DEVELOPMENT OF A COMPUTERIZED SAILPLANE PERFORMANCE ANALYSIS

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ABSTRACT

A specialized computer program was developed for the calculation of sailplane performance in wings-level and turning flight. The basis of the program was an existing non-linear lifting-line technique which permitted calculation of wing characteristics throughout the entire useful range of angles of attack using tabular airfoil section data which included Reynolds number effects. This wing analysis was coupled with appropriate fuselage and empennage models in an iterative solution which calculated performace in a trimmed condition by adjusting aircraft attitude and tail lift until the equations of vertical force and pitching moment equilibrium were satisfied. The program was checked against published flight test data with very satisfactory results.

INTRODUCTION

Ideally, a sailplane designer would like to be able to specify a geometric configuration for his proposed aircraft and then be able to quickly assess the effect on performance brought about by changes to the configuration. Unfortunately, this ideal is not often realized. While analytical tools exist, their use may involve compromising between spending hours on tedious calculations to achieve good answers, or making simplifying assumptions that sacrifice accuracy for speed.

The modern high-speed digital computer, however, is changing this picture. The computer, properly used, opens the way for analytical approaches such as iterative solutions and matrix methods that are very time-consuming when done manually. A well-conceived, computerized performance analysis program, if intelligently used, can be a powerful tool for the designer of sailplanes as well as other aircraft. The development of such a computer program is the subject of this paper.

The Sailplane Mathematical Model

The calculation of sailplane performance in terms of L/D ratio and rate-of-sink hinges on the determination of total aircraft drag in equilibrium gliding flight. Drag may be calculated by assuming a parabolic variation of the drag coefficient for the complete aircraft, which takes the form

$$C_D = C_{D_O} + \frac{C_L^2}{\pi ARe} \qquad (1)$$

The value of C_{D_O} is the sum of profile drag coefficients for the aircraft at $C_L = 0$, and e is for a factor accounting for the variation of drag with C_L^2. Note that this value of e is representative of the entire ship and is not to be confused with the classical span efficiency factor for the wing alone. The profile drag contributions of the wing, fuselage, and empennage are determined by the use of empirical drag data such as that published by Hoerner (Ref. 2). The total C_{D_O} is then increased by 5 percent to approximate interference drag. The value of e is also determined empirically by plotting measured sailplane C_D values against C_L^2. This yields a fairly linear curve, the slope of which is equal to $1/\pi eAR$ for the particular sailplane.

By sampling enough data for different sailplanes, it is possible to estimate the value of e for any untested sailplane. Despite its simplicity, this method, if used intelligently, can yield surprisingly good results for aircraft having nearly parabolic drag polars. See for example, a paper by Brown (Ref. 3) which compares drag polars calculated by this method to flight test data gathered by Bikle (Ref. 4).

The use of equation (1) and the method just discussed has its limitations, however, for design work. In the case of sailplanes the drag of the wing is, by far, the most significant component of total drag. It was, therefore, considered essential for this study to use an analysis capable of separating the effects of wing parameters other than aspect ratio, such as taper, twist, and airfoil properties.

The classical way of treating wing analysis is to use lifting-line theory. Methods using this theory have been developed by Prandtl, Glauert, and others, and any fundamental text on aerodynamics, such as Kuethe and Schetzer (Ref. 4), will have a detailed treatment of their techniques.

The fundamental equation of lifting-line theory is the integral expression for induced angle of attack at any point y_1 on the span given by

$$\alpha_i = \frac{180b}{\pi 8\pi}\int_{-b/2}^{b/2} \frac{d\left(\frac{c_\ell c}{dy}\right)}{y_1 - y}\,dy \, . \tag{2}$$

The difficulty in solving this equation is that it contains the lift distribution which is of course dependent on induced angle of attack.

