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AN EVALUATION OF A LEE WAVE FORECASTING NOMOGRAM

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A. INTRODUCTION

Accurate predictions of the occurrence and strength of mountain lee waves are important for the planning of both general and commercial aviation activities over mountainous regions. Associated turbulence activity may cause hazardous flight conditions. From a different point of view, sailplane pilots desire accurate predictions of lee waves in order to take advantage of wave-induced lift for maximum altitude gains and long distance flights. In the present paper, the problem of forecasting lee wave occurrence and strength is examined from the perspective of the soaring forecast.

The author's interest in this problem stems from a recent investigation into the nature of low level turbulent zones associated with mountain lee waves (Fingerhut and Lester, 1973). During the course of that research, it was noted that the application of a widely used lee wave forecast nomogram (Harrison, 1957) gave coarse and, occasionally, incorrect predictions of the wave. In an effort to understand the cause of these inaccuracies, the derivation of the Harrison nomogram was re-examined with more and better data. This investigation and the applications of its results to soaring forecasts are discussed in the following sections.

B. THE LEE WAVE FORECAST NOMOGRAM

Harrison (1957) initially developed his nomogram to predict lee waves and wave induced turbulence with available synoptic data. The site of his study was the Denver, Colorado area in the lee of the Front Range of the Colorado Rocky Mountains.

Wave activity data were derived from commercial and military pilot reports and some special probing flights by uninstrumented military aircraft. A lee wave case was identified whenever one or more of the following criteria were satisfied:

1. One or more characteristic lenticular clouds appeared overhead or to the west (of Denver, Colorado).
2. A strong foehn wall was visible along the west side of the Continental Divide.
3. Appreciable clear air turbulence was reported in the lee of the mountains between Long's Peak and Mt. Evans under non-frontal conditions and westerly winds.

A wave was assumed to be "moderate or strong" whenever one or more reports were received of appreciable turbulence or strong updrafts at flight level (otherwise the wave was classified as weak). Seventy-seven lee wave cases were classified in this manner for the period July 1, 1956 through June 30, 1957.

Wave occurrence and strength (as defined above) were found to be strongly related to the maximum winds at Denver between 10,000 and 18,000 feet ASL and to the cross-mountain sea level pressure difference between Denver and Grand Junction, Colorado, as shown in Figure 1.

Since the initial development of the Harrison nomogram in the mid 1950's, aviation forecasters have adopted the technique to predict lee waves in many areas (e.g., George, 1960). In a later publication, Harrison (1966) presented pressure pairs* for twenty mountain wave regions across the United States. Perhaps the widest use of the nomogram with a few minor modifications (Figure 2), has been made by the Global Weather Central (GWC) of the United States Air Force which makes mountain wave turbulence forecasts for 48 areas throughout the northern hemisphere (Burnett, 1970).

C. A RE-EVALUATION OF THE HARRISON NOMOGRAM

Two important questions related to forecasts of soarable waves based on the Harrison nomogram alone are: (1) How much confidence can one have that a wave will occur when a wave is predicted? and (2) How is the predicted "wave strength" related to the available lift?

1. Confidence in Wave Predictions.

In the development of his original diagram, Harrison (1957) considered only "wave days". That is, days on which the nomogram predicted a wave and a wave did not occur, were not taken into account. In order to place a confidence value on the nomogram predictions, it was necessary to examine these non-wave days.

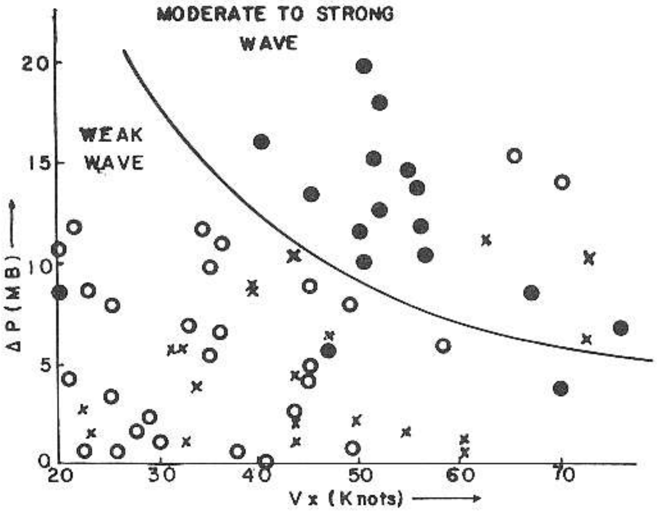

FIGURE 1. Verification of Harrison's Lee Wave Prediction Nomogram. Open circles indicate reported weak cases. Solid circles indicate strong wave cases. ΔP: Grand Junction Sea Level Pressure minus Denver Sea Level Pressure. V_x: Maximum winds 10,000 to 20,000 ft ASL. (From Harrison, 1957) see FIG 3 FOR DESCRIPTION OF X's

