IS A "SPEED RING" NECESSARY

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ABSTRACT

This paper presents a variation of the problem of maximizing cross-country speed. The intent is to emphasize that the variometer reading alone is sufficient for accomplishing this objective. The maximizing problem is formulated in terms of a sailplane parameter called the 'p' parameter. The variation of this parameter is investigated for both real and hypothetical sailplanes. The data used for actual sailplanes is a portion of that which has been published by Bikle. Sink rates to be used during an interthermal glide are computed for a range of soaring conditions. The sink rates may then be used to infer the applicability of the approximate rule "fly at a down indication equal to the achieved rate of climb in the last thermal". It is shown that this rule is reasonable for the sailplanes considered. Finally, certain properties of the 'p' parameter are identified; in particular, a high-speed limit is approached which suggests that a sailplane can achieve a cross-country speed of two-thirds of the air-speed if atmospheric sink and wind are negligible.

INTRODUCTION

The question of the speed to fly between thermals in a well instrumented, high performance sailplane frequently arises. A neophyte poses the question: how does one apply the technique to club sailplanes, most of which do not have speed rings. Of course, speed rings could be fitted to most variometers at a modest investment for a club. However, the speed ring is really an unnecessary complexity; the variometer indication alone can be used to maximize cross-country speed. In fact, flying by airspeed could contribute error if the proper calibration is not known.

It will be shown that the approximate rule "fly at a down indication equal to the achieved rate of climb in the last thermal" is indeed quite good and could be applied for nominal conditions of soaring where climbs of 200 to 700 feet per minute are achieved.

MAXIMIZING CROSS-COUNTRY SPEED

It has been amply demonstrated (MacCready, et al) that an optimum airspeed exists for maximizing cross-country speed. This optimum speed to fly depends on thermal strength, sink between thermals, and the glide polar of the sailplane. Carmichael[1] summarized much of the existing work on the subject. The approach in this paper is somewhat different than Carmichael's as airspeed is virtually excluded as a predominant parameter.

Maximizing average airspeed will maximize average ground speed independent of wind speed and direction so the wind is not a consideration for interthermal glide speed. The ground speed made good does of course depend on wind speed and is given by the relation

$$1)\quad V_{eff} = \frac{Vt_g - V_w(t_g + t_c)}{(t_g + t_c)}$$

where Vt_g is the distance flown relative to the air while gliding, and $V_w(t_g + t_c)$ is the distance the air mass translates relative to the ground during both the climb and glide. Equation (2) is based on the assumption that altitude h lost during a glide will be gained in the next thermal; t_g and t_c are replaced by h/w_g and h/w_c respectively, and equation (1) becomes:

$$2)\quad V_{eff} = \frac{Vw_c - V_w(w_c + w_g)}{(w_c + w_g)}$$

Given a climb rate w_c, we wish to select a variometer reading w_g that will maximize V_{eff}. Applying the maximizing condition

$$\frac{dV_{eff}}{dw_g} = 0 = \frac{[w_c(dV/dw_g) - V_w]}{(w_c + w_g)} - \frac{[Vw_c - V_w(w_c + w_g)]}{(w_c + w_g)^2}$$

In general, $(w_c + w_g) \neq 0$ so the maximizing condition becomes:

$$w_c + w_g = V(dw_g/dV)$$

The down indication between thermals is the sum of sailplane sink relative to the air, w_s, and atmospheric sink w_a.

$$w_g = w_a + w_s$$

If the atmospheric sink is assumed nearly constant:

$$dw_a = 0,\ dw_g/dV = dw_s/dV$$

3) $w_c + w_g = V(dw_s/dV$

The right hand side of equation (3) is strictly a sailplane characteristic which can be determined from the sailplane polar. It is convenient to write equation (3) as:

4) $(w_c + w_g) = (w_s)(V/w_s)(dw_s/dV) = (w_g - w_a)(V/w_s)(dw_s/dV)$

EVALUATION OF THE GENERALIZED PERFORMANCE PARAMETER

The parameter $p = (v/w_s)(dw_s/dV)$ is a non-dimensional performance parameter and must be obtained from the sailplane polar of w_s (sink speed) versus V (air speed). Because w_s and V are functionally related by the performance polar, the parameter $(v/w_s)\cdot(dw_s/dV)$ may be thought of as a function of either w_s or V. For our purposes, it is convenient to consider it a function of w_s. For this paper, the parameter p was calculated using some of the most realistic sailplane data available; that as presented by