A solution for the special case of an elliptical lift distribution, which is of particular interest because it has the minimum induced drag, was first obtained by Prandtl. A more general solution was obtained by Glauert, who used Fourier series analysis and assumed a linear variation of lift coefficient with angle of attack. Anderson (Ref. 5) applied Glauert's methods to obtain solutions for a variety of wing planforms which were presented by Anderson in the form of charts and tables and also by Abbott and Von Doenhoff (Ref. 6). While Anderson's solutions can be very useful, they also have limitations for design work because they are not completely general (linear twist and taper are required) and a linear lift curve slope is assumed. Additionally Anderson's methods are not particularly suited to computerization.

Another wing analysis, however, was found to be well-suited both to design work and to use in a computer program. This method, developed by Sivells and Neely (Ref. 7), is a distillation of the work of several persons, primarily Multhopp, and permits calculation of the lift distribution on a wing by means of successive approximations using actual nonlinear section data. Sivells and Neely handle the integration of equation (2) by expressing the spanwise c_ℓ distribution (after the example of Glauert) as a Fourier series and using harmonic analysis to arrive at an expression for α_i. This expression for the induced angle of attack at any point on the span is a summation of terms composed of the spanwise c_ℓ distribution and a series of multipliers that are a function of the number of spanwise stations and independent of planform.

The use of the method is an iterative process well suited to machine computation. A first approximation for the c_ℓ distribution is made (an average of an elliptical and a chord length distribution is satisfactory.) Next the α_i is computed at each spanwise station. These α_i's are used to determine the local α and then the c_ℓ at each station is found from airfoil section data. The calculated c_ℓ distribution is then compared to the assumed distribution. If they are different the assumed distribution is adjusted and the process repeated until calculated and assumed c_ℓ's agree within a prescribed tolerance. After the angle of attack and c_ℓ distribution along the wing are calculated, c_{d_o} and c_m are easily obtained for each spanwise location. The values of C_L, C_{D_o}, C_{D_i}, and C_M for the entire wing are calculated by spanwise integration. This integration is handled by harmonic analysis and each of the aerodynamic coefficients for the wing is expressed as a summation in terms of the section coefficient involved and an appropriate multiplier.

The method of Sivells and Neely seemed to be an excellent choice for the wing analysis in a sailplane performance program, because lifting-line techniques have proven to be very accurate for small sweep angles and high aspect ratios which are typical of sailplane wings. The capability of this method to use actual airfoil section data to calculate wing performance for non-linear lift curve slopes and in the vicinity of maximum lift coefficient was also important because most thermalling flight occurs at high angles of attack just below stall where the lift curve slope is decidedly non-linear. And finally, the method is well suited to computerization because it was developed for manual computation using the early mechanical calculators and can be made to run very rapidly on a high speed digital computer.

The wing model requires two types of data. First, the airfoil section data consist of tables of lift, drag, and moment coefficients as functions of angle of attack and Reynolds number. Also required are configuration data such as span, area, aspect ratio, chord distribution and twist distribution.

The remainder of the mathematical model, while less sophisticated than the wing, can be expected to yield the necessary accuracy.

The fuselage aerodynamic model consists of equations for lift and moment coefficient variations as functions of angle of attack and data table "look-up" for drag coefficient versus angle of attack. Data for the drag table are considered to contain wing/fuselage interference effects and may be estimated or obtained from wind tunnel data such as reported by Althaus (Ref. 8).

The empennage model is composed of horizontal and vertical surfaces. The latter contributes to the total drag only in the form of a constant profile drag coefficient referenced to the surface area. The horizontal surface enters into the lift and pitching moment balances in trim as well as the total drag. It is defined by an area, a moment arm, a profile drag coefficient, and an effective aspect ratio. During trim iterations the stabilizer lift coefficient to trim the aircraft is calculated which then allows the stabilizer drag coefficient to be calculated as a parabolic variation with the lift coefficient.

Additional details of the mathematical model are treated in the next section which discusses the computer program.

Programming the Mathematical Model

The sailplane performance program, "SAILPER," is written in FORTRAN IV Language. It consists of a main program and five subroutines. The flow diagram of Fig. 1 illustrates the major functions of the program.

