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PLANFORM EFFECTS ON THE INDUCED DRAG OF UNTWISTED WINGS

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The elliptical planform was shown by Prandtl (Ref. 1) to be the optimum shape for producing minimum induced drag. Planforms with straight leading and trailing edges are generally preferred for practical reasons however, and judicious design can make the induced drag penalty of non-elliptical shapes comparatively small. The existing literature such as Ref. 2 is mainly devoted to wings of simple taper and relatively low aspect ratio which are characteristic of powered aircraft. In the present work, computer solutions using lifting line theory were obtained for aspect ratios up to 30 and the three common alternatives to the elliptical planform: a) single taper, b) double taper, and c) rectangular inboard section combined with a tapered outer section, termed here the outer taper.

The general relation for the induced drag coefficient of an untwisted wing is

$$C_{D_i} = \frac{C_L^{\,2}}{\pi\ AR}\ (1 + \delta)$$

The influence of aspect ratio is obvious, although as discussed in Ref. 3, profile drag, weight and expense also increase with increasing aspect ratio. The optimum aspect ratio also depends on C_L, i.e. speed, and the sailplane designer is thus faced with a compromise. The induced drag factor δ is a function of planform shape and to a lesser degree aspect ratio, but for a well-designed wing δ is small and therefore a minor factor in selecting the aspect ratio.

The calculated values of δ for the single taper planform are shown in Fig. 1 which is essentially an extension of the usual powered-airplane data to the higher aspect ratio appropriate to sailplanes. It should be noted that older airplane data (such as in Ref. 2) may be somewhat approximate since relatively few points were calculated for each curve before the advent of modern computers.

Calculations of δ for the double taper planform involved four variables: aspect ratio, spanwise location of taper ratio

