = \frac{\frac{1}{12} - \frac{K}{16}}{1 - \frac{K}{2}}$$

Note that as the value of K varies from 0 to 1, the wing planform varies from rectangular to triangular. The taper ratio, λ , equals 1-K. In terms of λ , the formula becomes

$$k_X^2 = \frac{1 + 3\lambda}{24 \ (1+\lambda)}$$

Case 2 - Weight of each chordwise element proportional to area of airfoil section.



$$I_X = \int y^2 \, dm \qquad m = \int dm$$

$$dm = \rho_w s_a \, dy$$

$$s_a = s_{a_0} (1 - \frac{y}{y_e})^2$$

hence

$$I_X = \rho_w s_{a_0} \int_0^{b/2} y^2 (1 - \frac{y}{y_e})^2 \, dy$$

let

$$b/2 = K y_e$$

$$I_X = \rho_w s_{a_0} \left[\frac{y^3}{3} - \frac{2y^4}{4y_e} + \frac{y^5}{5y_e^2} \right]_0^{K y_e} = \rho s_{a_0} y_e^3 \left[\frac{K^3}{3} - \frac{K^4}{2} + \frac{K^5}{5} \right]$$

also

$$m_w = \rho_w s_{a_0} \int_0^{K y_e} (1 - \frac{y}{y_e})^2 \, dy$$

$$= \rho_w s_{a_0} \left[y - \frac{y^2}{y_e} + \frac{y^3}{3y_e^2} \right]_0^{K y_e} = \rho_w s_{a_0} y_e \left[K - K^2 + \frac{K^3}{3} \right]$$

$$I_X = m_w K_X^2$$

hence

$$K_X^2 = \frac{I_x}{m_w} = y_e^2 \frac{\left[\frac{-K^3}{3} - \frac{K^4}{2} + \frac{K^5}{5} \right]}{\left[K - K^2 + \frac{K^3}{3} \right]}$$

$$k_X^2 = \frac{K_X^2}{b^2} = \frac{1}{4K^2 y_e^2}$$

hence

$$k_X^2 = \frac{1}{4K^2} \frac{\left[\frac{K^3}{3} - \frac{K^4}{2} + \frac{K^5}{5} \right]}{\left[K - K^2 + \frac{K^3}{3} \right]} = \frac{\frac{1}{12} - \frac{K}{8} + \frac{K^2}{20}}{1 - K + \frac{K^2}{3}}$$

In terms of the taper ratio, λ ,

$$k_X^2 = \frac{1 + 3\lambda + 6\lambda^2}{40(1 + \lambda + \lambda^2)}$$

TABLE 1. CHARACTERISTICS USED IN CALCULATIONS

b	18.29 m	(60 ft)
ρ	1.226 kg/m^3	(,00238 $slugs/ft^3$)
S	12.99 m^2	(140 ft^2)
W	4057 N	(912 lb)
Aspect Ratio	25.8	
m	413.7 kg	(28.3 sl)
μ	1.41	
Wing Weight	2225 N	(500 lb)
k_X	.241	