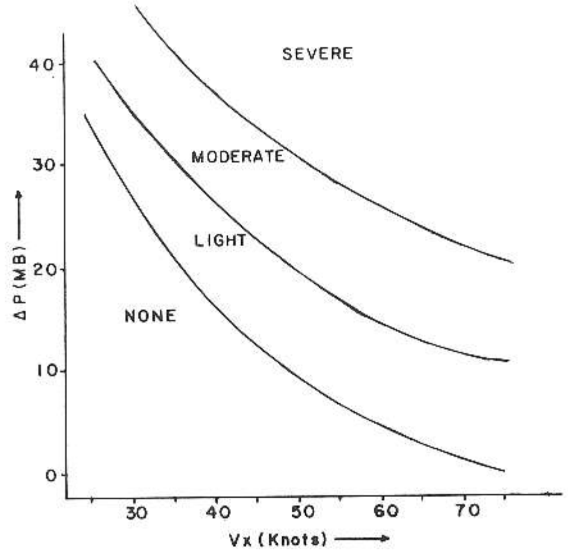

FIGURE 2. GWC Wave Prediction Nomogram. The layer through which the maximum wind is computed depends on the height of the mountain. Intensities refer to turbulence associated with the lee wave (redrawn from Burnett, 1970).

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*"Pressure pairs" refers to the two stations which are utilized to determine the cross mountain sea level pressure gradient for input into the Harrison nomogram.

For the period January 1, 1957-June 30, 1957, the appropriate pressure gradients and maximum wind speeds (10,000-18,000 feet ASL) were tabulated from available weather records for 116 non-wave days. Surface data for 1230 GMT (0530 MST) and rawinsonde data for 1500 GMT (or 1200 GMT) were utilized. The results of combining Harrison's wave days with the non-wave days are shown in Table 1.

TABLE 1. CONTINGENCY TABLE FOR THE PREDICTION OF LEE WAVE OCCURRENCES.

		Observed	
		Wave	No Wave
Forecast	Wave	45	20
	No Wave	0	96

Percent correct	88%
Skill score	0.73
Level of significance (X^2 Test)	99.9%
Sample	161 days

The accuracy of the forecast of wave occurrence is notably high as are the skill and level of significance based on the Chi-squared test. The tentative conclusion is that Harrison's nomogram is a highly dependable device for forecasting lee wave occurrence (assuming, of course, that sea level pressure differences and winds aloft are forecasted accurately).

The confidence level presented above applies to all forecasted wave cases, irrespective of strength. Table 2 presents the monthly frequency of lee waves as a function of strength for the period of Harrison's (1957) study. Assuming that these statistics show the typical annual distribution of lee wave occurrence and strength, it is apparent that the large majority of lee waves are "weak" (65% of the total in Table 2). Therefore, annually, most wave cases will fall in the lower left hand section of the Harrison nomogram where pressure differences and wind speeds are marginal. Figure 3 illustrates this situation. The problem for the soaring forecaster is obvious: He has much less confidence in predicting the more frequent, weak cases than moderate or strong cases.

TABLE 2. FREQUENCIES OF WAVES AS A FUNCTION OF MONTH AND REPORTED WAVE STRENGTH (derived from Harrison, 1957)

Strength	Weak	MDT	Strong	Total
Month (1956-1957)				
JUL				
AUG	1			1
SEP				
OCT	10	1		11
NOV	8	2		10
DEC	4	5	1	10
JAN	5	3	4	12
FEB	7	5	2	14
MAR	4		1	5
APR	3	1		4
MAY	1	1		2
JUN	7	1		8
TOTAL	50	19	8	77