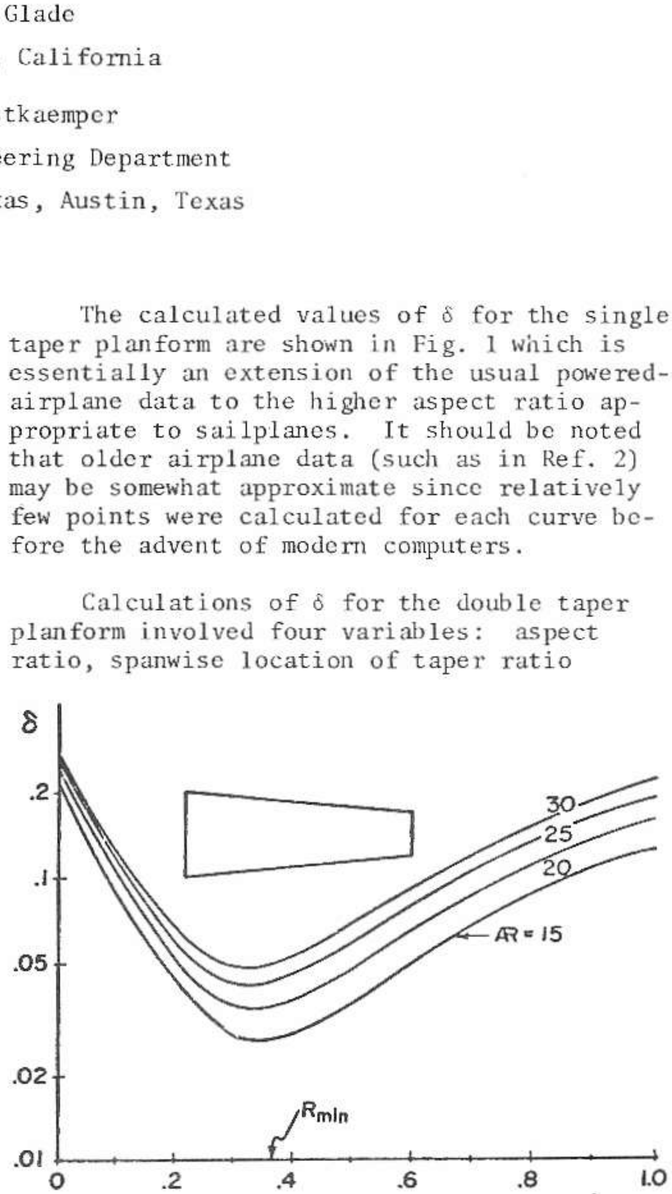

Figure 1. Induced Drag Factor for Single Taper Wings

change (TRC), and the taper ratio of the inner and outer panels. The outer taper planform is a special case of the double taper since it is a double taper with a taper ratio of one on the inner panel. In examining the large amount of data for δ resulting from this multiplicity of parameters, it was noted that there was a strong correlation between δ and the deviation of the practical planform shapes from the elliptical shape. This correlation is similar to the Schrenk approximation (Ref. 4) which states that the spanwise lift distribution for plain tapered wings is proportional to the average of the tapered and elliptical planform-area distributions. Therefore a method was formulated to analytically minimize the geometric difference between a realistic planform shape and the elliptical one. The problem was defined as the minimization of the residual which results from an integration over the span of the difference in chord lengths of the two shapes. The integral used was:

$$I = \int_{y=0}^{y=b/2} \left(\frac{C_{ellipse}}{b} - \frac{C_{practical}}{b} \right)^2 \frac{dy}{b/2}$$

After substitution of the appropriate geometric relations for the planform shapes, the result was of the form

$$I = \left(\frac{2}{AR}\right)^2 [f(TRC, \lambda_{in}, \lambda_{out})] = \left(\frac{2}{AR}\right)^2 [R]$$

The residual term R in brackets was independent of aspect ratio and was minimized numerically on a computer.

SINGLE TAPER

For the single taper planform a taper ratio of $\lambda = 0.376$ gave the minimum residual R, i.e., most closely matched the elliptical planform. As seen in Fig. 1, this value of λ is quite near that for minimum δ, indicating that the planform having the minimum residual also tends to have minimum induced drag.

OUTER TAPER

With the outer taper planform the spanwise position of taper-ratio change (TRC) was constrained to range from zero to one and the taper ratio was calculated which gave the minimum residual. The variation of the residual with TRC is shown in Fig. 2 which also indicates the value of λ for each TRC position. The optimum condition for this planform is an outer panel taper ratio of 0.302 with the taper starting at 47.8% of the semispan.

The combinations of TRC and λ of Fig. 2 were then used as input parameters in the lifting-line program to calculate the induced drag factor δ. The results are presented in Fig. 3 for several aspect ratios; also included are limited data from the initial parametric study in which a somewhat coarse grid of λ was used at each of the six values of TRC. (The scatter among these data is attributed to the grid size). It is evident that the residual method tends to predict the value of λ which gives minimum induced drag for each TRC position.

DOUBLE TAPER

In a similar manner, the computer was used to determine the inner and outer panel taper ratios to minimize R for the double taper planform with TRC position as a speci-

