SOME NOTES ON SOARING FLIGHT OPTIMIZATION THEORY

by

Dr. Risto Arho

Department of Aeronautics and Astronautics

Massachusetts Institute of Technology

Cambridge, Massachusetts, U.S.A.

SUMMARY

General soaring flight optimization techniques are briefly reviewed. The straight dolphin mode is examined and an optimal flight policy on course of successive dolphin mode elements is suggested. Also significant recent references in the field are given.

NOTATION

d	differential operator
w	sailplane's vertical velocity with respect to ambient air
v	sailplane's airspeed
x	distance along the element
λ	Langrange - multiplier
c(x)	vertical velocity of the air as function of x
Δh	altitude change in traversing the element (= terminal altitude - initial altitude)

Upward direction is considered positive, downward negative in transitions as well as in *all* velocities.

INTRODUCTION

In recent years there has been a substantial increase of interest and research in the field of soaring flight optimization. The problems of achieving maximum cross-country speeds and maximum distances are fascinating and challenging. Even the very first step, i.e. problem formulation in a real case, calls for theoretical as well as practical intuition.

On an actual soaring racing course the ultimate goal of the pilot would be to follow a flight policy that brings him from the starting to the finishing line in minimum time. On a given day there might exist one or more flight policies that yield the absolute optimum. At this point it is logical to ask: Does the pilot have any means intentionally to pick up the policy or a policy that yields the absolute optimum? The answer is a solid no. The answer may depress a "scientist pilot", but it surely cheers up the "natural pilot", who has perhaps already become anxious about the increasing talk of the application of scientific methods and technical instruments within soaring.

And really — the problem of finding the flight policy that minimizes the flight time through a general course contains too many independent variables, constraints and stochastic aspects combined with the existence of several different available flight modes to allow a feasible rigorous mathematical approach in practice. Consequently, although the global (absolute) optimum at least in a certain sense exists, it is out of the ability of any pilot intentionally to perform the correct series of dolphin motions, circlings (thermalling), essings and deviations from the straight course that perhaps are needed to produce the optimum. However, it is reasonable to speak about optimizing an element of the course, i.e. a clear-cut, short part of the course. Also some elements may be combined and at least a relative optimum of the combination can be found. Accordingly it seems to the author that as to a soaring flight optimum it is perspicuous to speak about a "kind of optimum" a term that was used by the author already in Reference 1.

For the optimization of the circling mode we have the well-known result of MacCready. As for the straight dolphin mode References 2, 3, 4, and 5 should be consulted. Furthermore the mode of essing (alternate right-and left-hand turns, no complete circles) is introduced by Metzger and Hedrick in Reference 3. It is evident that the circling mode is optimal when strong thermals are clearly localized. If the lifting regions are not extremely localized, but occur over some significant percentage of the course, it usually is optimal to utilize the straight dolphin mode. Under less favorable lifting conditions than those necessary for straight dolphining the essing mode may prove to be profitable.

In this paper we will be examining the straight dolphin mode and suggest an optimal flight policy on course of successive dolphin mode elements. To obtain the optimal solution for the dolphin element we have to know the atmospheric lift-sink distribution along the element. However, as Irving in Reference 5 quite correctly states, "the pilot has no powers of prophecy." But in order to be able to utilize the developed theory in practice we have to cope with this "prophecy" affair. We shall outline here a method that will compensate the estimation errors in successive steps.

DOLPHIN ELEMENT ESTIMATE

By Reference 2 we assume that any polar equation can be approximated by the polynomial

$$w = Av^2 + Bv + C \tag{1}$$

where A, B and C are constants. Then further by Reference 2 we obtain for the optimal dolphin speed (neglecting winds) the expression

$$v = v_{opt}(x,\lambda) = \sqrt{\frac{1/\lambda + C + c(x)}{A}} \tag{2}$$

$$A < 0,\ C < 0,\ \lambda < 0$$

Finally, the altitude constraint is given by

$$\int_{x_1}^{x_2} \left[Av + B + \frac{C + c(x)}{v} \right] dx = \Delta h \tag{3}$$

where the integration is executed over the element. Substitution of v from Eq. 2 to Eq. 3 and prescribing Δh allow the determination of parameter λ by Eq. 3. For details the interested reader should consult Reference 2. It is convenient to note that, after substitution of v from Eq. 2, Eq. 3 takes the form