	AIRSPEED - KNOTS							
	40	50	60	70	80	90	100	110
	Sink Rate (W_s) Feet Per Minute And Computed parameter $p = (V/w_s)(dw_s/dV)$							
Kestrel	148 -2.678	132 0.996	168 1.532	219 1.913	287 2.105	372 2.374	495 3.098	672 2.923
Diamant	122 -1.657	131 1.337	168 1.415	219 2.142	307 2.844	435 3.011	598 3.044	803 3.123
Phoebus C	134 -1.594	134 1.264	184 2.054	257 2.234	347 2.269	458 2.502	609 2.881	790 2.216
Cirrus	138 -2.550	136 1.228	173 1.473	230 2.252	319 2.537	430 2.580	577 3.052	766 2.374
T-6	130 -0.793	140 1.115	179 1.540	236 2.114	326 2.675	450 2.704	590 2.454	758 3.120
Phoebus A	151 -2.269	152 1.461	207 1.845	282 2.166	380 2.257	497 2.360	655 2.967	890 3.234

TABLE 1. Computed Value of the Parameter p.

Bikle[2]. Sink rate at air speeds of 40 through 110 knots, in 10-kt increments, was used to define the polars of six sailplanes.

Initially, an attempt was made to determine the parameter p graphically by drawing a tangent to the polar at various points and measuring the slope (dw_s/dV). It was found that significant variations in the result, following repeated trials, were obtained using this technique, especially at the extreme ends of the polar. It was decided to abandon the graphical technique as being too subjective. Instead, the polar of a particular sailplane from 40 to 110 knots was represented by a seventh-order polynomial using Newton's forward formula. The polynomial goes through each of the eight given data points defining the polar points and the slope of the polynominal is analytically calculated at each of these points. The results of these computer calculations are given in table (1) and figure (2). The results are not gratifying in that the curves for different sailplanes show variations that are difficult to explain and suggest that this parameter is sensitive to small errors in the data. Differentiations of discretely defined functions in general present difficulties.

OPTIMUM DOWN INDICATION BETWEEN THERMALS

An "outer envelope" and an "inner envelope" were drawn to bracket the p - parameter for all six sailplanes investigated. (figure 1). The equation:

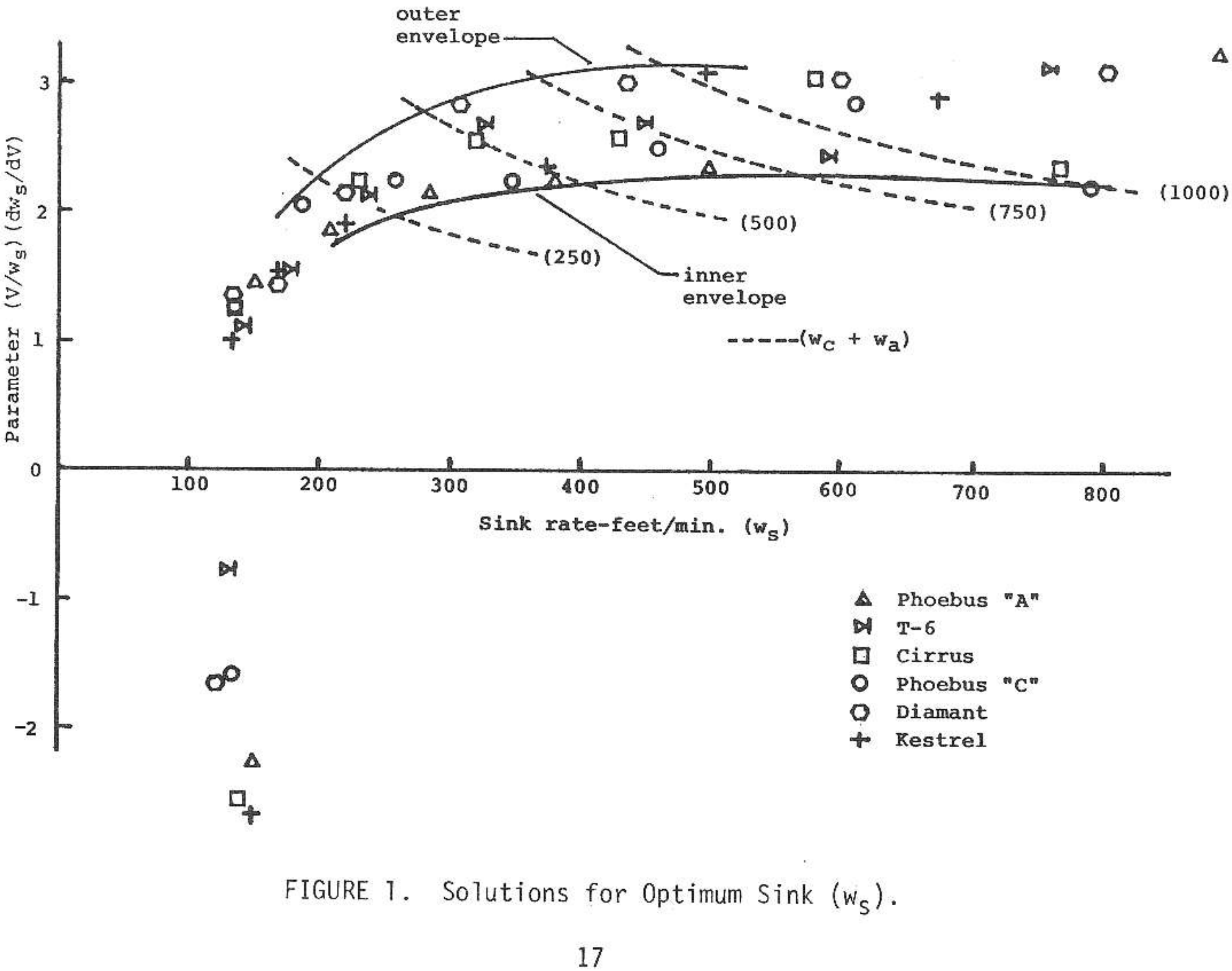

FIGURE 1. Solutions for Optimum Sink (w_s).

$$w_c + w_g = w_s P$$

is re-written in the form:

$$(w_c + wa) = (w_s(p-1)$$

and is cross-plotted on figure (1) for selected values of $(w_c + w_a)$. Intersection of these curves of constant $(w_c + w_a)$ with the p-envelope curves define the limits of w_s for the data presented. Table (2) gives the glide

BASIC SOLUTIONS FROM FIGURE 1

$(w_c + w_a)$ fpm	w_s Outer Envelope	w_s Inner Envelope
250	200	260
500	280	400
750	370	570
1000	470	820

Optimum Glide Solution (W_g) When Atmospheric Sink (w_a) is 20% of Achieved Climb Rate (w_c)

Climb (w_c) fpm	Atm. Sink (w_a) fpm	Optimum Glide $(w_g) = (w_s + w_a)$ Outer Envelope	Inner Envelope	Average
208	42	242	302	272
417	83	363	483	423
625	125	475	695	595
834	166	636	986	811

Same as Above But With (w_a) as 25% of (w_c)

Climb (w_c) fpm	Atm. Sink (w_a) fpm	Optimum Glide $(w_g) = (w_s + w_a)$ Outer Envelope	Inner Envelope	Average
200	50	250	310	280
400	100	320	500	440
600	150	520	720	620
800	200	670	1020	845

TABLE 2. Optimum Glide Solutions Based on Figure 1.

variometer reading to be used based on the envelopes, and in addition, presents an average of the two envelope extremes. It will be demonstrated that this average, which is close to what one might have using a best fit curve through all the data points, is very close to a theoretical result which will be presented.

Table (1) includes some assumptions as to atmospheric sink between thermals. Under normal cumulus conditions, it is generally agreed that there is sinking of the air between the thermals and that the strength of the sink increases with an increase in thermal strength, although other factors, such as the area coverage and number density of thermals, are also very important. The selected values of w_a = 20% and 25% of w_c are considered reasonable by this author. Considering the uncertainty in the estimate of w_c (our average climb rate) and variations experienced during the interthermal slide, the differences between the appropriate w_g and w_c of table (1) indeed appear small.

ANALYTIC FORMULATION OF THE P- PARAMETER

Referring again to the data of Bikle, the curves of drag coefficient versus lift coefficient squared are reasonably straight lines for a lift coefficient range of about 0.1 to 0.8. This means the classic model of drag variation; $C_D = C_{D_e} + C_L^2/\pi ARe$ can be applied to sailplanes. Using this model of drag variation, it can be shown that the p-parameter takes the form:

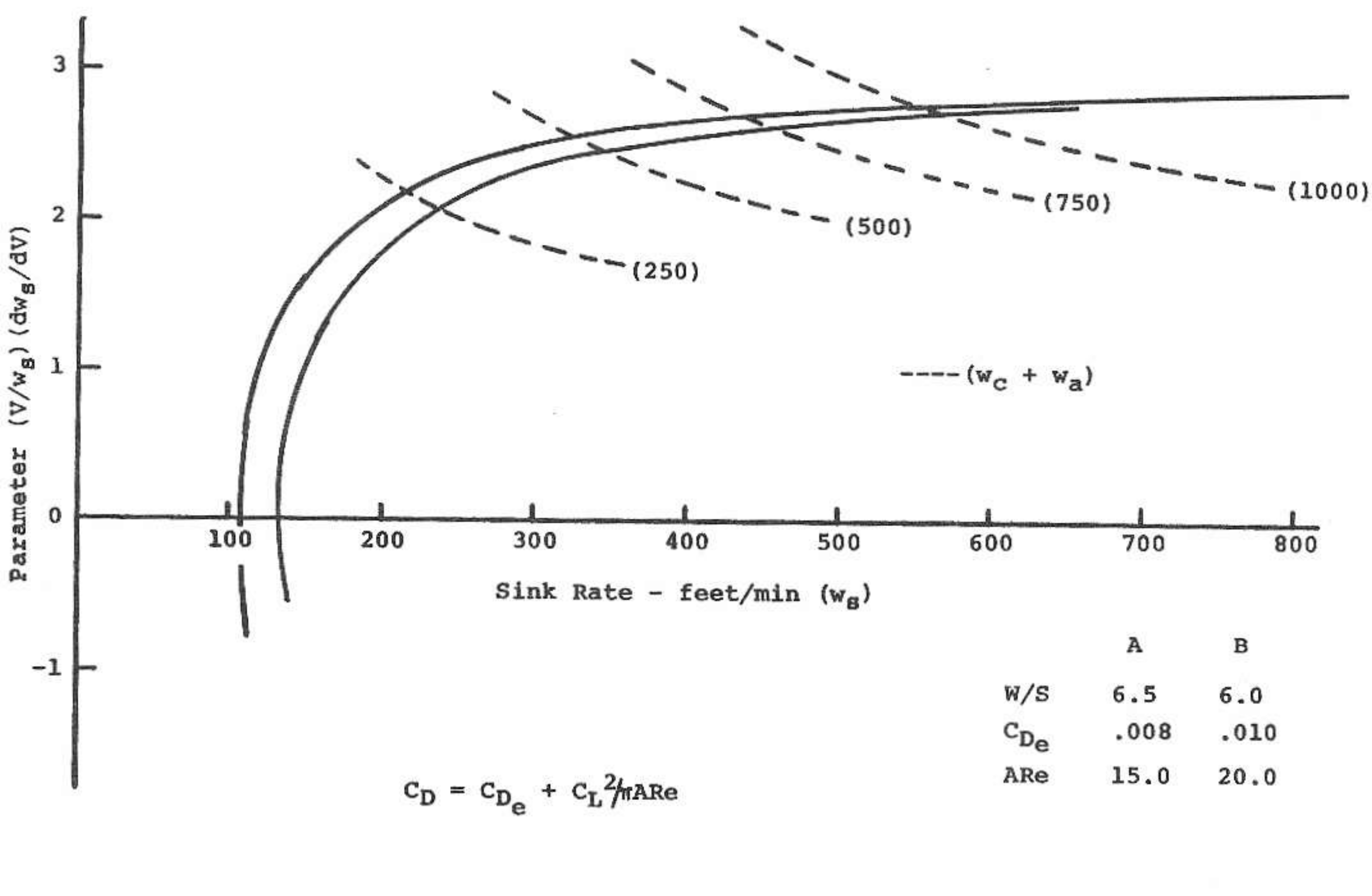

FIGURE 2. Solution for Optimum Sink (w_s) Based on Model Drag Relation.

$$p = 1 + \left[\frac{2}{1 + \frac{1}{\pi ARe}}\right]\left[\frac{C_{D_e} - \frac{C_L^2}{\pi ARe}}{C_{D_e} - \frac{C_L^2}{\pi ARe}}\right]$$

$$\doteq 1 + 2\left[\frac{C_{D_e} - \frac{C_L^2}{\pi ARe}}{C_{D_e} + \frac{C_L^2}{\pi ARe}}\right]$$