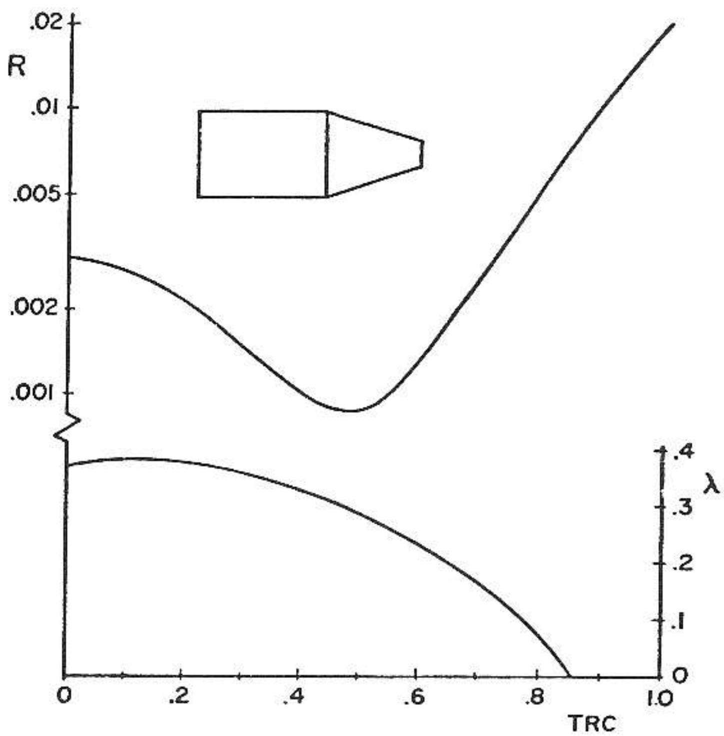

Figure 2. Residual and Taper Ratio for Outer Taper Wings

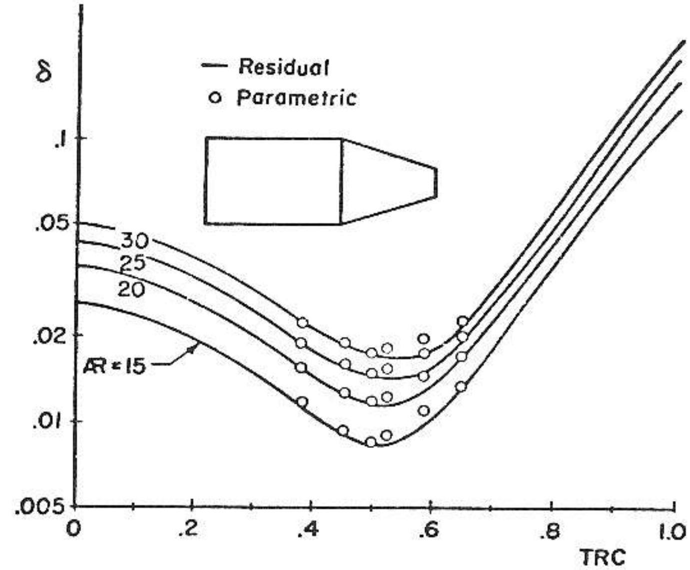

Figure 3. Induced Drag Factor for Outer Taper Wings

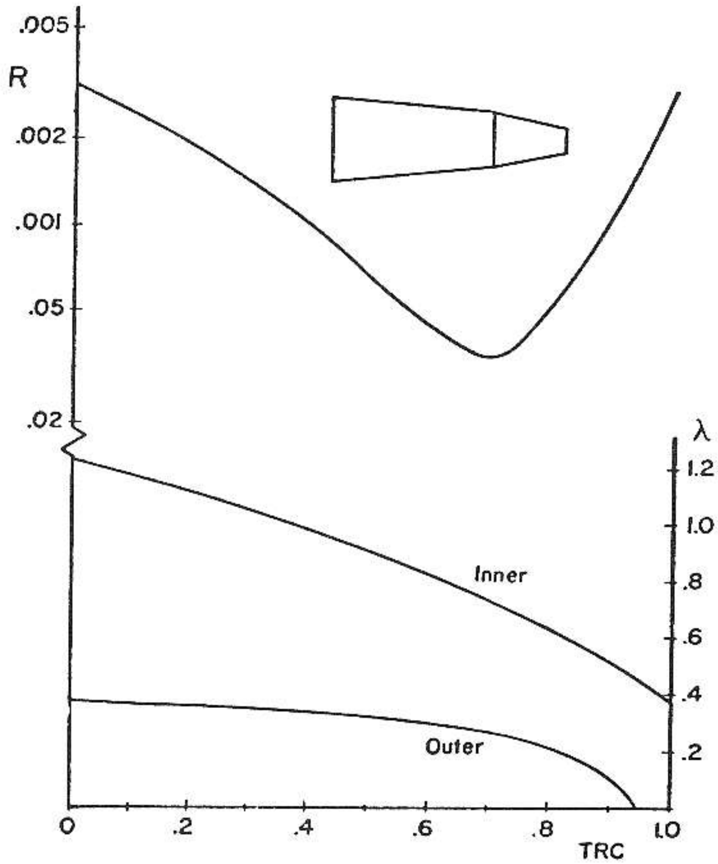

Figure 4. Residual and Taper Ratios for Double Taper Wings

fied input condition. Fig. 4 shows the variation of the minimum residual R with TRC and the two corresponding values of λ for each TRC location. The best combination of parameters for this planform is a taper change at 69.5% semi-span, an inner panel taper ratio of 0.741 and an outer panel taper of 0.278.