$$\int_{x_1}^{x_2} f\left[c(x),\ \lambda \right] dx = \Delta h \tag{4}$$

Accordingly in order to determine λ we have to know c(x) in $[x_1, x_2]$, i.e. the lift-sink distribution along the element. Although the pilot has "no powers of prophecy", he however has the ability to estimate. We denote the estimate of the lift-sink function in the interval by $\tilde{c}(x)$. This estimate could be based for instance on the isolated thermal model or on the four-cell, blended-core thermal structure, both of which are introduced by Gedeon in Parts I and II of Reference 4 respectively. Also other kinds of updraft profiles could be utilized. As a matter of fact, thermal conditions might sometimes appear to be hazy enough to justify the application of constant $\tilde{c}$ (mean value) across the element as a first approximation. The author wants to use the opportunity to remark that the sinusoidal updraft-downdraft profile, used by the author in Reference 2, does not reflect the author's view of a practical thermal model, but is utilized to show clearly the behavior of pulling up in lift and diving through down.

As to the element, it can of course consist of one or more thermals. A longer element produces more inaccuracies into $\tilde{c}(x)$, but on the other hand it allows the pilot better to concentrate on actual flying.

After having fixed $\tilde{c}(x)$ we insert it in the altitude constraint, i.e. Eq. 4, and have

$$\int_{x_1}^{x_2} f\left[\tilde{c}(x),\ \lambda\right] dx = \Delta h \qquad (5)$$

where Δh is prescribed. Solving Eq. 5 for the Lagrange-multiplier yields us $\bar{\lambda}$, an estimate of the correct λ. Consequently we have coped with the "prophecy affair." It is however left to establish a procedure that compensates the inherent errors of this method. Now flying through the element the pilot is able to find at every point the actual up- or downdraft (at least in theory). With $\bar{\lambda}$ and the actual atmospheric vertical velocity distribution $c(x)$ he has the following airspeed function to obey:

$$\bar{v}(x) = \sqrt{\frac{1/\bar{\lambda} + C + c(x)}{A}} \qquad (6)$$

Obeying the airspeed function given by Eq. 6 he actually comes out of the element with an altitude change $\int_{x_1}^{x_2} f[c(x),\ \bar{\lambda}]dx$. This change generally is not equal to the prescribed change Δh. Let us denote this change by $\Delta\bar{h}$. Accordingly

$$\int_{x_1}^{x_2} f\left[c(x),\ \bar{\lambda}\right] dx = \Delta\bar{h} \text{ (in general } \Delta\bar{h} \neq \Delta h) \qquad (7)$$

Here $\bar{\lambda}$ is the fixed estimate just found (thus Eq. 7 is *not* supposed to be solved for $\bar{\lambda}$).

It should be noted at this point that the solution given by $\bar{\lambda}$ and $\bar{v}(x)$ (and flown by the pilot) is also an optimum solution. It is the solution by which the actual distribution $c(x)$ of the element is traversed in minimum time, when the change of altitude is $\Delta\bar{h}$. If $\Delta\bar{h} \approx \Delta h$, the pilot has virtually succeeded in solving the problem he had intended to solve. Even if the differences do not match, he has the possibility later to compensate the error. In the above treatment no attention has been paid to other sources of error except the use of $\tilde{c}(x)$ in the determination of the Lagrange-multiplier.

SEQUENCE OF OPTIMAL DOLPHIN ELEMENTS

Irving in Reference 5 chooses for his constraint the demand for zero total change of energy height in traversing the element. Metzger and Hedrick, Reference 3, apply the condition $\Delta h = 0$. Although any feasible Δh can be applied according to the purpose of the element involved in the flight strategy, it might in general be beneficial to maintain a constant base level in a cloudstreet. Consequently we consider a sequence of consecutive dolphin elements. For each element the pilot can prescribe Δh_i before starting to traverse the element. We suggest that his prescriptions would look like the following

$$\Delta h_1 = 0,\ \Delta h_{i+1} = -\sum_{j=1}^{i} \Delta\bar{h}_j,\ i = 1,\ldots,n-1 \quad (\Delta h_i \text{ feasible}) \qquad (8)$$

Here n = number of elements in the sequence. Of course $\sum_{j=1}^{i} \Delta\bar{h}_j$ is simply the actual deviation from the initial base level at the end of the i:th step.