As indicated earlier input values are read in which define the aerodynamic characteristics of the wing airfoil section, as well as the aerodynamics and geometry of the fuselage and empennage. Additional data are also necessary to specify iteration and error limits and the flight conditions for which the sailplane performance is to be calculated.

After the input data are read a number of preliminary calculations are made. For example, the multipliers used in the lifting-line method are calculated and stored. Local velocities and Reynolds number at each spanwise station are computed. Using the spanwise twist distribution, local angles of attack are expressed as functions of the angle of attack at the fuselage centerline.

The iterations which produce a trimmed flight condition for a given velocity are initiated by estimating a wing C_L and corresponding angle of attack. The angle of attack estimate provides a first approximation of the local angle of attack at each spanwise station and leads to a c_ℓ distribution. The lifting-line subroutine is entered to obtain an α_i distribution and in turn a c_ℓ distribution. The initial estimated and calculated distributions are then compared station by station. If the difference is greater than the error bound at any station, the assumed c_ℓ is adjusted and another iteration is made with the adjusted c_ℓ distribution. This process continues until the difference is less than the error bound at every station.

The resulting c_ℓ distribution is then integrated to find the wing C_L for the estimated angle of attack. This C_L is compared to the estimated value and if the difference is greater than the error bound, an adjustment is made to angle of attack and the process repeated until the difference is within limits.

The C_L, α_i distribution, and c_ℓ distribution obtained by this process are used to calculate C_{D_i}, C_{D_o}, and C_M for the wing.

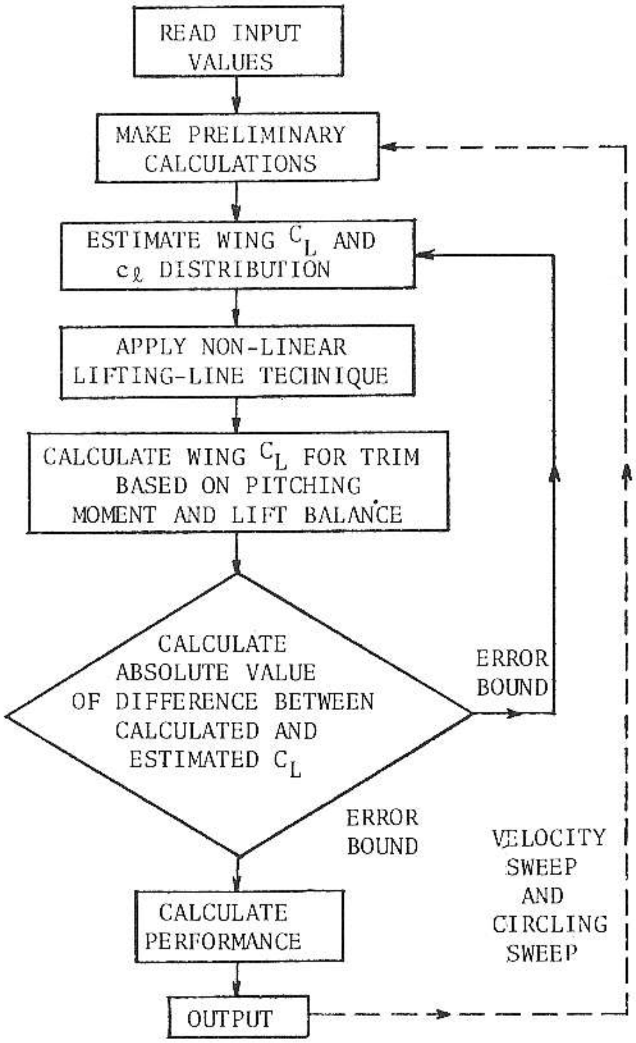

Figure 1. Flow diagram of performance program for sailplanes.

Using these values, along with the C_L and C_M of the fuselage, a required C_{L_t} for the horizontal tail is obtained to balance the aircraft in pitch. This C_{L_t} for trim is then used to compute a wing C_L required for a vertical force balance. This wing C_L is compared to that calculated by the lifting-line method, and if the difference is greater than the error bound, the aircraft attitude is adjusted and the process repeated until both vertical force and pitching moment balances are achieved.

