THE EFFECT OF ERRORS IN INTER-THERMAL SPEED ON THE AVERAGE CROSS-COUNTRY SPEED

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INTRODUCTION

From the familiar construction of Fig. 1, it will be seen that if the average achieved rate of climb is v_c then the optimum gliding speed between thermals will be U_1. If, however, the sailplane is actually flown at U_2, there will be a loss in average speed δU_x. In principle, this loss in speed for a particular situation can be assessed by performing graphically the construction shown in the diagram. In practice, it is difficult to do so accurately: one is trying to assess a second-order error and the result obtained is very sensitive to inaccuracies in the plotted performance curve and the practical difficulty of locating the point of tangency.

It therefore seemed useful to derive a general analytical expression enabling the error to be easily assessed under any conditions, for purposes such as examining the effects of errors in variometer indications.

Throughout this note, all speeds are assumed to be "equivalent".

The Dimensionless Performance Curve

As Goodhart (Ref. 1) has noted, a good approximation to the drag coefficient of a sailplane in a given configuration is

$$C_D = C_{D_o} + KC_L^2 \qquad (1)$$

In this expression, the second term is not simply due to the vortex drag: it also includes a contribution due to the dependence of the profile drag on lift coefficient and wing Reynolds number. In total, this second term is often 25% or more in excess of the vortex drag contribution.

If equation (1) applies, it may be shown (Refs. 2, 3) that the performance curve corresponds to the dimensionless expression

$$\overline{v}_s = \tfrac{1}{2}\,(\overline{U}^3 + 1/\overline{U}), \qquad (2)$$

where $\overline{U} = U/U_o$

and $\overline{v}_s = v_s/v_{s_o}$.

$\overline{U}$ and $\overline{v}_s$ are therefore dimensionless speeds obtained by dividing the actual forward speed and rate of descent by the values appropriate to $(L/D)_{max}$.

The optimum condition of Fig. 1 corresponds to the criterion

$$\frac{d\overline{v}_s}{d\overline{U}} = \frac{\overline{v}_c + \overline{v}_s}{\overline{U}} \quad \text{at } \overline{U} = \overline{U}_1 , \qquad (3)$$

where $\overline{v}_c = v_c/v_{s_o}$.

Equation (3) is the normal MacCready criterion expressed in terms of the dimensionless quantities.

In general, if the sailplane is flown at U between the thermals, the average cross-country speed will be:

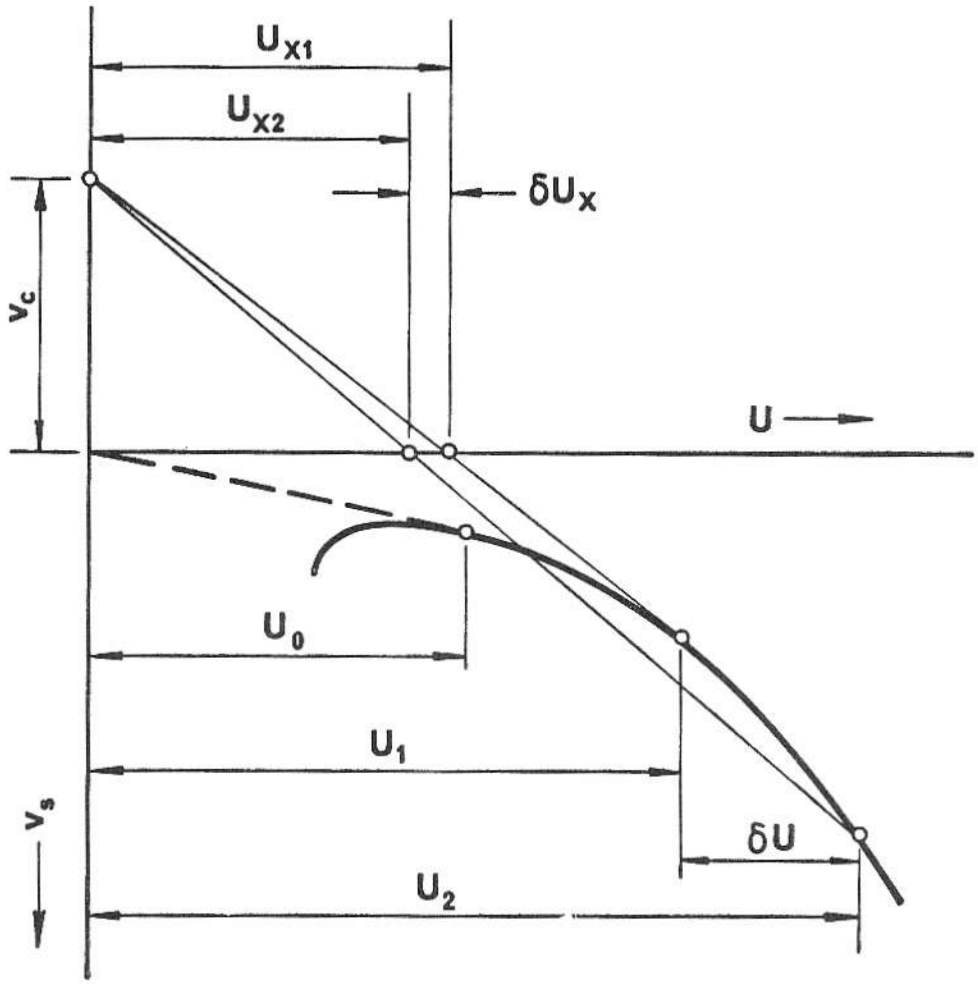

FIGURE 1.

$$\overline{U}_x = \frac{\overline{U}\,\overline{v}_c}{\overline{v}_c + \overline{v}_s} , \qquad (4)$$

where $\overline{U}_x = U_x/U_o$.

It then follows, from equations (2), (3) and (4) that, when $\overline{U} = \overline{U}_1$, the corresponding dimensionless cross-country speed will be

$$\overline{U}_{x_1} = \frac{2\overline{U}_1(\overline{U}_1^3 - 1/\overline{U}_1)}{(3\overline{U}_1^3 - 1/\overline{U}_1)} , \qquad (5)$$

and $\overline{v}_c = \overline{U}_1^3 - 1/\overline{U}_1$. (6)

Effect of Flying at the Non-Optimum Speed

It also follows from (2), (4) and (6) that, if the sailplane is flown at U_2 during the glide with the same average rate of climb v_c, the average cross-country speed becomes

$$\overline{U}_{x_2} = \frac{2\overline{U}_2(\overline{U}_1^3 - 1/\overline{U}_1)}{2(\overline{U}_1^3 - 1/\overline{U}_1) + (\overline{U}_2^3 + 1/\overline{U}_2)} . \qquad (7)$$

If, in equation (7), we now put $\overline{U}_{x_2} = \overline{U}_{x_1} + \delta\overline{U}_x$ and $\overline{U}_2 = \overline{U}_1 + \delta\overline{U}$, we can expand this expression in the usual fashion.

After some manipulation, we compare it with equation (5) and find that

$$\frac{\delta\overline{U}_x}{\overline{U}_x} = - \left[\frac{3\overline{U}_1^3 + 1/\overline{U}_1}{3\overline{U}_1^3 - 1/\overline{U}_1}\right] \left[\frac{\delta\overline{U}}{\overline{U}_1}\right]^2 . \quad (8)$$

Reverting to dimensional quantities, this can also be written

$$\frac{\delta U_x}{U_x} = - \left[\frac{3U_1^4 + U_o^4}{3U_1^4 - U_o^4}\right] \left[\frac{\delta U}{U_1}\right]^2 . \quad (9)$$

So, knowing the optimum glide speed U_1 and the speed for $(L/D)_{max}$ in this configuration, U_o, the effect of errors in U_1 may be found.

If equation (9) is written

$$\frac{\delta U_x}{U_x} = - E \left[\frac{\delta U}{U_1}\right]^2 , \quad (10)$$

then E is easily found for various values of U_1/U_o, i.e. of $\overline{U}_1$. It will be seen from Fig. 2 that, for likely values of $\overline{U}_1$, it lies between 1.1 and 1.3. Under typical circumstances, a 10% error in U_1 will lead to a loss of about 1.2% in average cross-country speed.