Accordingly, every step of the sequence is optimized separately with individual $\tilde{c}_i(x)$ and $(x_2 - x_1)_i$. Naturally the same thermal model could be applied throughout varying only the model parameters. By prescribing Δh_i by Eq. 8 the pilot clearly tries to maintain the initial base level. The relatively strong correction procedure represented by Eq. 8 should remain the only intended corrective element of the method. That is, a possible deviation from the initial base level at the end of a step shall definitely not affect the selection of the model parameters of the thermal estimate of the subsequent step. Double correction could easily result in nonoptimal level oscillation.

The above scheme to try to maintain a constant base level in a sequence of optimal dolphin elements can not be claimed to be the solution that absolutely minimizes the flight time through the sequence. The scheme possesses also another drawback, i.e. the approximate optimal airspeed is discontinuous on the boundary of two successive elements. From Eq. 6 we have

$$d\bar{v} = -\frac{d\bar{\lambda}}{2A\bar{v}\bar{\lambda}^2} \qquad (9)$$

where $d\lambda = \lambda_{i+1} - \lambda_i$. In general the λ's are different and accordingly the approximate optimal airspeed has somewhat different values at the end of step i and at the beginning of step i = 1. A small correction maneuver by the stick is acceptable, but should $d\tilde{v}$ on the boundary appear really remarkable, the λ_{i+1} determined by Eqs. 8 and 5 should not be utilized. In this case the pilot ought to apply simply a $\tilde{\tilde{\lambda}}_{i+1}$, which is between λ_i and λ_{i+1} and with which the difference $d\lambda = \tilde{\tilde{\lambda}}_{i+1} - \lambda_i$ would not enforce $d\tilde{v}$ (given by Eq. 9) too large. With this discontinuity in mind we also note that it might be advantageous to select elements with boundaries at maximum down current locations since there $\tilde{v}$, which occurs in the denominator of the right hand side of Eq. 9, assures its largest values thus cutting down $d\tilde{v}$.

CONCLUSIONS

In this survey we have represented mainly general ideas of dolphin mode soaring. Also the effects and compensation of the estimation error of c(x) have been discussed. We have not paid any attention to the effects of other error sources. And, however, in practice the pilot would encounter the difficulty of obeying his airspeed function (Eq. 6), coping with the discontinuity of this approximate optimal airspeed on the element boundary, etc. It is also clear that the method featured here would necessitate a tiny computer on board, if applied literally. The prescribed Δh_i and the model parameters of $\tilde{c}_i(x)$ as input quantities the computer would yield λ_i by solving Eq. 5 by iteration. Further the computer would calculate the airspeed function $\tilde{v}_i(x)$ (Eq. 6) by means of the polar constants A, C, the just fixed $\tilde{\lambda}_i$ and the actual $c_i(x)$ at points along the element. In addition to proper flying the pilot's main task would be in estimating the model parameters of the thermal distribution of the element ahead. It should be noted also that we have not included in this discussion any of the various constraints which may affect dolphin soaring tactics, too. Nevertheless, although the theoretical ideas put forth in this treatment are not very easy to follow in practice in the present phase of soaring art, they provide the pilot at least with qualitative know-how on sustained dolphin techniques. The other benefit comes in the form of provoking further research along the paths outlined here. A closer look might be thrown at the stability of the correction procedure represented by Eq. 8. Also smoothing down the airspeed discontinuity on the element boundary would be worth a specific study.

REFERENCES

1. Arho, R.: "Distance Estimation Error and Stationary Optimal Gliding," *Aero-Revue*, 8/1973, pp. 443-444.
2. Arho, R.: "Optimal Dolphin Soaring as a Variational Problem," *Acta Polytechnica Scandinavica*, Me 68, 1972; also *Technical Soaring*, No. 1, Vol. III, 1973.
3. Metzger, D. and Hedrick, K.: "Optimal Flight Paths for Soaring Flight," *AIAA/MIT/SSA 2nd Int. Symposium on the Technology and Science of Low Speed and Motorless Flight*, AIAA Paper No. 74-1001, 1974. Also *Journal of Aircraft*, Nov. 1975.
4. Gedeon, J.: "Dynamic Analysis of Dolphin-Style Thermal Cross-Country Flight, Part I:" *Technical Soaring*, No. 1, Vol. III, 1973, "Part II:" *Technical Soaring*, No. 3, Vol. III, 1974.
5. Irving, F.: "Cloud-Street Flying," *Technical Soaring*, No. 1, Vol. III, 1973.