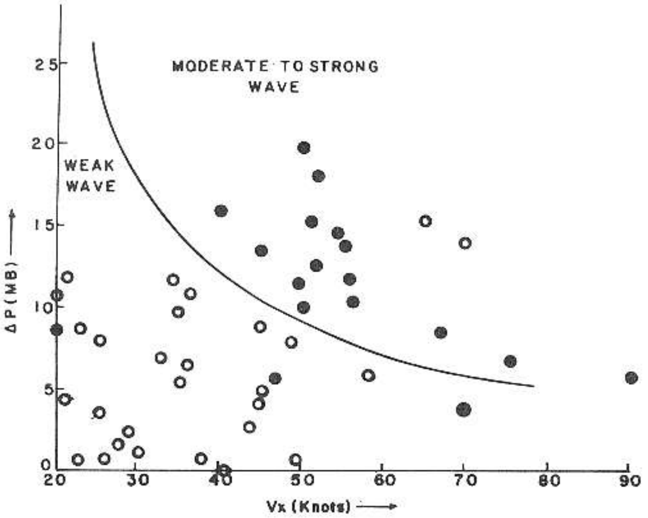

FIGURE 3. Same as Figure 1, But With 'no-wave' Cases Plotted (indicated by 'x'). The probability of wave occurrence is zero for $\Delta P < 0$, and/or $V_x < 20$ kts.

Although this problem cannot be solved, the confidence in the forecasts can be expressed quantitatively through the analysis of the wave and no-wave data of Figure 3. Figure 4 shows the results of such an analysis in the form of isopleths of probability of wave occurrence. It is interesting to note that the probability of lee waves is nearly independent of wind speed in the 30 to 55 knot range and, beyond 55 knots, probabilities clearly decrease with wind speed. It is also noted that the line which separates weak and moderate to strong waves intersects the probability isopleths at relatively large angles. This feature is related to the procedure that Harrison used to determine wave strength.

2. Wave Strength Versus Available Lift.

In developing and applying the Harrison nomogram, forecasters have had one primary aim: to forecast turbulence associated with the lee wave. In fact, one of the primary measures of "wave strength" in each of Harrison's (1957) cases was the occurrence of clear air turbulence. Thus, in the nomogram (Figures 1-4) "wave strength" technically should be called "turbulence strength". This differentiation is critical for the soaring forecaster who wishes to apply Harrison's nomogram, because he must either relate turbulence intensity to a useful estimate of available lift or he must modify the prediction nomogram in such a manner that the desired forecast may be made directly.

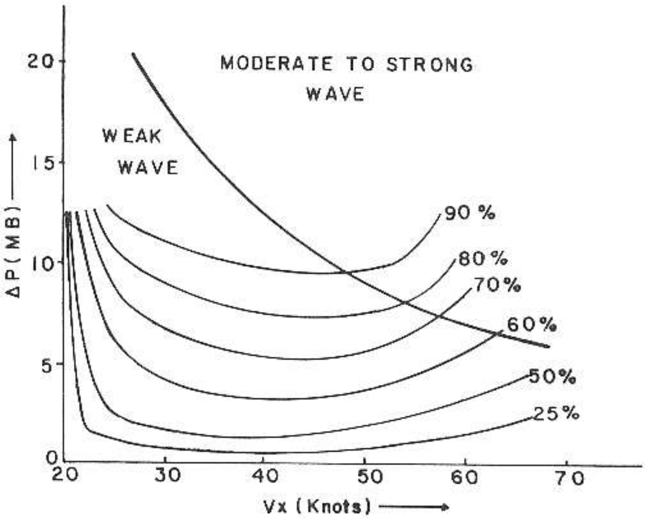

FIGURE 4. Harrison Nomogram Showing Isopleths of Probability of Wave Occurrence

Although it is known from detailed studies of lee waves that the areal extent of heavy turbulence in lower levels is roughly proportional to the amplitude of the lee waves, it is also known that locally severe patches of turbulence can occur in lee waves that are relatively weak (Fingerhut and Lester, 1973). Therefore conventional pilot reports of turbulence, in addition to being subjective, are not necessarily representative of the strength of a lee wave in terms of lift for sailplanes.

A better estimate of the soarability of waves can be derived from direct measurements of wave characteristics by either instrumented aircraft or radar-tracked balloons. Although data of these types are not regularly collected, several short-term, intensive lee wave studies have been carried out over the past twenty years resulting in the production of a moderate-sized sample of detailed lee wave data. In the present study, 44 lee wave cases were selected from the reports and other publications of the Sierra Wave Project (SWP) and the Colorado Lee Wave Program (CLWP) in an attempt to further refine Harrison's nomogram. Table 3 lists the cases, sources of data and other pertinent information.