Example A Standard-Class glider has a speed for $(L/D)_{max}$ of 46 knots (85 km/h). It is being flown under conditions such that the optimum glide speed is 69 knots (128 km/h). The maximum average cross-country speed would then be 39.5 knots (73 km/h). Since $\overline{U}_1 = 1.5$, E is 1.14.

If the glide speed differs from the optimum by 10% (i.e. 7 knots, 13 km/h), the loss in average speed will be 1.14% (i.e. 0.45 knots, 0.84 km/h).

Conclusions Since we are considering departures from an optimum condition, it follows that errors in gliding speed will have a second-order effect on the loss in average speed, as shown by the foregoing analysis. The example indicates that quite noticeable errors in gliding speed have only a small effect. However, championships are won or lost by quite small margins so one concludes that, whilst it is worth making reasonable efforts to optimise the gliding speed, it is hardly worth going to great trouble to achieve precision. In any case, the available data (e.g. on average rate of climb) is not usually sufficiently accurate to enable one to do so.

It is much more profitable to cultivate one's skill and judgement in the pursuit of improved rate of climb, since this quantity has a first-order effect on the average speed.

It may be shown from the foregoing equations that, under optimised conditions, an increase in rate of climb δv_c improves the average speed by δU_x where

$$\frac{\delta U_x}{U_x} = \left[\frac{U_1^4 + U_o^4}{3U_1^4 - U_o^4}\right] \left[\frac{\delta v_c}{v_c}\right] , \quad (11)$$

or

$$\frac{\delta U_x}{U_x} = F \left[\frac{\delta v_c}{v_c}\right] . \quad (12)$$

F varies from 1 to 1/3, depending on $\overline{U}_1$, as shown in Fig. 3. In the case considered in the example above, it would have a value of 0.43. An increase in average rate of climb of 3%, i.e. from 3.73 knots (1.92 m/s) to 3.84 knots (1.98 m/s), would more than offset the loss in average speed due to errors in gliding speed.

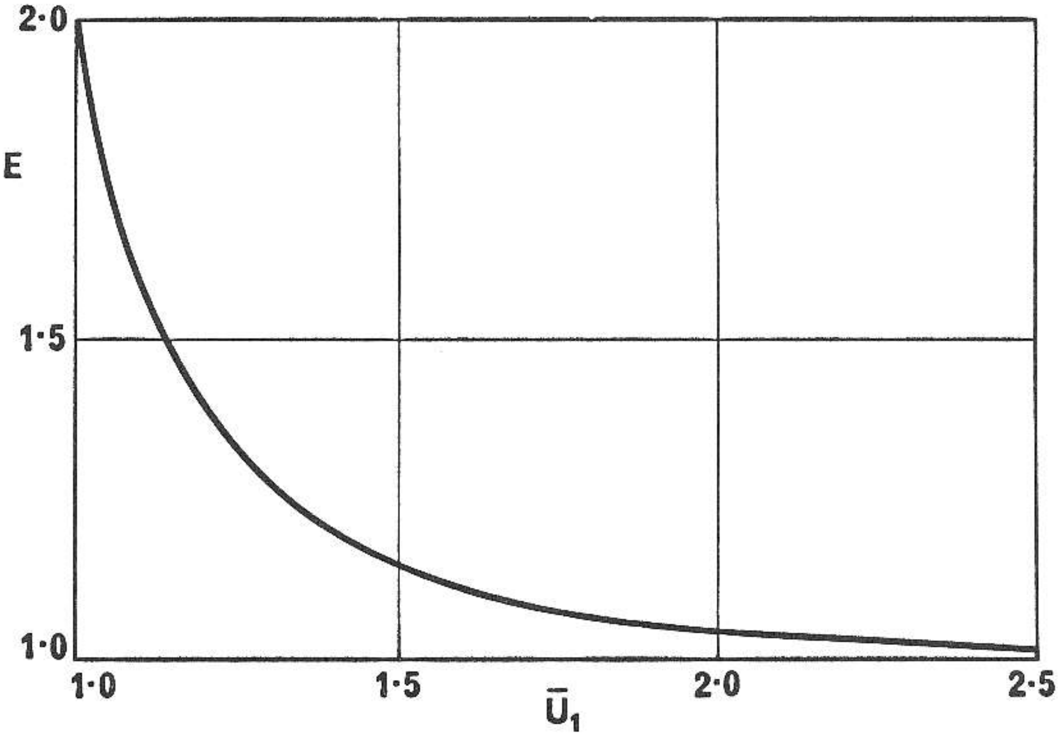

FIGURE 2.