After the trim condition is determined, performance parameters are calculated and a comprehensive trim summary is printed.

Next the velocity is incremented and the process is repeated for the next airspeed until the desired velocity sweep is complete or the program execution is terminated for some other reason.

An added feature of the program is the circling performance option. This option simulates the degraded performance of turning flight by trimming at a higher gross weight. This weight is obtained by multiplying the actual weight by the load factor required for the given turn radius and velocity. Further realism is furnished by including the effects of spanwise variations of velocity and Reynolds number which are of particular significance in tight radius turns.

Using the increased weight and other corrections, circling performance is calculated in the same fashion as for wings-level flight. The inclusion of a thermal model allows calculation of rate of climb performance in thermal updrafts of various diameters and strengths. The thermal model used is one suggested by Marsen (Ref. 9), and is described by the equation

$$V_{th} = V_o \cos \frac{\pi R}{D_o} \qquad (3)$$

where:

V_{th} = updraft velocity at radius R

V_o = updraft velocity at R=0

R = turn radius

D_o = thermal diameter.

The computed rate of sink calculated for the trim then may be subtracted from the updraft velocity to yield an achieved rate of climb in a given thermal.

Evaluating the Performance Program

As the program "SAILPER" was developed, checks were made to insure that each subroutine was free from errors in logic. When possible, this consisted of running check cases for which the answer was already known or could be easily obtained. Check cases were readily available for the lifting-line method by Sivells and Neely because the documentation contains tables of all the multipliers and an example for one angle of attack which is worked out in considerable detail. The program's answers agreed with this example. A further evaluation was made by comparing wing data calculated by the lifting-line subroutine to experimental wing data obtained from NACA wind tunnel tests and presented by Sivells and Neely. The wing chosen for this check case was of tapered planform with a taper ratio of 2.5, an aspect ratio of 10.05, and a twist of -3.5 degrees. The root airfoil section was a NACA 4420 which varied linearly to a NACA 4412 at the tip. Fig. 2 shows the comparison of calculated and experimental data.

Despite some difficulties in modeling the experimental wing because of the varying airfoil section from root to tip, and problems with airfoil data accuracy, the calculated wing data generally showed very good agreement with experimental data. It was particularly encouraging to note that C_D vs C_L, which is most important to performance, was very close even for this relatively low (for sailplanes) aspect ratio. Agreement between calculated and experimental data would be expected to improve, as is characteristic of a lifting-line method, for the higher aspect ratios that are typical of sailplanes.

When the accuracy of the wing analysis had been established, the final check was to compare calculated data from the total performance program with performance data measured by flight testing. Some "fine-tuning" of the mathematical model was expected to result from this comparison.

The continuing flight test program conducted by Paul Bikle (Refs. 10, 11, 12 and 13) under the auspices of the Flight Test Committee of the Soaring Society of America provided good data for almost twenty different sailplane types, and appeared to offer several possibilities for comparison with calculated performance. Upon closer examination, however, some difficulties became apparent. Several aircraft had to be eliminated because the airfoil section used varied from root to tip, a variation the program was not able to model. Others had to be ruled out because the airfoil used was either unknown or insufficient section data was available. Most of the remainder were unsatisfactory because there was simply not enough published data on the geometry to permit a good configuration definition.