Cross-mountain sea level pressure differences for each wave case were calculated for the "pressure pairs" (including appropriate corrections for distance variations) recommended by Harrison (1966). Pressures were corrected with the reported pressure tendency to the central time of the aircraft or balloon flights prior to the computation of pressure differences.

The maximum wind between 10,000 and 20,000 feet above sea level (ASL)* was taken from the sounding

TABLE 3. LIST OF OBSERVATIONAL PERIODS, PRIMARY MODES OF OBSERVATION, AND SOURCES OF INFORMATION FOR SWP AND CLWP.

	Cases	Project[1]	Mode of Observation[3]	Source of Information
27 Nov 51- 30 Mar 52	10	SWP	A	Holmboe and Klieforth (1957)
29 Mar 55- 25 Apr 55	3	SWP	A	Holmboe and Klieforth (1957)
8 Dec 66- 28 Mar 67	13	CLWP	B	Vergeiner and Lilly (1970)
15 Feb 68- 25 Feb 68	5	CLWP	A	Kuettner and Lilly (1968) Fingerhut and Lester (1973)
13 Feb 70- 24 Mar 70	10	CLWP	A	Lilly (1971) Lilly et al. (1971)
8 Dec 70- 20 Jan 71	2	CLWP[2]	A	Fingerhut and Lester (1973) Julian and Zipser (1971)
11 Jan 72	1	CLWP[2]	A	Lilly and Zipser (1972)
TOTAL	44			

[1] SWP: Sierra Wave Project

CLWP: Colorado Lee Wave Program

[2] Special Windstorm Studies

[3] A: Aircraft and/or radar-tracked sailplanes

B: Radar-tracked balloons

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*In work subsequent to his 1957 study, Harrison extended the layer in which the maximum wind was determined from 10,000 to 18,000 feet to 10,000 to 20,000 feet.

which was closest in time and space for each case. Two slight modifications were introduced in the maximum wind determinations. First, only upstream soundings were used because of their regular availability and because they were assumed to be more representative of the air mass before it was disturbed by lee waves. Second, in an effort to make the treatment of wind profiles as objective as possible, all winds were resolved into components perpendicular to the crest of the mountains. Forty-three of the 44 wind soundings were taken within 5 hours of the central time of the lee wave flights and within 350 km of the lee wave region.

Figure 5 shows the 44 wave cases plotted on the Harrison nomogram. As was expected from the last section, the occurrence of the large majority of wave cases (irrespective of strength) was predicted correctly. If errors of 1 mb in pressure difference and 3 mps in wind speed are allowed, all of the occurrences are correctly indicated by the diagram. It should be noted, however, that there are no "no-wave" cases plotted (i.e., no cases where aircraft or balloons were flown and waves were not observed).

As a first approximation of wave strength (not turbulence intensity), independent estimates were acquired for 24 cases from the references cited in Table 3. These wave strength estimates are those of the individual investigators and are plotted in Figure 6. The most outstanding feature of the diagram is the tendency for moderate cases to occur well into the region identified as 'weak' in the Harrison nomogram.

