A SPEED RING FOR CLOUD STREET FLYING

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SUMMARY

A modification of the MacCready speed ring is proposed. The modified ring, which could be added as a second scale to a standard ring, is designed to give the pilot optimum speeds to fly between straightaway climbs. In order to arrive at a simple arrangement, it is necessary to assume a fixed value for airspeed in the straight climb. Assuming climbs at the airspeed for minimum sink rate corresponds to a practical strategy. A numerical example is provided.

INTRODUCTION

The classical MacCready speed ring provides optimum cruise speeds between circling climbs. Flying cross-country using extended areas of lift, without circling, is now feasible as a result of advances in glider performance. A modified speed ring may be useful under such straight climb circumstances. A previous paper[1] by Kauer and Junginger presented such a modification. That paper assumed a sinusoidal model for atmospheric vertical velocity. Another such modified speed ring is developed in this paper using a square-wave model for atmospheric vertical velocity, and other assumptions.

SYMBOLS

h = Altitude change, positive for gain. h_{cl} - climb leg, h_{cr} - cruise leg

t = Elapsed time for flight leg. t_{cl} - climb leg, t_{cr} - cruise leg

V = Speed along course on cruise leg

V_{at} = Air mass vertical rate in cruise, negative for sink

V_{av} = Total (glider sink plus thermal) climb rate, positive up

V_{cl} = Speed along course on climb

V_m = Airspeed for minimum sink rate in still air

V_s = Glider sink rate in still air, negative down

V_t = Air mass vertical rate on climb leg, positive for lift

V_x = Average cross-country speed

γ = Flight path angle, positive for climb

* = Starred values are optimums, maximizing average cross-country speed

THE ATMOSPHERIC MODEL

An idealized square-wave model is assumed for the cloud streets and areas of sink between them. A uniform upward vertical velocity is assumed under the cloud street, while a uniform sink velocity is assumed between the cloud streets. The glider is flown in a straight climb along the course in lift, and in a higher speed cruise along the course in sink.

SOLUTION FOR MAXIMUM AVERAGE CROSS-COUNTRY SPEED

The average cross-country speed is

$$V_x = \frac{(Vt_{cr} + V_{cl}\, t_{cl})}{(t_{cr} + t_{cl})}$$

A fundamental assumption is that a cycle consists of a climb leg followed by a cruise leg, and that the altitude at the end of each cycle is the same as at the start. The condition for no net altitude change after one cycle is

$$h_{cl} + h_{cr} = 0 \tag{2}$$

These altitude changes can be written as

$$h_{cl} = V_{av}t_{cl},$$

$$h_{cr} = (V_{at} + V_s)\ t_{cr} \tag{3}$$

Substituting Equation (3) into Equation (2)

$$V_{av}t_{cl} + (V_{at} + V_s)\ t_{cr} = 0,$$

or

$$t_{cl}/t_{cr} = -\ (V_{at} + V_s)/V_{av} \tag{4}$$

Now using Equation (1)

$$V_x = Vt_{cr}/(t_{cr} + t_{cl})$$

$$+ V_{cl}t_{cl}/(t_{cr} + t_{cl})$$

$$= V/(1 + t_{cl}/t_{cr})$$

$$+ V_{cl}/(1 + t_{cr}/t_{cl})$$

Using Equation (4)

$$V_x = V/(1 - (V_s + V_{at})/V_{av})$$

$$+ V_{cl}/(1 - V_{av}/(V_s + V_{at}))$$

$$= \frac{(VV_{av} - V_{cl}\ V_s - V_{cl}\ V_{at})}{(V_{av} - V_s - V_{at})} \tag{5}$$

Equation (5) is differentiated with respect to the cruise speed V and the derivative is set equal to zero, to find the maximum cross-country speed V_x*. Note that while V_x and V_s are functions of V, V_{av}, V_{cl}, and V_{at} are not.

$$\frac{\partial V_x}{\partial V} = \frac{(V\ V_{av} - V_{cl}V_s - V_{cl}V_{at})\ (\partial V_s/\partial V)}{(V_{av} - V_s - V_{at})2}$$

$$+ \frac{V_{av} - V_{cl}\ (\partial V_s/\partial V)}{V_{av} - V_s - V_{at}} \tag{6}$$