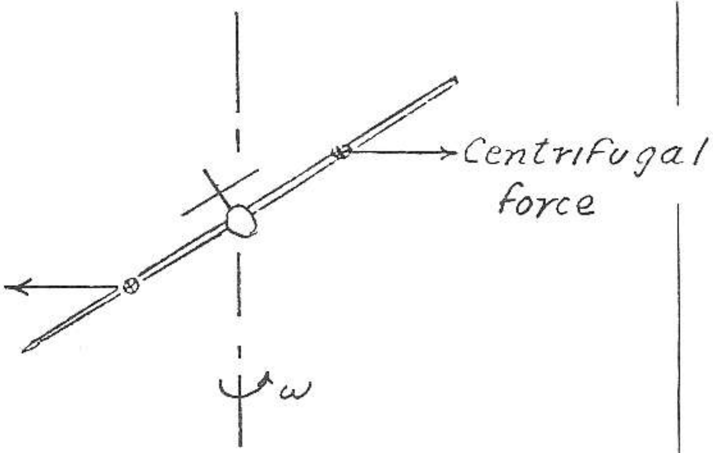


FIG. 1. ILLUSTRATION OF SOURCE OF GYROSCOPIC ROLLING MOMENT ACTING ON A GLIDER IN A STEADY TURN

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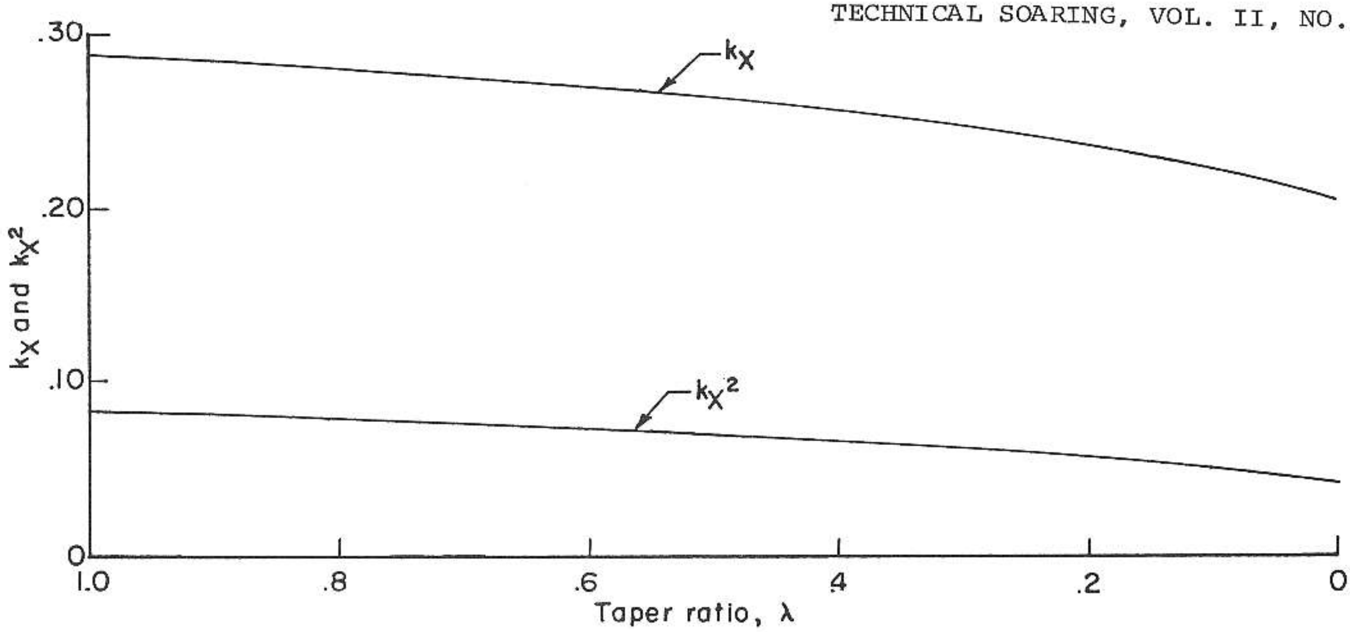

FIG. 2. VALUES OF RATIO OF RADIUS OF GYRATION ABOUT X AXIS TO WING SPAN, K_X, AND OF K_X^2 AS A FUNCTION OF TAPER RATIO, FOR TAPERED WINGS WITH CONSTANT THICKNESS OF SKIN AND STRUCTURAL MEMBERS