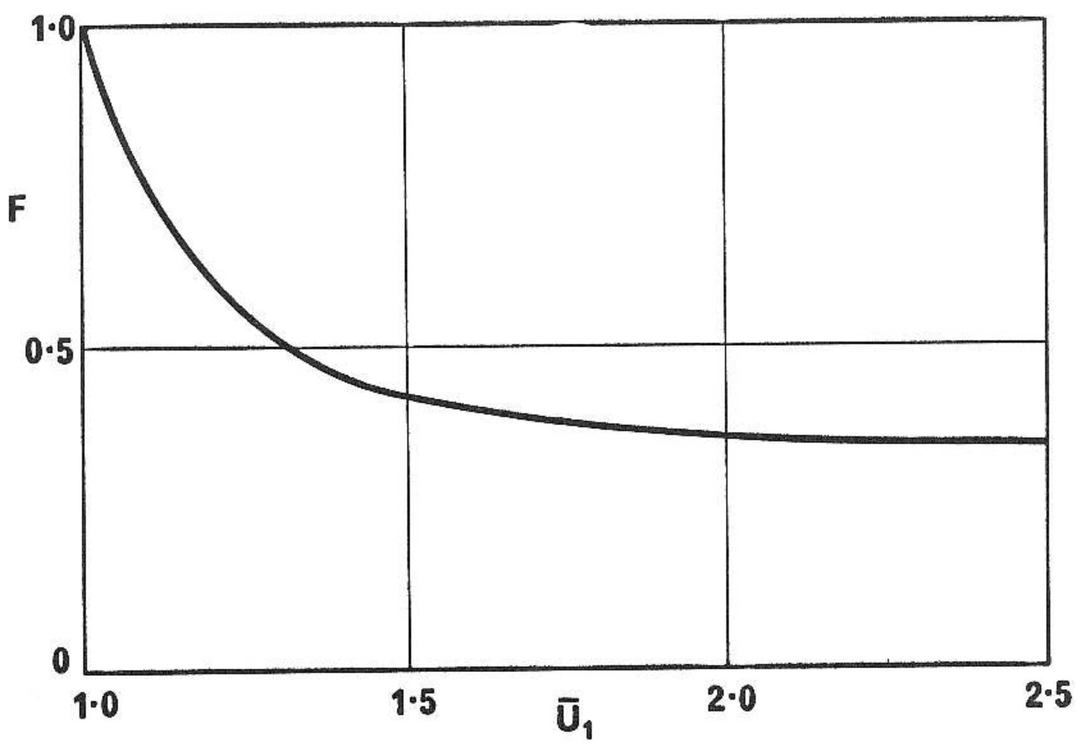

FIGURE 3

REFERENCES

1. Goodhart, H. C. N. "A Note on the Measurement of the Induced Drag Factor (k) of a Glider". OSTIV Publication XI.
2. Irving, F. G. "An Analysis of Goodhart's Figure of Merit". Aero Revue, May 1958.
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SYMBOLS

All speeds are 'equivalent'.

U	forward speed
U_o	the value of U at $(L/D)_{max}$
U_1	optimum gliding speed for a given v_c
U_2	a non-optimum gliding speed
δU	$U_2 - U_1$
U_x	average cross-country speed
U_{x_1}	U_x corresponding to $U = U_1$ } For the same v_c.
U_{x_2}	U_x corresponding to $U = U_2$ } For the same v_c.
δU_x	$U_{x_2} - U_{x_1}$
$\overline{U}$	U/U_o . Similarly, all of the above quantities are rendered dimensionless, e.g. $\overline{U}_x = U_x/U_o$, etc.
v_s	rate of descent in still air
v_{s_o}	v_s at $(L/D)_{max}$
$\overline{v}_s$	v_s/v_{s_o}
v_c	average achieved rate of climb
$\overline{v}_c$	v_c/v_{s_o}
E, F	functions of $\overline{U}_1$.