Setting Equation (6) to zero gives

$$(V^*V_{av} - V_{cl}\ V_s^* - V_{cl}\ V_{at})\ (\partial V_s/\partial V)^*$$

$$+ V_{av}\ (V_{av} - V_s^* - V_{at})$$

$$- V_{cl}\ (\partial V_s/\partial V)^*(V_{av} - V_s^* - V_{at}) = 0$$

Solving for $(\partial V_s/\partial V)^*$

$$\left[\frac{\partial V_s}{\partial V}\right]^* = \frac{V_s^* + V_{at} - V_{av}}{V^* - V_{cl}} \tag{7}$$

Equation (7) is the desired result. It reduces to the standard MacCready solution, as required, for a zero speed along course on the climb leg, or $V_{cl} = 0$. In the general case, the speed along course on the climb leg is an arbitrary parameter. Some considerations on the choice of climb speed follow.

CHOICE OF SPEED ON CLIMB LEG

Climbing along course at the airspeed for minimum sink rate V_m would seem to make best use of the lift. However, that precise climb speed will not in general satisfy the condition for no net altitude change expressed by Equation (2). Aside from that mathematical condition there are practical situations where lower or higher airspeeds would be better. A lower airspeed would be better if lift was strong and the pilot was trying to incline the flight path upwards as steeply as possible without regard to maximizing average cruise speed. This case is illustrated as follows: The flight path angle is

$$\gamma = (V_s + V_t)/V \tag{8}$$

The maximum or steepest climb angle is found in the usual way be differentiating Equation (8) and setting the differential to zero.

$$\partial\gamma/\partial V = \frac{V \frac{\partial V_s}{\partial V} - (V_s + V_t)}{V^2} = 0 \quad (9)$$

This gives

$$V_{max\gamma} = (V_s + V_t)/(\partial V_s/\partial V) \quad (10)$$

Equation (10) shows that for large positive values of the air mass vertical rate V_t, the glider is flown on the "back side" of the sink rate versus airspeed curve, where $\partial V_s/\partial V$ is positive. This maximizes the climb angle without regard for average cruise speed.

A different condition leads to higher climb speeds along course V_{cl} than the airspeed for minimum sink V_m. This is the case in which climbing at V_m causes the maximum desired or attainable altitude to be reached before leaving the area of lift. It is asserted without proof that a better policy in that case is to climb at an airspeed higher than V_m, reducing the climb rate but increasing average cruise speed. In a practical sense the pilot is unlikely to be able to choose the climb speed increment above V_m that would cause maximum altitude to be reached just before leaving the area of lift, so that a conservative policy of climbing at no higher speeds than V_m seems indicated. Furthermore, the numerical example presented subsequently shows that for the case in which the pilot wishes to maximize climb angle, airspeeds only slightly below V_m correspond to quite large air mass vertical velocities. Thus, it is concluded that for the practical application of the street flying optimum given by Equation (7) the climb speed V_{cl} can be taken as the airspeed for minimum sink V_m.

TABLE 1

AIRSPEEDS THAT MAXIMIZE CLIMB ANGLE

(STANDARD-CLASS GLIDER, SPEED IN KNOTS)

V	V_s	$\frac{\partial V_s}{\partial V}$	V_t (Eq. 10)
40	-1.3	0.036	2.74
45	-1.2	0	1.20
50	-1.4	-.044	-.80
60	-2.0	-.060	-1.60

SAMPLE CALCULATION

The previous results are illustrated for data representing a modern standard-class glider in Tables 1 and 2. Table