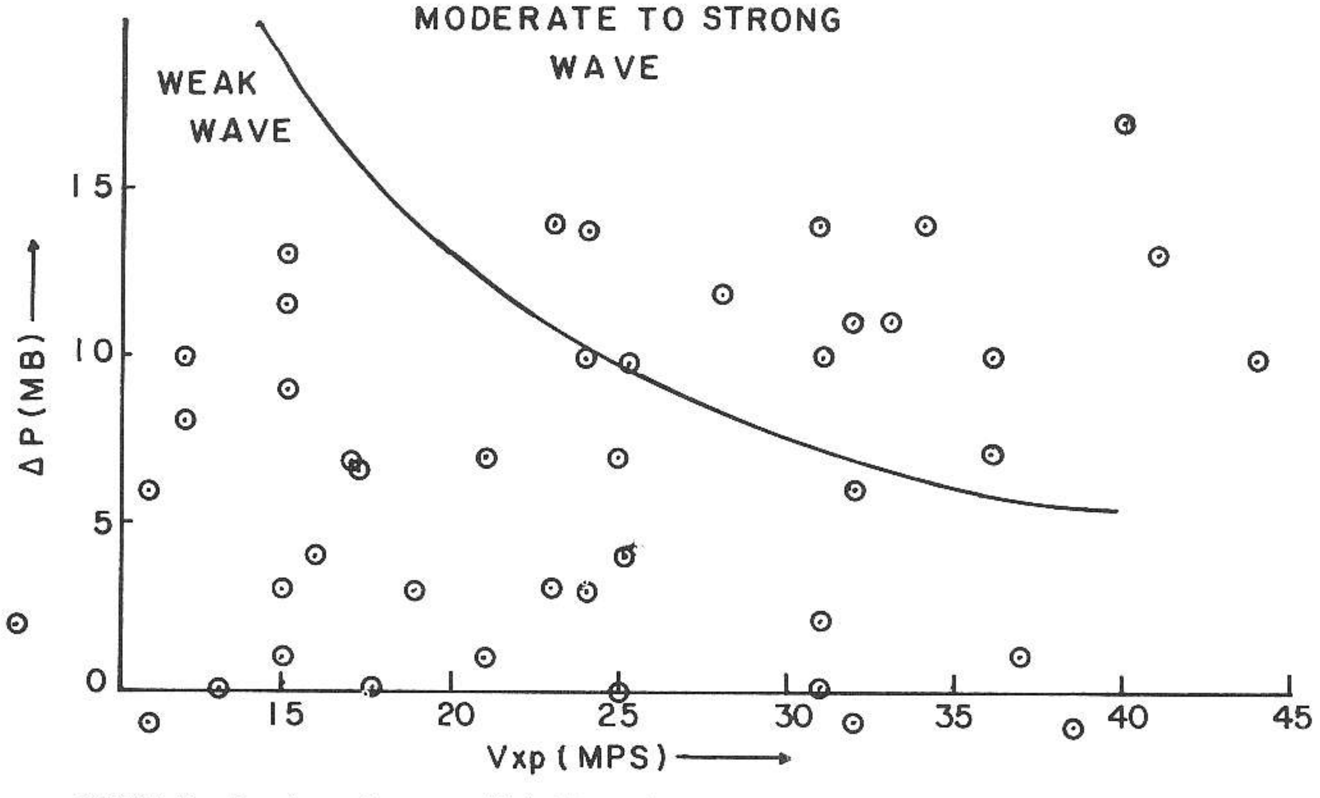

FIGURE 5. Harrison Nomogram With SWP and CLWP Data. Lee waves were observed in all cases. For CLWP cases, ΔP is the sea level pressure difference between Grand Junction and Denver, Colorado. ΔP for SWP is the difference between the sea level pressures at Fresno, California and Tonopah, Nevada plus 2 mb. V_{xp} is the maximum wind component perpendicular to the ridgeline between 10,000 and 20,000 feet.