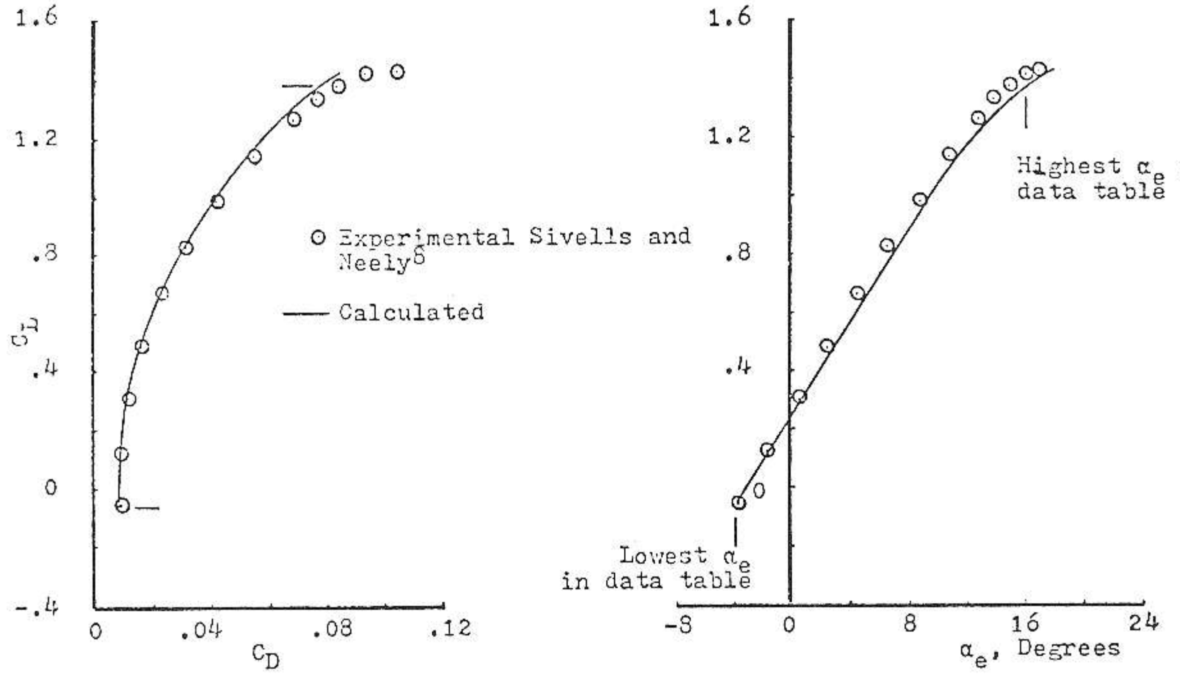

Figure 2. Comparison of non-linear lifting-line technique with experimental data.

One sailplane, the T-6 owned by Bikle himself, not only had been more extensively tested, but could be modeled because there was adequate configuration and airfoil data available.

The T-6 is a modified, longer-winged version of the all-metal HP-14T designed and sold in kit form by Richard Schreder. The airfoil is a Wortmann FX61-163 which is modified in the aft 20 to 30 percent by filling in the cusp on the lower surface. This modification produces an effective decrease in camber which alters lift and drag for a given angle of attack. Tests done by Bikle indicated that the effect on lift was to shift the angle of attack for zero lift about 2 degrees in the positive direction and decrease maximum lift about 15 percent. Additionally, section profile drag measured with a traversing probe was equivalent to that of the unmodified section with about 4 degrees of upward flap deflection. These corrections were applied to section data for the FX61-163 airfoil presented by Althaus (Ref. 14). Moment coefficient data was also modified as appropriate for a 4 degree upward flap deflection.

Aerodynamic data for the fuselage was estimated in the cases of drag and pitching moment. Lift of the fuselage was neglected because it was considered to have a rather small effect on performance or trim, as indicated by Perkins and Hage (Ref. 15), and no really good data were available.

Drag data were estimated based on tests of several typical sailplane fuselage shapes reported by Althaus (Ref. 8). These fuselages were bodies of revolution and although some differences were apparent, shape no. 1 was considered to be representative of the T-6 configuration.

Moment data were determined for the shape no. 1 fuselage based on Multhopp's method in the USAF DATCOM (Ref. 16).