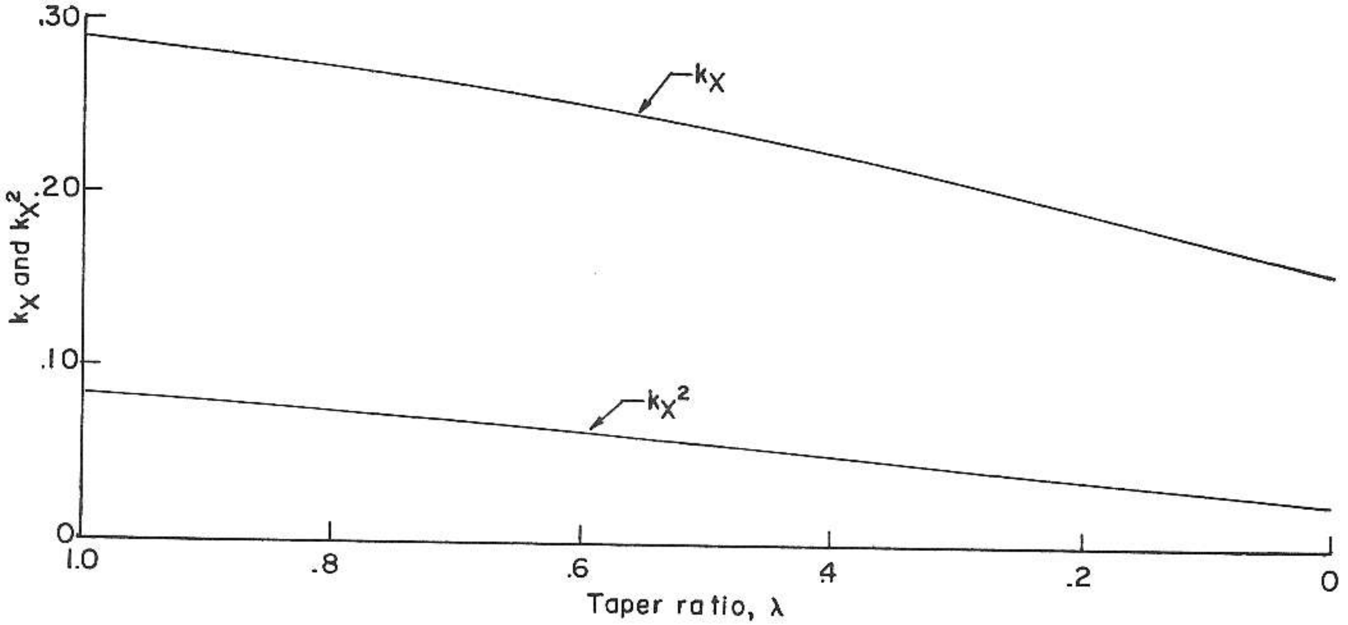

FIG. 3. VALUES OF RATIO OF RADIUS OF GYRATION ABOUT THE X AXIS TO WING S SPAN, K_X, AND OF K_X^2 , AS A FUNCTION OF TAPER RATIO, , FOR TAPERED WINGS WITH THICKNESS OF SKIN AND STRUCTURAL MEMBERS OF PROPORTIONAL TO CHORD

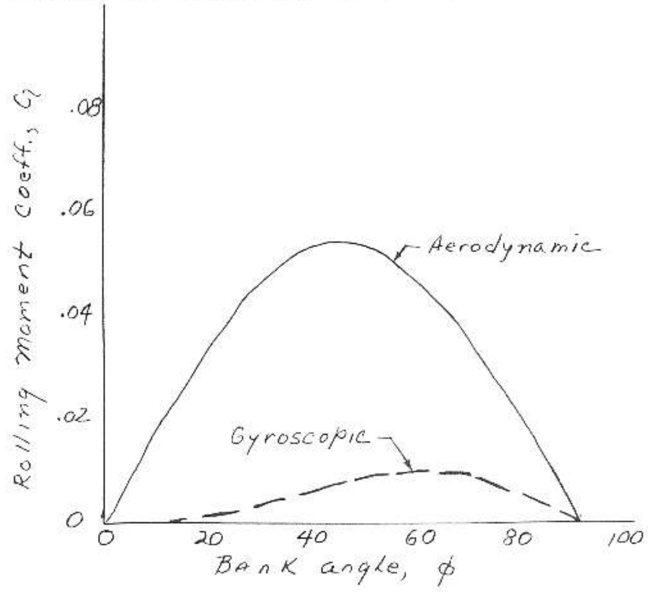

FIG. 4. VARIATIONS WITH BANK ANGLE OF AERODYNAMIC ROLLING-MOMENT COEFFICIENT DUE TO YAWING VELOCITY AND GYROSCOPIC ROLLING-MOMENT COEFFICIENT DUE TO TURNING; $C_L = 1.6$. (MOMENTS HAVE OPPOSITE SIGN)

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