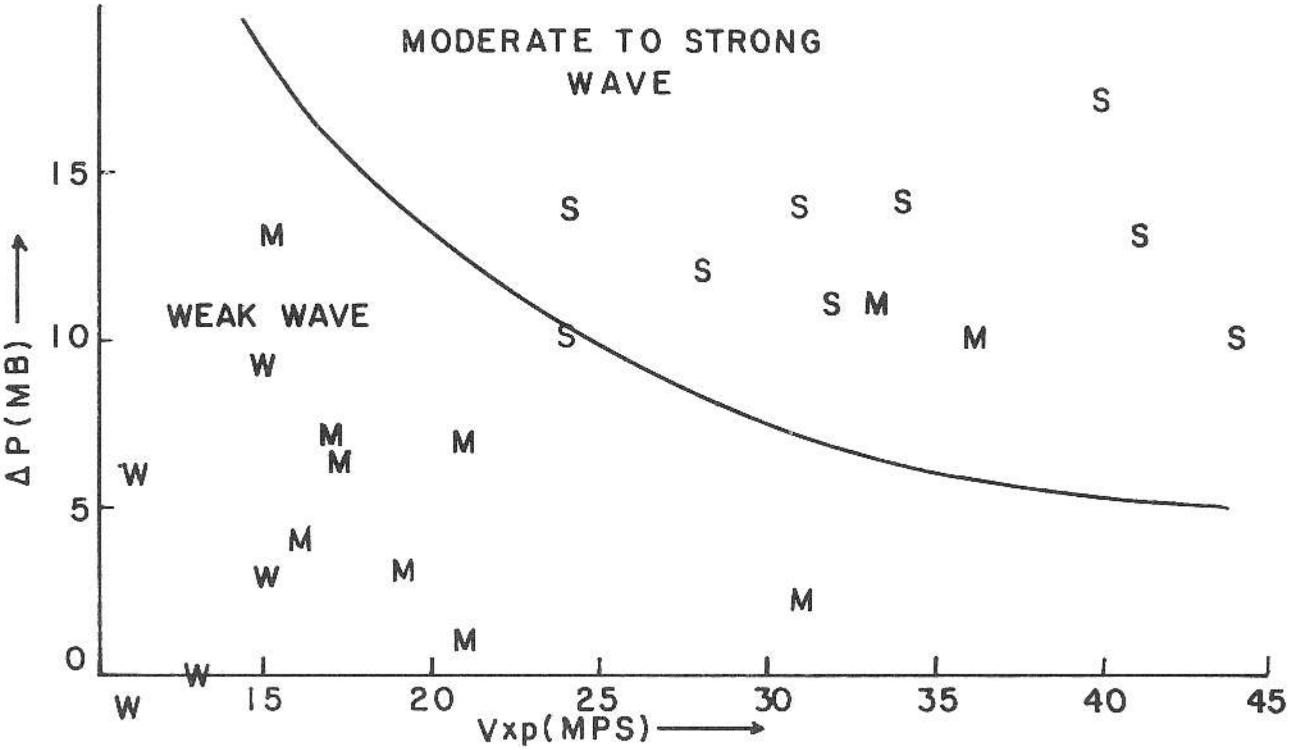

FIGURE 6. Same as Figure 5 With Subjective Strength Classifications assigned by SWP and CLWP investigators. S: Strong; M: Moderate; W: Weak wave.

Because of the inherent subjectiveness of many of the estimates plotted in Figure 6, in addition to the relatively small sample, an objective strength criterion developed by SWP investigations (Holmboe and Klieforth, 1957) was applied to the CLWP data. The criterion are shown in Table 4.

TABLE 4. WAVE STRENGTH CRITERION UTILIZED IN SWP (from Holmboe and Klieforth, 1957). λ: wave length, $2A_x$: maximum double amplitude in the troposphere, W: vertical velocity.

Wave Strength	λ(km)	$2A_x$ (km)	W (mps)
Strong	13-32	1.22-2.44	9.1-18.3
Moderate	8-13	0.61-1.22	4.6-9.1
Weak	4-8	0.15-0.61	1.5-4.6

Some problems were experienced in the application of the criterion in Table 4. First, the wave length (λ) category almost always resulted in a strong classification. The length of the primary wave was measured whenever possible. Second, the vertical velocity (W) was not given in all cases and often had to be estimated from the isentropic analyses and available wind soundings. Most vertical velocities fell in the weak and moderate categories of Table 4. The maximum double amplitude ($2A_x$) was measured with the most confidence and, therefore, was weighted heaviest in the wave strength determinations for the CLWP data. The results of the objective intensity classification are shown in Figure 7. In general, the objective method either increased the subjective estimate (Figure 6) or did not change it (e.g., from weak to moderate).

From Figures 6 and 7 it appears, therefore, that the 'strength' of lee waves (as defined in Table 4) is such that the line which separates "weak" from "moderate to strong" (turbulence) in Harrison's diagram should be designated as the boundary between moderate and strong waves for wave soaring forecasts. Furthermore, a second line should be added to separate weak and moderate waves.

In order to determine a more quantitative measure of wave strength, available vertical velocities (W) and wave double amplitudes (2A) were averaged over several sectors of the Harrison nomogram and subsequently analyzed to produce fields of mean values of those parameters. The results of these analyses have been combined with the frequency of occurrence analysis (Figure 4) in a wave soaring prediction nomogram (Figure 8).

The shaded area in Figure 8 represents frequencies of lee wave occurrence of less than 50%. The "weak", "moderate" and "strong" wave areas were selected on the basis of the results of the strength classifications derived from SWP and CLWP data (Figures 6 and 7). Isopleths of $\bar{w}$ = 2.5 and 4.0 mps and $\overline{2A}$ = 1 km have been included to enhance the strength estimate.

In order to utilize Figure 8 for a lee wave region other than the Denver, Colorado, pressure differences must be corrected to a distance compatible to the Denver-Grand Junction, Colorado separation (about 320 km). As a first approximation this may be done simply by multiplying the observed pressure difference by the ratio between the Denver-Grand Junction distance and the actual distance. The need for further refinement of the correction should become obvious as one gains experience with the diagram. Some guidance may be found in Harrison (1966).