Initial comparison runs showed the calculated performance to be slightly conservative, although the shape of the rate of sink polar, and the stall characteristics were quite similar. Because of the relatively greater uncertainty about the fuselage drag data, attention was focused there in attempting to correlate with the flight test data. The first adjustment was simply to change the wing incidence of the math model from .5 degrees to 2 degrees. This helped somewhat in moving the calculated curve closer to flight test data. This incidence change was justified by noting on the profile view of the T-6 that the forward fuselage was drooped and the angle measured between the wing chord and a reference line drawn from the nose of the fuselage to the tail was approximately 2 degrees rather than .5 degrees as quoted in the specifications. The final adjustment made to the data was to reduce the interference drag contribution of the fuselage, particularly at high angles of attack. This seemed appropriate because flow visualization pictures by Bikle show somewhat cleaner flow at the wing-fuselage junction of the T-6 than was observed on the wind tunnel models.

Fig. 3 presents the final comparison between the performance calculated by "SAILPER" for the T-6 and that measured in flight test

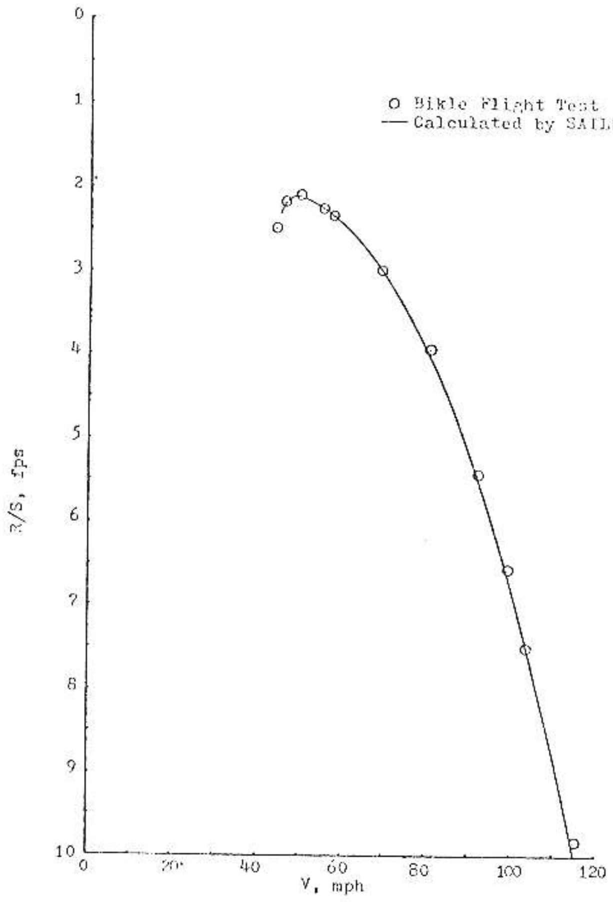

Figure 3. Comparison of calculated and flight test values.

by Bikle. The agreement was nearly perfect after the rational adjustments mentioned above were made. It should be noted that even before the fuselage drag values were adjusted, the maximum deviation was on the order of 5 percent.

SUMMARY

The specialized performance program for sailplanes, called "SAILPER," was developed based on an existing non-linear lifting-line method for wing analysis. This iterative solution method allowed 3-D wing characteristics to be calculated for the entire useful range of angles of attack using tables of airfoil section data which included Reynolds number effects. Fuselage aerodynamic data were represented by equations for lift and pitching moment variations with angle of attack while drag variations were modeled by a data table. The horizontal tail drag was expressed as a parabolic variation with the tail lift required to trim the aircraft. The vertical tail drag coefficient was not varied.

A trim condition was achieved by iterating on aircraft attitude until the equations for vertical force and pitching moment equilibrium were satisfied for the velocity, weight, and c.g. specified. Performance in terms of rate of sink and L/D ratio was then computed.

A circling option, which allowed performance calculation in turns of any radius, was also developed. The circling performance model iterated to a trim condition at the load factor necessary for equilibrium flight at the desired turn radius and velocity. Spanwise variations of velocity, dynamic pressure, and Reynolds number in the turn, as well as an asymmetric spanwise loading of the wing were included in the math model.

The validity of the lifting-line technique as well as the entire performance program was checked using experimental data. Correlation with experimental data was highly satisfactory.

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