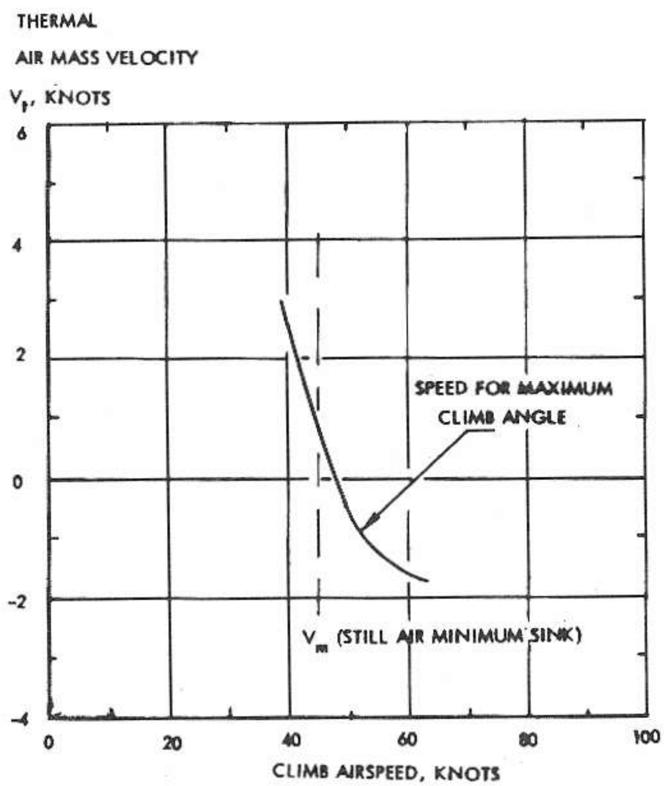

FIGURE 1. Variation of Speed for Maximum Climb Angle with Thermal Strength

TABLE 2

STREET AND CONVENTIONAL SPEED RING PARAMETERS
(STANDARD-CLASS GLIDER, SPEED IN KNOTS)

V	V_s	$\frac{\partial V_s}{\partial V}$	$\frac{\partial V_s}{\partial V} V$	$V-45$	$\frac{\partial V_s}{\partial V} x(V-45)$
45 (V_m)	-1.2	0	0	0	0
50	-1.4	-0.044	-2.20	5	-0.22
60	-2.0	-.060	-3.60	15	-.90
70	-2.6	-.080	-5.60	25	-2.00
80	-3.6	-.106	-8.50	35	-3.71
90	-4.8	-.146	-13.15	45	-6.59
100	-6.2	-.174	-17.40	55	-9.58
110	-8.3	-.224	-24.60	65	-14.58

1 gives numerical results from the application of Equation (10), defining the airspeed for maximum climb angle. Those results, plotted in Figure 1, show that quite strong thermals correspond to climb airspeeds only slightly lower than V_m. This is why it is unlikely that a pilot might wish to climb at airspeeds less than V_m, the speed for minimum sink rate.

Table 2 gives sample results from the application of Equation (7). The climb speed V_{cl} is taken as V_m, or 45 knots. The right-hand column lists variometer readings corresponding to specific optimum cruise airspeeds. For comparison, the fourth column lists variometer readings corresponding to optimum cruise airspeeds for the conventional case involving circling climb. Graphical solutions of Equation (7) for both street and circling climb cases are presented in Figure 2.

SPEED RING SCALE FOR STREET FLYING

A two-scale speed ring can be prepared from Equation (7), as illustrated in Figure 3 (p. 14). The inner scale is the conventional or MacCready scale. The outer scale gives optimum cruise speeds for street flying. Both scales in Figure 3 are designed for a hypothetical variometer calibrated at 10 degrees of arc per knot. The inner and outer ring markings are found by laying out the first column of Table 2 against the fourth and sixth columns, respectively. The street scale calls for strikingly higher cruise airspeeds than the conventional scale, for the same average climb rates.

The relative indications of the two speed ring scales are illustrated in a numerical example. Assume an average climb rate V_{av} of two knots, or about 200 feet per minute. Assume further an air mass vertical rate in cruise V_{at} of -0.5 knots (sink). The triangular index of Figure 3 is set at 2.0. A cruise speed of 65 knots is read on the inner scale, corresponding to a total sink rate of 2.8 knots. This sink rate is the 2.3 knots read from Figure 2 at 65 knots plus the air mass rate of -0.5 knots. A cruise speed of 96 knots is read on the outer scale, corresponding to a total sink rate of 6.2 knots. This is the sum of -5.7 knots read from Figure 2 plus the air mass rate.