For mountain ranges with mean heights which are lower than the Rockies or Sierra Nevada, it is recommended (e.g., Burnett, 1970) that the wind speed maximum be derived from the 3 km (10,000 foot) layer above the peaks. For example,

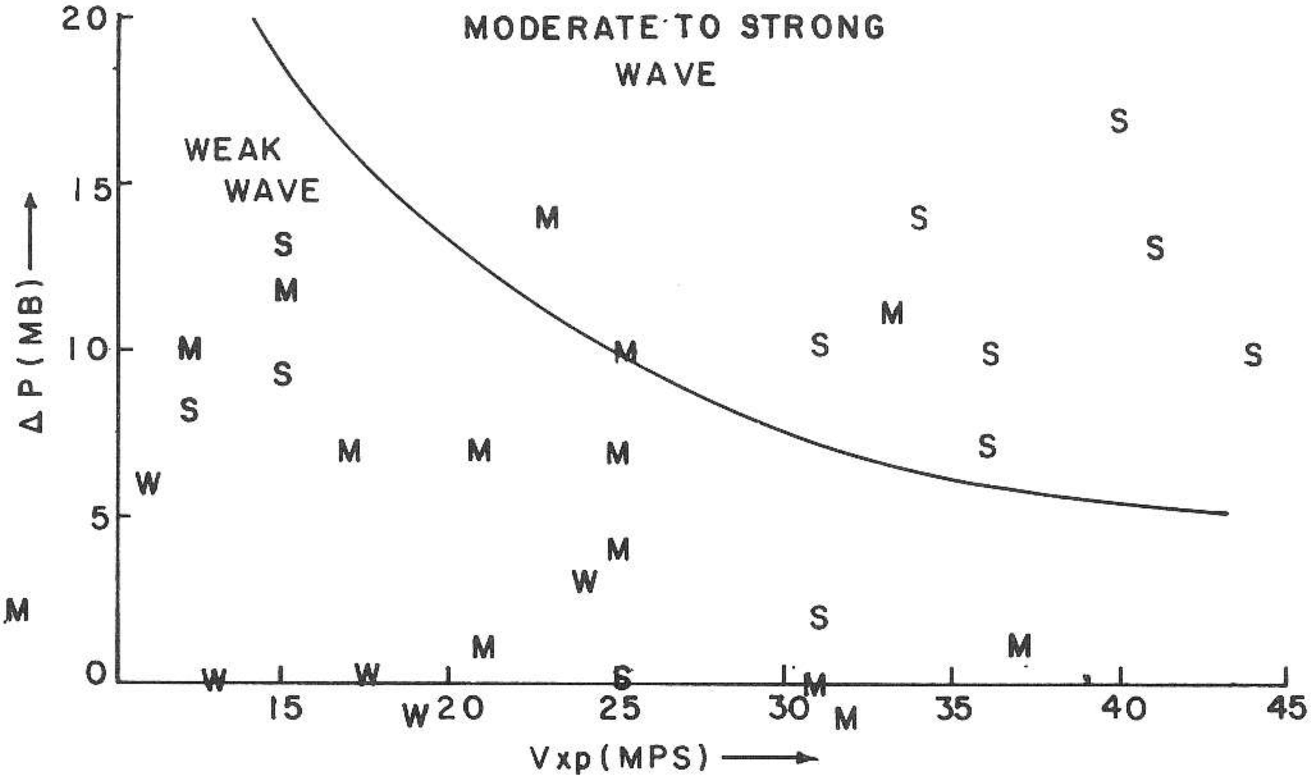

FIGURE 7. Same as Figure 6 With Objective Strength Classifications. See text for details.