The two taper ratios shown in Fig. 4 for each TRC position were then used in the lifting-line program to calculate δ, the results being shown in Fig. 5 for AR = 15. Again the symbols are data from the initial parametric study using a grid of taper ratios for each of 8 specified values of TRC. It is evident that the grid was not sufficiently fine to include the taper ratio combinations to give the true minimum value of δ. The inflections of the two sets of data do occur at the same TRC position, however. Thus for the double taper planform the residual method also tends to identify the two values of λ which minimize δ for each TRC location.

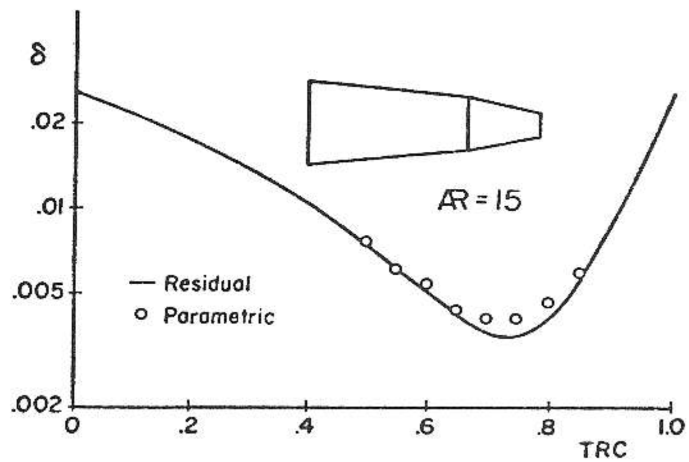

Figure 5. Induced Drag Factor for Double Taper Wings

CONCLUDING REMARKS

Planforms which most closely approximate the ellipse are seen to have the lowest induced drag factor δ, the double taper being the best of the three considered. Various planforms may be readily compared by calculating the residual parameter defined earlier, there being a good correlation between δ and the residual. In the present work, the location of taper-ratio change was specified as an input variable and the computer program enabled the determination of the taper ratios to best approximate the elliptical shape. (Constraining the location of taper change appeared consistent with the design process, although it is also practicable to constrain one or both of the taper ratios should this be desirable from a design standpoint).

Since the method of the residual gives planforms approaching the elliptical, the spanwise lift coefficients which result also tend to be constant. For the optimum cases of the three planforms studies, the tips were loaded from 4 to 6% higher than the average, C_ℓ, as seen in Fig. 6. Thus they present problems with roll control in near-stall conditions. This problem is discussed in Refs. 5 and 6, both of which suggest twist as a solution. Holighaus selects the double taper planform while Wortmann advocates the outer taper, in both cases with twist on the outer panel only. It is hoped that the material in this article may aid the designer in selecting initial planforms for further study and modification to meet the requirements of stall control, low drag, aileron span and construction ease. (Copies of the tabulated data are available from the second author, c/o Aerospace Engn., Univ. of Texas, Austin, Tx. 78712).

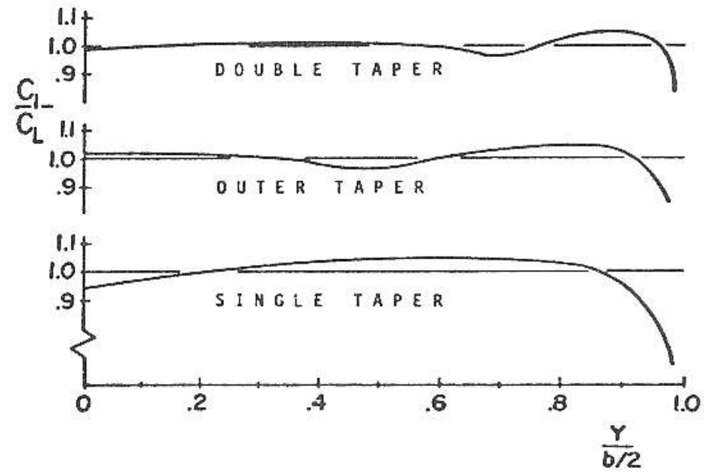

Figure 6. Spanwise Lift Coefficients for Optimum Wings

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