The second approximate form is applicable as for high performance sailplanes

$$\frac{1}{\pi ARe} << 1.$$

Several limits of the p-parameter are immediately apparent although not within the operating range of the aircraft:

a) High speed: $C_L \to 0$, $p \to 3$

b) Low speed: $C_L \to \infty$, $p \to -1$

The conditions of interest which are in the operating range of the sailplane are

c) Minimum Sink, $C_{D_e} = \frac{1}{3}(C_L^2/\pi ARe)$,

$p = 0$ (maximum $C_L^{3/2}/C_D$)

d) Best L/D, $C_{D_e} = C_L^2/\pi ARe$,

$p = 1$ (maximum C_L/C_D

e) $C_{D_e} = 3(C_L^2/\pi ARe)$,

$p = 2$ (maximum $C_L^{1/2}/C_D$)

As the glide speed for cross-country soaring lies between speed for best L/D and maximum speed, the theoretical value of the p-parameter must lie between 1 and 3. The p-parameter can be graphed versus air speed using the relations:

$$\tan \nu = -C_D/C_L, \; V = \sqrt{\frac{2W\cos\nu}{\rho S C_L}}$$

Figure (2) shows the variation of the p-parameter for two combinations of the parameters W/S (wing loading), C_{D_e} (zero lift intercept of drag coefficient) and ARe (effective aspect radio). It is observed that curves (A and B) of figure 2 are nearly the mean of the inner and outer envelope of figure (1). The C_{D_e} range of .008 to .010 is representative of high performance sailplanes as are effective aspect ratios of about 15 to 20.

SPEED LIMIT FOR STRONG CONDITIONS

Under very strong conditions, the parameter p approaches the value 3. Under the same limiting conditions, the solution becomes

$$w_c + w_g = 3w_s = 3(w_g - w_a)$$

or

$$w_g = \frac{1}{2}w_c + \frac{3}{2}w_a$$

The effective air speed becomes:

$$V_{eff} = V\left(\frac{w_c}{w_c + w_g}\right) = V\frac{w_c}{\frac{3}{2}(w_c + w_a)}$$

$$V_{eff} = \frac{2}{3}V\left(\frac{w_c}{w_c + w_a}\right)$$

If we neglect atmospheric sink w_a, the upper limit of effective airspeed becomes two-thirds of the flight speed between thermals. Under very strong conditions, the limit on V_{eff} becomes the airspeed we are willing to drive the glider, V. It appears that many sailplanes are already approaching this V_{eff} limit in contest triangles.

CONCLUSION

Performance data for six sailplanes has been evaluated in terms of a generalized 'p' parameter. This parameter has also been determined theoretically for two hypothetical sailplanes for which the polars exhibit a quadratic variation of drag coefficient with lift coefficient. Both of the hypothetical sailplanes' performance curves appear to fall almost centrally within the envelope of points for the six actual sailplanes.

The broad width of the envelope representing data for the six sailplanes could be in part due to error in the data, or, in the technique employed for computation of the 'p' parameter. Further study should be made in both analysis and collection of data.

The 'p' parameter is used to determine the variometer reading to be used during an interthermal glide such that cross-country speed will be maximized. A knowledge of atmospheric sink is presumed. It has been demonstrated that if atmospheric sink is assumed linearly related to the thermal strength, the approximate rule, "fly at a down indication equal to the achieved rate of climb in the last thermal" is a simple approach to achieving a near maximum speed.

Finally, certain properties of the 'p' parameter have been identified; in particular, a high speed limit is approached (very strong conditions) which yields a maximum cross-country speed of two-thirds of the airspeed if atmospheric sink and wind are negligible.

LIST OF SYMBOLS

h = Climb in thermal or loss of height between thermals

t_c = Time required to thermal h feet

t_g = Time in glide to lose h feet

w_a = Atmospheric sink between thermals

w_g = Rate of sink of sailplane between thermals (variometer reading)

w_c = Achieved rate of climb in thermal

w_s = Sailplane polar sink rate as a function of airspeed

V = Airspeed of sailplane

V_w = Wind speed (positive for headwind)

V_{eff} = Effective ground or cross-country speed

k = A constant in the approximation of a sailplane polar

p = $(V/w_s)(dw_s/dV)$ sailplane performance parameter

C_L = Lift coefficient

C_D = Drag coefficient

C_{D_e} = Zero lift intercept of the linear C_D vs C_L^2 representation of the drag polar

ARe = Effective aspect ratio

ν = Flight path angle

W = Weight

ρ = Air density

S = Wing area

REFERENCES

1. Carmichael, B.H., "Cross-Country Soaring Criteria Based on Thermal Strength", IAS Reprint, Twentieth Annual IAS Meeting, New York, N.Y., January 1952.

2. Paul Bikle, "Sailplane Performance Measured in Flight", Technical Soaring, Vol. 1, No. 3, January 1972.