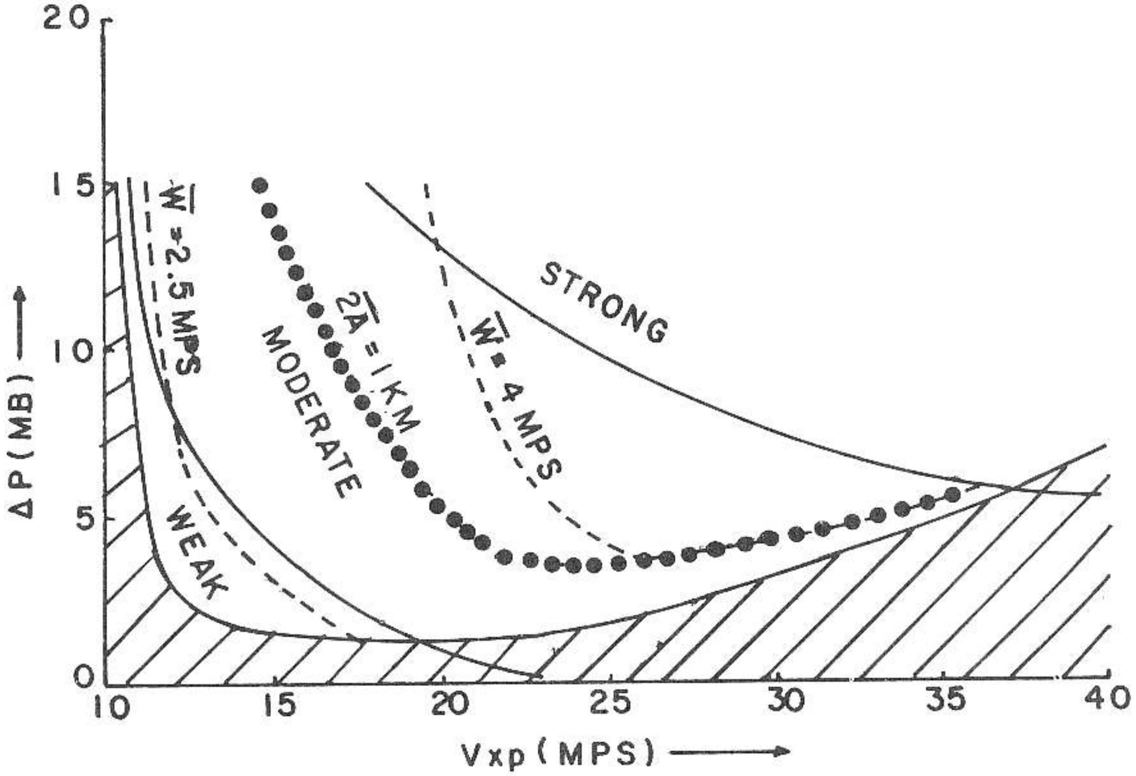

FIGURE 8. Wave Soaring Prediction Nomogram. Dashed lines: Mean vertical velocity isopleths; Dotted line: isopleth for mean double amplitude equal to 1 km. Vertical velocities and double amplitudes increase to the upper right of the diagram.

if the mountain tops are near 1.5 km, one would consider the layer 1.5-4.5 km for maximum winds. Experience will lead to modifications.

As a final point, it was noted during the course of the study that, for relatively high mountain ranges, (e.g., the Rockies), the 500 mb wind gave a quick (although slightly less accurate) estimation of the maximum wind for the layer 10,000 to 20,000 feet ASL. For some forecasts, the convenience of utilizing the 500 mb wind may outweigh the inaccuracies.

D. Conclusions and Recommendations

A re-evaluation of Harrison's (1957) lee wave prediction nomogram has resulted in the design of a diagram for wave soaring forecasts. The diagram is simple to use and may be adapted to any lee wave area. It takes into account both occurrence and strength of lee waves in such a manner that the wave soaring prospects can be assigned a rough confidence level (e.g., greater or less than 50% probability). Further refinement and modification of the diagram is recommended. The lack of sufficient data has not allowed the diagram to be verified statistically and, therefore, this should be done as data become available.

The accuracy of the diagram is strongly dependent on accurate forecasts of sea level pressure gradients and winds aloft in the area of anticipated waves. Thus, one should not be misled by the high levels of accuracy stated in the last section; as with most meteorological forecasts, the accuracy of the device will decrease as the length of the forecast period increases.

The accuracy of the diagram also depends on the frequency and period of observations. It is well-known that large changes in the strength of lee waves can take place in very short periods, for example, with a frontal passage (Fingerhut and Lester, 1973). The scatter and probabilistic nature of the data presented here reflect to some extent the fact that lee wave data are typically gathered over periods of a few hours and that associated upper air data may be a few hours and a few hundred kilometers removed from the time and place of the wave occurrence.

It should also be stressed that the wave soaring prediction nomogram is based on primarily mid and lower tropospheric data. Nothing can be said explicitly about the vertical extent of the waves predicted by the forecast aid.

Finally, the results of the present study suggest that more work should be done with the theory of lee waves. Primarily, it is not well-understood why the frequency of occurrence of lee waves should decrease for large wind speeds at a given ΔP.

E. Acknowledgments

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