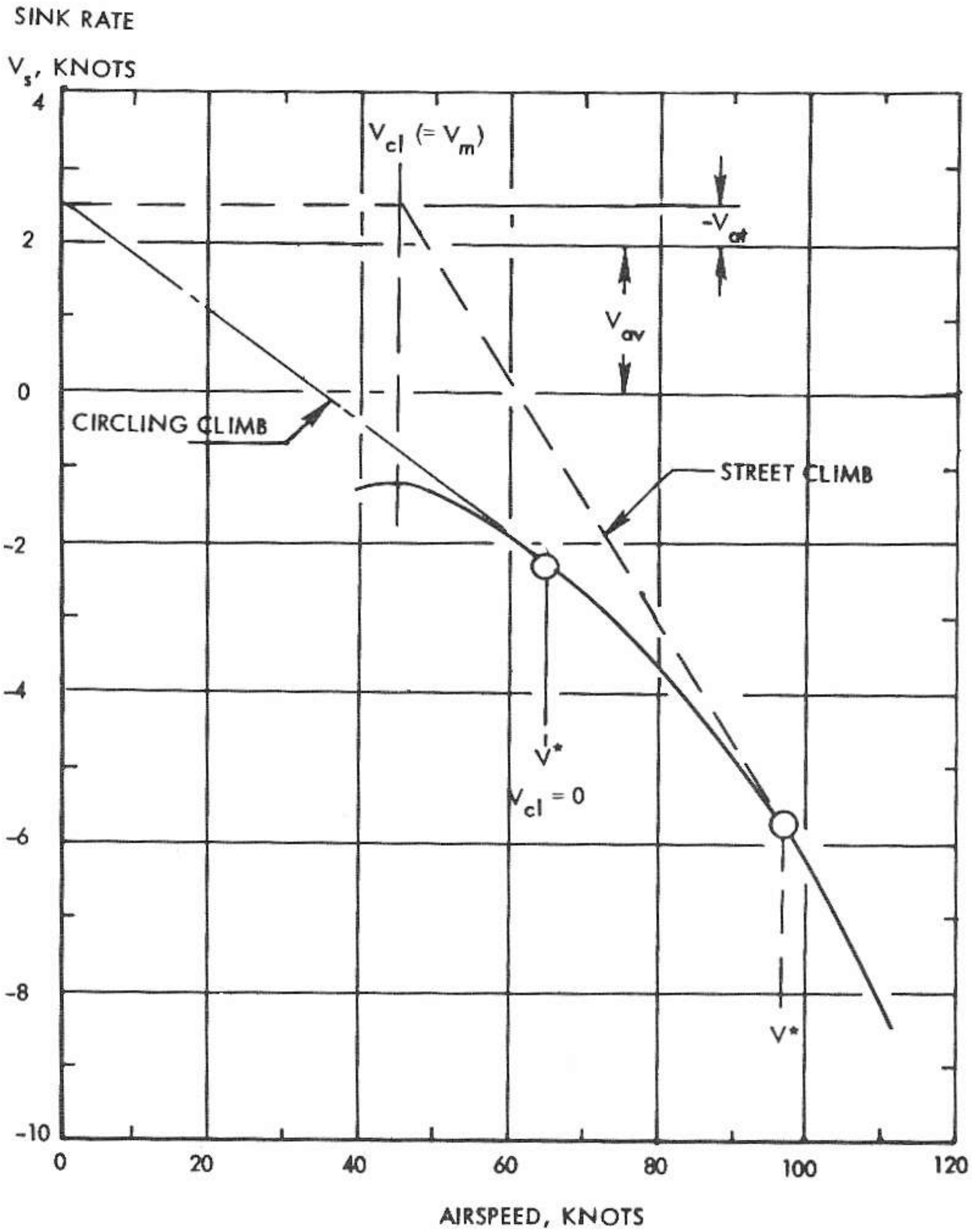

FIGURE 2. Graphical Solution for Optimum Cruise Speed V* Standard Class Glider

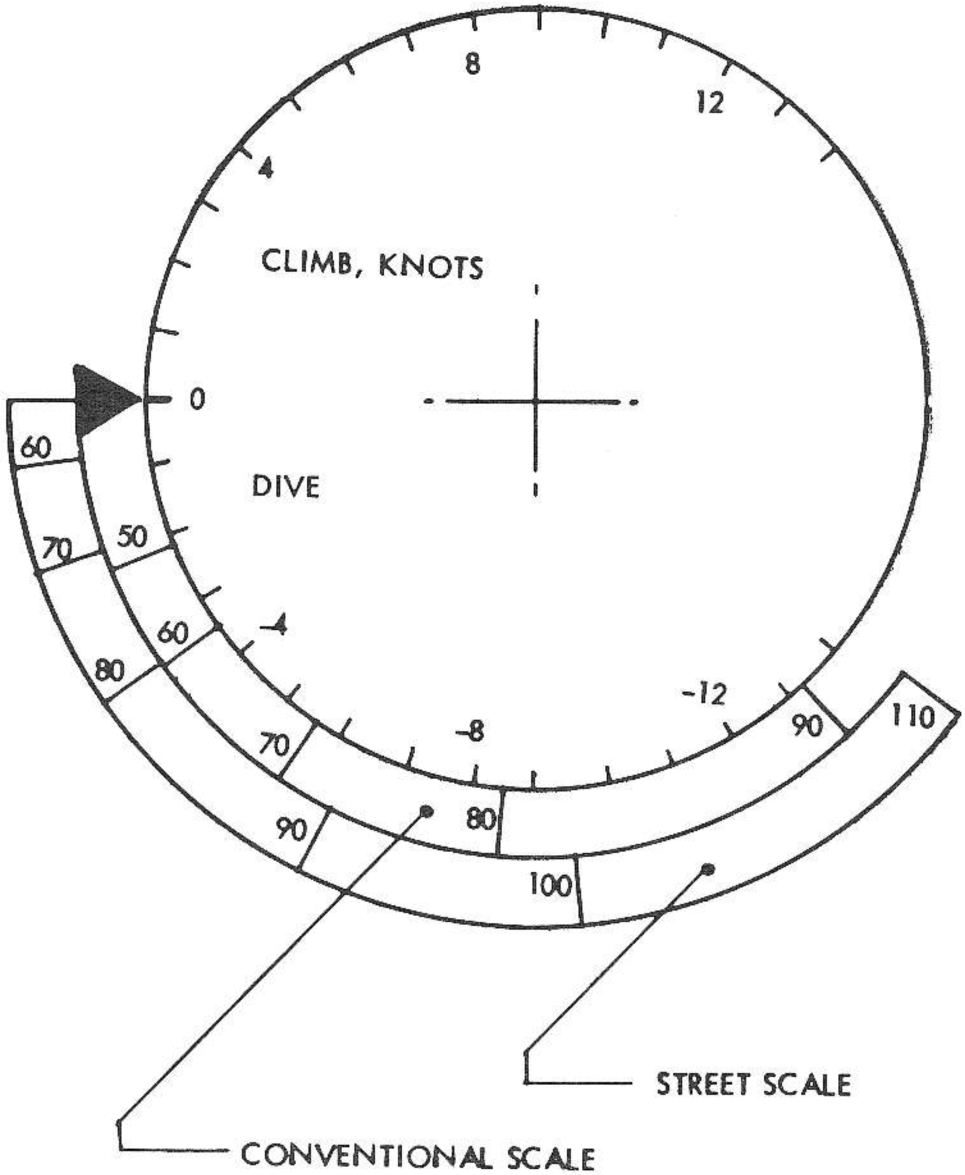

FIGURE 3. Two-Scale Speed Ring

ACKNOWLEDGEMENT

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REFERENCES

1. Kauer, E., and H.G. Junginger, "Soaring in Dolphin-Style", *Aerokurier* September - October 1973, pp. 658 - 660, 741 (in German).

2. du Pont, Steven, *New Soaring by the Numbers*, published by the author, 1974.